Geometric Scattering in Tele-manipulation of Port Controlled Hamiltonian Systems

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Abstract In this paper we study the interconnection of two port controlled Hamiltonian systems through a transmission line with delay. The contributions of the paper are firstly a geometrical, multi-dimensional, power consistent exposition of tele-manipulation of Intrinsically Passive Controlled (IPC) physical systems (Stramigioli 1998, Stramigioli et al. 1999), with a clarification on impedance matching, and secondly a system theoretic condition for the adaptation of a general port controlled Hamiltonian system with dissipation (PCHD system) to a transmission line. To the knowledge of the authors, the latter result in particular has never appeared in such a general form. Experimental results on an Internet implementation are also presented.

1 Introduction

A lot of work has been done in the field of tele-manipulation. Some of the contributions specifically address the problem of time delays due to the actual transmission through a line of non-neglectable length. The problem was firstly addressed in (Anderson and Spong 1989) for a one dimensional case, and then extended in (Niemeyer and Slotine 1991), where important considerations on the line causality and extensions with adaptation techniques are treated.

In this work we present a geometrical multi-dimensional case, which uses digital transmission of data in order to create a perfectly bilateral tele-manipulation system on a transmission line with varying, non-neglectable delays: the Internet.

2 Background

2.1 Generalized Hamiltonian systems

Almost[1] any lumped parameter physical system with independent states can be represented by the following generalized Hamiltonian equations (Dalsmo and van der Schaft 1999, van der Schaft 1999) with dissipation:

$$
\begin{cases}
\dot{x} = (J(x) - R(x))\dfrac{\partial H(x)}{\partial x} + G(x)u \\
y = G^T(x)\dfrac{\partial H(x)}{\partial x}
\end{cases}
\tag{1}
$$

where x is the state vector, H the Hamiltonian energy function, $J(x)$ the skew-symmetric Poisson tensor, $R(x)$ the symmetric, positive semi-definite, dissipation tensor, u the input and y the output.

2.2 Geometric scattering

Scattering variables are well known in network theory. At the knowledge of the authors, the first works which present scattering variables from a geometrical point of view are (Maschke and van der Schaft 1999, van der Schaft 1999). This way of defining scattering is conceptually important because it allows to define scattering variables of vector spaces which do NOT have a canonically defined inner product like $se(3)$ (Lončarić 1985, Duffy 1990). This allows to implement a tele-manipulating system from an intrinsically geometric point of view.

The main idea is as follows. Given any vector space V,

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[1]This point will be discussed later.

we can consider the vector space:

$$D := V \times V^*$$

On D there exist a canonical, symmetric, two co-variant tensor called $+pairing$. This symmetric, non-degenerate 2-form, is defined by the bilinear operation:

$$\langle (f_1, e_1), (f_2, e_2) \rangle_+ := \langle e_1, f_2 \rangle + \langle e_2, f_1 \rangle$$

where $(f_i, e_i) \in D$ and $\langle e_i, f_j \rangle$ denotes the intrinsic dual pairing. Using this tensor it is also possible to give a geometric definition of a Dirac structure which is a generalization of Poisson structures (Maschke and van der Schaft 1999, van der Schaft 1999, Maschke and van der Schaft 2000). Once dual bases for V and V^* are used, a matrix representation of this $(0,2)$ type tensor becomes:

$$T_{ij} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \qquad (2)$$

It is now possible to consider the singular values[2] of the tensor T_{ij} which turn out to be $+1$ and -1 (both with multiplicity equal to the dimension of V). Furthermore, due to the fact that the 2-form is not degenerate, there are two complementary subspaces associated to the singular values, namely:

$$\mathcal{S}^+ := \{(v, v^*) \; ; \; \langle (v, v^*), (v, v^*) \rangle_+ \geq 0\} \qquad (3)$$
$$\mathcal{S}^- := \{(v, v^*) \; ; \; \langle (v, v^*), (v, v^*) \rangle_+ \leq 0\}, \qquad (4)$$

