Landauer's erasure principle in a squeezed thermal memory

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Landauer's erasure principle states that the irreversible erasure of a one-bit memory, embedded in a thermal environment, is accompanied with a work input of at least $k_\mathrm{B}T\ln 2$. Fundamental to that principle is the assumption that the physical states representing the two possible logical states are close to thermal equilibrium. Here, we propose and theoretically analyze a minimalist mechanical model of a one-bit memory operating with squeezed thermal states. It is shown that the Landauer energy bound is exponentially lowered with increasing squeezing factor. Squeezed thermal states, which may naturally arise in digital electronic circuits operating in a pulse-driven fashion, thus can be exploited to reduce the fundamental energy costs of an erasure operation.

Energy dissipation is one of the key design factors in digital electronics today [1–3]. Smaller transistors operating at lower-voltages are a natural design choice that may reduce power consumption of central processing units. In 1961, Rolf Landauer argued that there exists a limit to which the power consumption of certain logical operations can be reduced. Landauer's principle states that the erasure (or reset) of one bit of classical information is necessarily associated with an entropy increase of at least $k_\mathrm{B}\ln 2$ and an energy input of at least $k_\mathrm{B}T\ln 2$ [4–11]. For the present generation of silicon-based digital circuits, the energy dissipation per logic operation is about a factor of 1000 larger than the Landauer limit, but is predicted to attain it within the next decades [1–3]. Thus, improvements in our understanding of energy dissipation in information-processing devices are of both scientific interest and technological relevance. Due to the ongoing miniaturization of digital electronic circuits non-equilibrium and quantum effects must be taken into account [12–16]. In this work, it is theoretically demonstrated that memory devices embedded in a squeezed thermal environment are unbounded by the Landauer limit. In these environments, thermal fluctuations show periodic correlations in time which can be exploited to reduce the minimum energy requirement for an erasure operation below the standard Landauer limit. This situation may naturally occur in digital electronic circuits operating in a pulse-driven fashion and, in future, could be exploited to build more energy-efficient electronic devices.

Squeezed thermal states are the classical analog of squeezed coherent states in quantum mechanics. Both are characterized by an asymmetric phase space density as opposed to the rotationally invariant phase space densities of coherent, thermal or vacuum states. A mechanical oscillator may be prepared in a squeezed thermal state [17–19] by a periodic modulation of the spring constant [20]. This leads to a state with reduced thermal fluctuations in one quadrature (e.g. momentum) and enhanced thermal fluctuations in the orthogonal quadrature (e.g. position) compared to the expected level of fluctuations at the given temperature. In the context of heat engines, squeezed thermal reservoirs have been proposed as a resource for work generation unbounded by the standard Carnot limit [21–27]. Due to the non-equilibrium nature of these reservoirs, this result does not violate the second law of thermodynamics. In recent work [28], we have demonstrated a physical realization of such an engine, in which the working medium consists of a vibrating nano-beam that is driven by squeezed electronic noise to perform work beyond the Carnot limit. We have furthermore demonstrated that a phase-selective thermal coupling allows to extract work from a single squeezed thermal reservoir, which is not possible with a standard thermal reservoir.

In this work, we propose and theoretically analyze a minimalist mechanical model of a one-bit memory subject to squeezed thermal noise. This memory consists of a single particle that is trapped in a harmonic potential. The trap can be spatially divided into two halfs by a partition in the trap center. If the particle resides on the left-hand side of the trap, the memory is regarded as being in the logical state '0'; if it is located on the right hand side, the memory is in the '1' state. We further assume that the particle is coupled to a squeezed thermal reservoir, which can be modeled by introducing a stochastic force $f = f(t)$ to its equation of motion, as described by the Langevin equation $m\ddot{x} = F(x) - c\dot{x} + f$. Here m denotes the mass of the oscillator, $F(x) = -m\omega^2 x$ describes the restoring force, and c denotes the viscous damping coefficient. The stochastic force $f(t)$ is synthesized from two independent noise signals $\xi_{1,2}(t) \in$ [-1;1] that are mixed with sine and cosine component of a phase reference at the oscillator eigenfrequency $\nu = \omega/2\pi$ [28]:

$$f(t) = a_0\left[\mathrm{e}^{+r}\xi_1(t)\cos(\omega t) + \mathrm{e}^{-r}\xi_2(t)\sin(\omega t)\right] \quad (1)$$

