

# Analysis of Equilibria for Generalized Market Sharing Games

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**Abstract.** We analyze the quality of several equilibria for generalized market sharing games. Generalized market sharing games model  $n$  selfish players selecting subsets of a finite set of items, where the payoff of an item is divided equally among all players choosing that item. The subsets that are feasible for any given player are allowed to be an arbitrary subset of the set of items. Market sharing games as studied in the literature are a special case of this, where the feasible subsets are defined by budget constraints. Market sharing games were studied by Goemans et al. [1]. They showed that the price of anarchy is at most 2 by showing that market sharing games fall into the class of valid utility games as defined by Vetta [2]. A closely related yet different model are set packing type of games as studied by de Jong and Uetz [3].

We show that generalized market sharing games are valid utility games as well. More interestingly, we sharpen the bound on the price of anarchy and show that it is exactly equal to  $2 - \frac{1}{n}$ . Our proof boils down to a smoothness-type of argument, which implies that our price of anarchy result extends also beyond Nash equilibria. We also show that the lower bounds are even attained for special cases, namely for symmetric and/or singleton generalized market sharing games.

Furthermore, we study the sequential version of the game where players choose their strategies in a full information setting, and analyze subgame perfect equilibria for these games. In order to be able to analyze subgame perfect equilibria, we introduce a class of games that we call *shared misery games*. For this class of games we set up a framework which can be used to provide upper bounds on the sequential price of anarchy. To exemplify how this approach works, we can for instance prove that any subgame perfect equilibrium in a symmetric singleton congestion game is also a Nash equilibrium in the simultaneous variant of the game. When applied to symmetric singleton generalized market sharing games, we thereby obtain the result that the sequential price of anarchy is also  $2 - \frac{1}{n}$ . For non-singleton or non-symmetric generalizations, the quality of subgame perfect equilibria may be worse; our work here is ongoing.

Finally, we also consider an egalitarian (max-min) social choice function, and observe that the sequential price of anarchy may differ significantly: For the symmetric singleton variant it is equal to 1, while in general it is equal to  $n$ .

## References

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