having the property that

$$D = \mathcal{S}^+ \oplus \mathcal{S}^-,$$

which implies that there is a unique way to express a power pair $(f, e) \in D$ as the sum of two elements $s^+ \in \mathcal{S}^+$ and $s^- \in \mathcal{S}^-$. Furthermore, by restricting $\langle, \rangle_+$ to $\mathcal{S}^+$ we obtain an inner product on $\mathcal{S}^+$, and by restricting $-\langle, \rangle_+$ to $\mathcal{S}^-$ we obtain an inner product on $\mathcal{S}^-$. After choosing any basis for V and the corresponding unique dual basis for V^*, it is then possible (see e.g. (van der Schaft 1999) to choose induced *orthonormal* bases for $\mathcal{S}^+$ and $\mathcal{S}^-$. The decomposition on the two subspaces can be then expressed as[3]:

$$\begin{cases} s^+ = \dfrac{1}{\sqrt{2}}(e + f) \\[2mm] s^- = \dfrac{1}{\sqrt{2}}(e - f) \end{cases} \qquad (5)$$

where, with an abuse of notation, s^+ indicates a numerical representation of the projection of the power pair (e, f) on the subspace $\mathcal{S}^+$ using the induced orthonormal basis and similarly for the negative subspace. It

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[2]Note that we cannot intrinsically use the eigenvalues of a two covariant tensor.

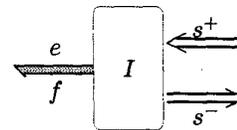[3]Note that the sum must be interpreted as a sum of real numerical vectors as expressed in the dual bases explained

Figure 1: *Scattering transformation.*

is then possible (see e.g. (Maschke and van der Schaft 1999, van der Schaft 1999) to see that

$$\langle e, f \rangle = \frac{1}{2} \|s^+\|^2 - \frac{1}{2} \|s^-\|^2$$

where the norms for $\mathcal{S}^+$ and $\mathcal{S}^-$ are induced by the above canonically defined inner products. This result is very important because it shows that we can algebraically write the power flow as the sum of a positive and negative power depending only on the two scattering variables. This can be interpreted as power going in opposite directions as shown in Fig. 1 where it is shown in bond graph notation that the power bond has indeed the same direction as the variables s^+ due to the accordance of sign. Intuitively s^+ can be thought of as a wave transporting power in the direction of the bond and s^- in the opposite one.

3 Causality and sign issues

It has been shown in (Anderson and Spong 1989) that in order to preserve passivity with a transmission line connecting two systems, the power port connected to the transmission line can be "coded and decoded" in scattering variables. The coded signal s^- can be sent on the line and it will be used by the other side as the incoming signal s^+. The total energy stored on the line is therefore the integral of the traveling signal. Since the variable s^+ is always an input for the two systems attached to the line, we have two causal possibilities:

- Computing e and s^- as a function of f and the incoming wave s^+

- Computing f and s^- as a function of e and the incoming wave variable s^+.

It has been shown in (Niemeyer and Slotine 1991) that there are multiple reasons for choosing the last of the two options. Furthermore, if we want to have a perfectly symmetric system, the causalities at both sides should be the same. A first possibility, from a purely causal point of view, would be to let the line behave as a gyrative action[4] in such a way that for the line length and delay tending to zero, the effort supplied by one side would become the input flow of the other and vice versa. Unfortunately such a system cannot work because in steady state, when the master and slave do

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[4]In a gyrative action, the effort on one side would be a function of the flow on the other side and the other way around.

not move, the velocities should be zero ($f = 0$), but at the same time, we want that a reflection of a force different from zero could take place if necessary (that is, $e \neq 0$). Since with a symplectic connection the two variables f and e are equal, this is not possible. This implies that the systems at both sides should have an *impedance* causality, and that the line in the limit of its length tending to zero should *not* behave as a gyrative action.