The squeezed thermal reservoir modeled by $f(t)$ is characterized by an overall amplitude a_0 and a squeezing parameter r that tunes the imbalance between the two orthogonal quadratures. The latter introduces non-vanishing temporal correlations in the stochastic force at the frequency of the drive. Corresponding phase-space probability distributions are presented in Fig. 1 and demonstrate a reduction in thermal fluctuations in the squeezed quadrature and an enhancement of fluctu-

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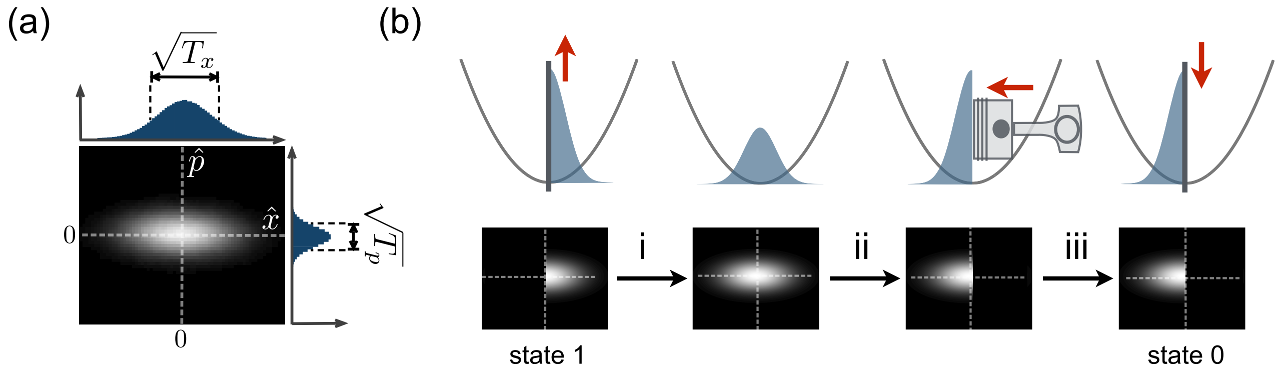

Figure 1. (a) Phase space diagrams of a squeezed thermal state. With increasing squeezing factor r, thermal fluctuations in one quadrature are reduced at the cost of enhanced fluctuations in the orthogonal quadrature. The variances of the line integrated distributions (shown in blue) correspond to two temperatures T_x, T_p that govern the thermal fluctuations of the system. All numerical results in this work have been obtained by numerically integrating the Langevin equation of a harmonically confined particle subject to the stochastic force given in eq. (1) using the Runge-Kutta method (fourth order) with constant time steps. (b) Scheme to erase one bit of classical information in a squeezed thermal memory: (i) removal of partition and free expansion, (ii) isothermal compression at constant squeezing ($T=$ const, $r=$ const) (iii) insertion of partition. This inititalizes the bit in state 0 regardless of the initial conditions.

ations in the anti-squeezed quadrature. The quantities $\hat{x}$ and $\hat{p}$, corresponding to the two axes of the phase-space plots, may be regarded as two orthogonal quadratures co-rotating with the driving force (rotating frame). Another valid interpretation is to regard $\hat{x} = x\sqrt{m\omega/\hbar}$ and $\hat{p} = p/\sqrt{\hbar\omega m}$ as dimensionless instances of the actual physical position x and momentum p (laboratory frame). In this case, the diagram in Fig. 1a represents a stroboscopic phase-space probability density measured at equidistant points in time t_0, $\nu^{-1}+t_0$, $2\nu^{-1}+t_0$, ..., where t_0 sets the relative timing of the observations with respect to the stochastic force $f(t)$. For the remainder of this work, we restrict our presentation to the laboratory frame. An important consequence of this is that any interaction with the system has to be performed in a stroboscopic fashion. A spatial compression, for example, needs to be divided into a sequence of smaller compression steps that have to be executed with the desired timing t_0.

Squeezed thermal states can be understood in terms of a generalized Gibbs ensemble [28, 29]. The thermal fluctuations of the two orthogonal quadratures $\hat{x}$ and $\hat{p}$ are controlled by two different temperatures T_x and T_p, which take the role of state variables:

$$\rho_{\text{sq}}(\hat{x},\hat{p}) \propto \exp\left(-\frac{\hbar\omega\hat{x}^2}{2k_{\text{B}}T_x} - \frac{\hbar\omega\hat{p}^2}{2k_{\text{B}}T_p}\right) . \quad (2)$$

An effective temperature T of the system may be defined as $T=\sqrt{T_x T_p}$. A consequence of this definition is that an isothermal squeezing operation ($T=$ const) does not increase the entropy of the state [28].