From this we conclude we have to choose exactly the same scattering mapping on both sides, and connect the departing wave of one side to the incoming wave of the other side. This has an important consequence: if the line length and its delay are tending to zero, then we get a causal inconsistency since 'the line' should supply the same power variable at both sides, namely the flow f and this value should come from somewhere. By writing down the equations it would be possible to see that this would correspond to an algebraic loop with no delays and implies that the energy storage of a finite length line "fixes" the causal problem exactly as a mass would do to connect two physical systems through springs.

4 Line Impedance adaptation

The scattering mapping relation reported in Sect. 2.2 is somehow canonical. This means that the system attached to the power port (e, f) of the transmission line reported in Fig. 1 feels an identity impedance. To see this we can proceed as follows.

Since T_{ij} of Eq. (2) is a quadratic form, we cannot directly consider the eigenvalues of its representing matrix since this would not be a coordinate free operation as it is well known in tensor calculus. In order to do this, we first need to "higher" one of its indexes(Dubrovin *et al.* 1992). For this purpose, we consider a characterizing impedance z_{ki} which is a 2-covariant tensor in V. It is then possible to define the following tensor of type $(2, 0)$ in D:

$$Y^{ki} := \begin{pmatrix} z_{ki}^{-1} & 0 \\ 0 & z_{ki} \end{pmatrix}$$

It is now possible to study the eigenspaces of the $(1, 1)$ tensor

$$S_j^k := Y^{ki}T_{ij} = \begin{pmatrix} 0 & Z \\ Z^{-1} & 0 \end{pmatrix}$$

where Z is a numerical representation of the tensor z_{ki}. It is possible to see that, if we choose an identity impedance $Z = I$, we get two eigenvalues equal to the singular values $+1$ and -1, with eigenspaces $\mathcal{S}^+$ and $\mathcal{S}^-$. Furthermore, as an extra check, it can also be easily seen by supposing no power coming from the line ($s^+ = 0$) and looking at another causal form of Eq. (5):
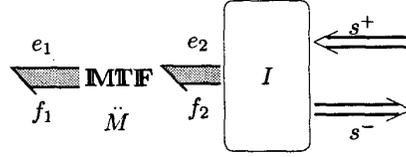
$$e = \sqrt{2}s^+ - f$$

Figure 2: *Scaling of impedance.*

How can we then model, in a power consistent way, an impedance line different from the identity ? This can be done by using what is called in bond graphs a *transformer*. The resulting scheme is given in Fig. 2. The equation characterizing a transformer with matrix transformation M are

$$\begin{aligned} f_1 &= Mf_2 \\ e_2 &= M^T e_1 \end{aligned} \tag{6}$$

where in our case M is a square, in general time-varying, non-singular matrix. The impedance seen at the (e_1, f_1) port is the matrix Z_1 such that $e_1 = Z_1 f_1$ and substituting the transformer's equations:

$$e_1 = M^{-T}e_2 = M^{-T}f_2 = (MM^T)^{-1}f_1$$

which implies that $Z_1 = (MM^T)^{-1}$. This is a trivial, well known result in network theory.

Z_1 is a positive definite, symmetric, 2-contravariant tensor, and therefore:

$$P = e_1^T f_1 = f_1^T Z_1^T f_1 \geq 0 \tag{7}$$

A question arises: is it possible to find an **MTF** of Fig. 2 such that the impedance seen from (e_1, f_1) can get any symmetric desired value Z?
The answer is given by a well known result, namely that given any symmetric, positive semidefinite matrix Z there exists always a symmetric matrix N such that

$$Z = NN^T = N^2$$

With this result we can state that *all* meaningful impedances (symmetric and positive definite) Z can be generated by a proper choice of a transformer $M = N^{-1}$.
This implies that since a generic scattering transformation can be expressed by

$$\begin{cases} s^+ = \dfrac{1}{\sqrt{2}}(e_2 + f_2) \\ s^- = \dfrac{1}{\sqrt{2}}(e_2 - f_2) \end{cases}$$

using Eq. (6)

$$\begin{cases} s^+ = \dfrac{1}{\sqrt{2}}(N^{-1}e_1 + Nf_1) \\ s^- = \dfrac{1}{\sqrt{2}}(N^{-1}e_1 - Nf_1) \end{cases}$$

Eventually, we obtain the scattering transformation for a generic, multidimensional impedance Z:

$$\begin{cases} s^+ = \dfrac{1}{\sqrt{2}}N^{-1}(e_1 + Zf_1) \\ s^- = \dfrac{1}{\sqrt{2}}N^{-1}(e_1 - Zf_1) \end{cases} \qquad (8)$$

As we shall see hereafter, Z is a fundamental parameter for the line, which characterizes the wave variables s^+, s^-, and directly effects the system behavior.

It is important to know that, in a real analog transmission line like a coaxial cable or a twisted pair, the impedance is obviously a physical characteristic of the line which we cannot influence. On the other hand, in a digital transmission line like the one considered in this work, only data are sent and the scattering mapping of Fig. 2 corresponds to an algorithmic implementation which codes and decodes the sent and received data. Future work will formally analyse the correctness of this analogy.

5 Impedance Matching

Impedance matching is a well-known problem in transmission lines. The energy received from the line has to be absorbed by master and slave systems. Once the impedance Z seen at the power port of Fig. 3 is chosen (model of the line), a system with the "same impedance" needs to be connected at the end of the line to avoid waves reflections. This guarantees continuity of impedance with respect to the line.

A general system theoretic condition for matching of a general physical system connected to a line as in Fig. 3 can now be stated as follows:

Principle 1 *The system seen at the scattering side of the transformation of Fig. 3 and having s^+ as input and s^- as output has to be of relative degree ≥ 1 (that is, the system should have no direct feedthrough).*

This implies that there should not be an algebraic relation between the waves s^+ and s^-, which is exactly equivalent to the idea of undiscriminated reflection of power. In intuitive terms, the power should be first somehow "processed" by the master (resp. slave) before some information is sent back to the slave (resp. master).

Now, we want to investigate what conditions Principle 1 imposes on a generic PCHD system, as treated in Sect. 2.1, connected at the end of the line as in Fig. 3. Since we consider port controlled generalized Hamiltonian systems (both master and slave sides), we have:

$$\begin{cases} \dot{x} = (J(x) - R(x))\dfrac{\partial H(x)}{\partial x} + G(x)f \\ e = G^T(x)\dfrac{\partial H(x)}{\partial x} \end{cases} \qquad (9)$$
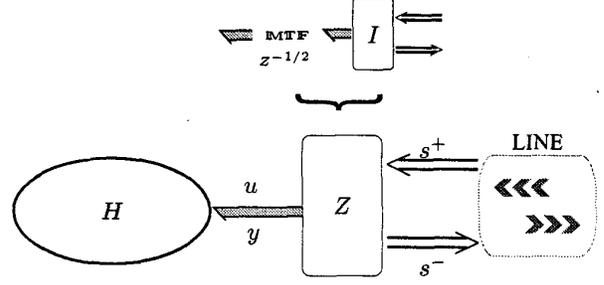

Figure 3: *Power interconnection with an Hamiltonian system.*

From the scattering transformation of Eq. (8), we can obtain the port variables as function of the wave variables:

$$s^+ + s^- = \sqrt{2}N^{-1}e \;\; \Rightarrow \;\; e = \frac{1}{\sqrt{2}}N(s^+ + s^-)$$

$$s^+ - s^- = \sqrt{2}Nf \;\; \Rightarrow \;\; f = \frac{1}{\sqrt{2}}N^{-1}(s^+ - s^-)$$

and thus the Hamiltonian system of Eq. (9) is transformed to:

$$\begin{cases} \dot{x} = (J(x) - R(x))\dfrac{\partial H(x)}{\partial x} + \\ \qquad \dfrac{1}{\sqrt{2}}G(x)N^{-1}(s^+ - s^-) \\ e = G^T(x)\dfrac{\partial H(x)}{\partial x} = \dfrac{1}{\sqrt{2}}N(s^+ + s^-) \end{cases}$$

The new system having as input s^+ and as output s^- is thus given as:

$$\begin{cases} \dot{x} = (J(x) - R(x)- \\ \quad G(x)N^{-1}N^{-1}G^T(x))\frac{\partial H(x)}{\partial x} + \sqrt{2}N^{-1}G(x)s^+ \\ s^- = \sqrt{2}N^{-1}G^T(x)\frac{\partial H(x)}{\partial x} - s^+ \end{cases}$$

Hence we conclude that the input s^+ is directly fed through to the output s^-. This implies that any power arriving from the line is sent back independently of the state of the system connected to the line. Thus the Hamiltonian system of Eq. (9) does not satisfy Principle 1, and is not general enough for impedance matching.

Hence, in order to meet Principle 1, we have to enlarge the class of PCHD systems. We do this by considering PCHD systems of the extended form

$$\begin{cases} \dot{x} = (J(x) - R(x))\dfrac{\partial H(x)}{\partial x} + G(x)f \\ e = G^T(x)\dfrac{\partial H(x)}{\partial x} + B(x)f, \end{cases} \qquad (10)$$

with $B(x) \geq 0$ a newly added dissipation matrix.
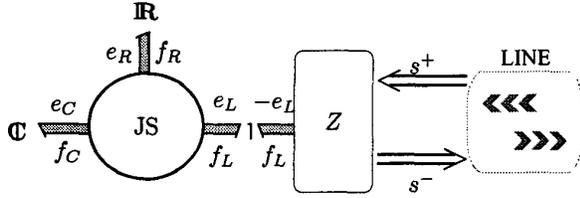In this case we obtain, using Eq. (8) and Eq. (10), the

Figure 4: The interconnection description

new output equation:

$$s^- = \left(\frac{B(x)}{\sqrt{2}}N^{-1} + \frac{1}{\sqrt{2}}N\right)^{-1} G^T(x)\frac{\partial H(x)}{\partial x} + Fs^+ \tag{11}$$

where

$$F = \left(B(x)N^{-1} + N\right)^{-1}\left(B(x)N^{-1} - N\right)$$

which implies, using Principle 1, that to have impedance matching we must have:

$$B = NN = Z$$

since this implies $F = 0$.

Thus, a system of the extended PCHD form Eq. (10), for which $B(x)$ is equal to the impedance Z of the line to which it is connected, guarantees the matching condition expressed in Principle 1, and eliminates any discriminate reflection of power.

Using the previous setting, it is even possible to give a measure for the level of matching if this is not perfect. This can be easily done considering the induced norm of F. From Eq. (11) it is possible to see that F is a mapping of the following form:

$$F : \mathcal{S}^+ \to \mathcal{S}^- \ ; \ s^+ \mapsto Fs^+$$

and since $\mathcal{S}^+$ and $\mathcal{S}^-$ are normed spaces, we can define the following induced norm for F:

$$\|F\| := \sup_{s \neq 0}\frac{\|Fs\|_-}{\|s\|_+}$$

If this norm is zero, perfect matching is obtained, if this norm is 1 perfect reflection takes place.

5.1 An interconnection approach

The previous conclusion can be obtained similarly using the framework of Dirac structures. For simplicity, we consider a network structure with reference to Fig. 4 relating three ports: the power port connected to the line (f_L, e_L), the power port connected to a dissipating component (f_R, e_R) and a power port connected to a storage element (f_C, e_C). The network structure can

be expressed by a linear mapping of the form:

$$\begin{pmatrix} e_L \\ e_C \\ e_R \end{pmatrix} = \begin{pmatrix} D_L & G_1 & G_2 \\ -G_1^T & D_C & G_3 \\ -G_2^T & -G_3^T & D_R \end{pmatrix} \begin{pmatrix} f_L \\ f_C \\ f_R \end{pmatrix}$$

where D_L, D_C, D_R are skew-symmetric. A dissipating element of the system has characteristic equations of the form $e_R = Rf_R$ with R symmetric and positive semi-definite. This implies that

$$f_R = (D_R - R)^{-1}G_2^T f_L + (D_R - R)^{-1}G_3^T f_C$$

and therefore

$$\begin{pmatrix} e_L \\ e_C \end{pmatrix} = \begin{pmatrix} B & A \\ C & D \end{pmatrix}\begin{pmatrix} f_L \\ f_C \end{pmatrix}$$

where:

$$B := D_L + G_2(D_R - R)^{-1}G_2^T \tag{12}$$

$$A := G_1 + G_2(D_R - R)^{-1}G_3^T \tag{13}$$

$$C := -G_1^T + G_3(D_R - R)^{-1}G_2^T \tag{14}$$

$$D := D_C + G_3(D_r - R)^{-1}G_3^T \tag{15}$$

Applying the scattering transformation to the power port $(f_L, -e_L)$ it is possible to obtain:

$$s^- = -(N + N^{-1}B)(N - N^{-1}B)^{-1}s^+ + Kf_c$$

where

$$K = -\frac{1}{\sqrt{2}}(N + N^{-1}B)(N - N^{-1}B)^{-1}N^{-1}A$$
$$+ N^{-1}A.$$

This implies for Principle 1 that for adaptation we need to have $(N + N^{-1}B) = 0$ and therefore $B = -Z$ which implies:

$$D_L + G_2(D_R - R)^{-1}G_2^T = -Z$$

with Z symmetric. This implies that necessarily $D_L = 0$ and furthermore, if we suppose G_2 to be square and non-singular, that also $D_R = 0$, implying that:

$$R^{-1} = G_2^{-1}ZG_2^{-T}.$$

From the previous analysis, we can conclude that the adaptation is independent of the state of the system and only depending on the system interconnection and its dissipative term.

6 Spatial tele-manipulation

The presented theory can be used to passively implement spatial tele-manipulation. With this is meant that the developed theory is well posed in a coordinate free setting and therefore it is possible to choose, for example

$$V = se(3) \times \ldots \times se(3).$$

In this case the transmitted power variables will be a set of twists and their dual wrenches. To keep decoupled twists during transmission, the chosen line impedance should be of the form:

$$Z = \begin{pmatrix} Z_1 & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & Z_n \end{pmatrix}.$$

This can be used in complex tele-manipulation systems using Intrinsically Passive Controlled techniques like the ones presented in (Stramigioli et al. 1999) in which variations of geometric spring's length is controlled by a twist, an element in $se(3)$. The importance of the presented theory becomes relevant in these kind of applications where, the vector spaces used do not have any intrinsic inner product.
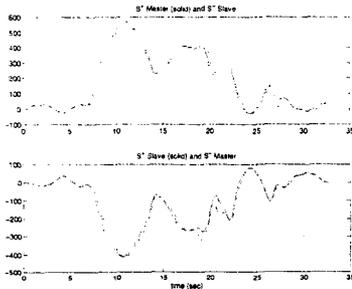

Figure 5: *Scattering variables*

7 Experiments

Real experiments have been implemented to verify the theory. These experiments used an Internet interconnection between the University of Bologna and the Delft University of Technology in order to create a real long, time varying delay. The experimental results indeed confirmed the theory. The only plot which is shown here is the one reported in Fig. 5 which shows the power waves s^+ and s^- and the virtual power transmitted through the network. A lot of other results have been obtained, but are not reported here for matter of space.

8 Conclusions

In this paper a general setting for tele-manipulation of Port Control Hamiltonian systems has been presented. A new system theoretic condition has been introduced which can be used to test if proper matching is taking place. A possible measure of matching has been also introduced.

It has been shown that the standard form of explicit port controlled Hamiltonian systems is not general enough to obtain matching and it must be extended by a feed-throw term. This can be shown more generally using directly a network structure as shown in Sect. 5.1.

The presented theory is important for the implementation of geometrical tele-manipulation where the vector space used $se(3)$ does not have an internal product.

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