The scheme to erase one bit of classical information in a squeezed thermal memory is shown in Fig. 1b. First, the partition is removed and the single-particle gas expands freely. In the second step, the gas is compressed by a piston keeping temperature T and squeezing r constant. In the last step, the partition is inserted in the center of the trap. This procedure initializes the memory in the state 0 regardless of the initial conditions. The general idea behind this scheme is to use squeezing as a means to reduce the occurrence of large positive momenta at the position of the piston. The latter reduces the pressure and, thus, the work required for the compression step. Note that this scheme relies on the notion of a spatially compressed squeezed thermal state. We will first discuss some subtleties and apparent difficulties associated to the latter.

Figures 2a,b show numerically obtained phase-space plots of a confined single-particle gas subject to squeezed thermal noise in the under-damped, critically damped, and over-damped regime. For purely harmonic confinement (Fig. 2a), the response of the gas to the squeezed noise is largely independent of the damping regime. This is markedly different in the presence of a piston (Fig. 2b). Collisions of the particle with the piston induce phase-shifts in the otherwise purely harmonic motion. In the under-damped regime, these phase-shifts destroy the correlation between particle motion and squeezed noise, which cancels the squeezing phenomenon. In the critically damped and over-damped case, the movement of the particle follows more directly the external force. The collisions with the piston perturb, but do not destroy the squeezing phenomenon. In the over-damped region, an additional effect comes into play, namely, that the particle tends to 'stick' to the piston, which leads to a strong enhancement of the probability density in this region. This effect can also be observed in Fig. 2c, which shows typical examples of the particle motion $x(t)$ in the various damping regimes. In the over-damped case, the particle tends to collide several times with the piston before it is

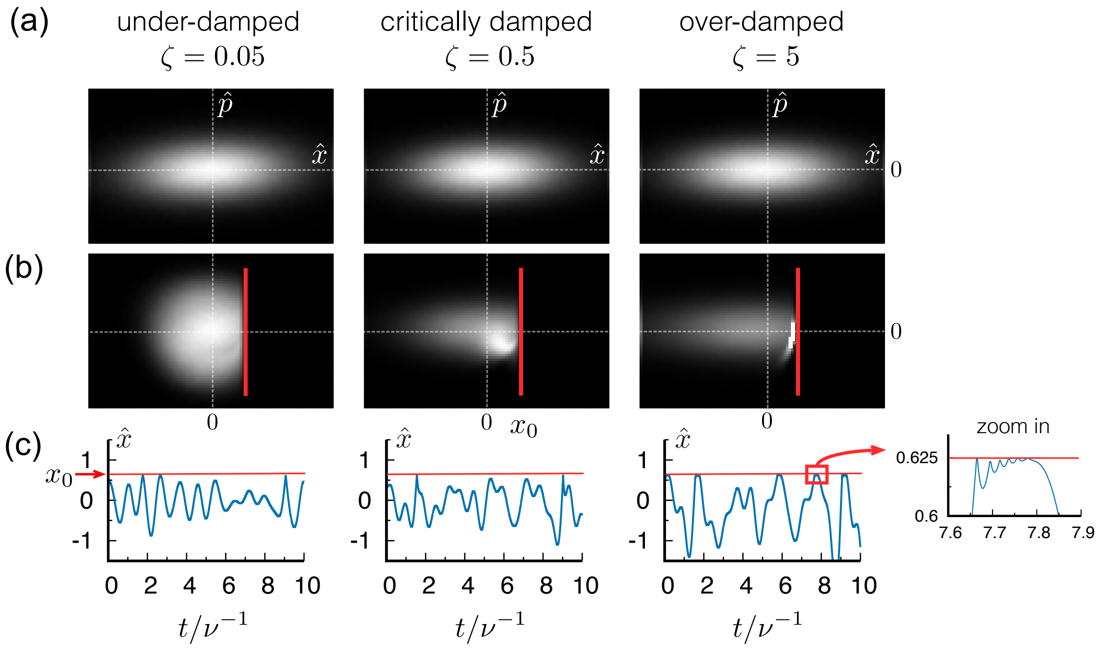

Figure 2. (a) Phase space probability densities of a harmonically confined particle subject to the squeezed thermal noise for three different dynamical regimes: under-damped motion ($\zeta = c/2m\omega = 0.05$), critically damped motion ($\zeta = 0.5$), and over-damped motion ($\zeta = 5$). The shown diagrams represent stroboscopic probability densities measured at equidistant points in time t_0, $\nu^{-1} + t_0$, $2\nu^{-1} + t_0$, ..., where $t_0 \simeq 0.25\,\nu^{-1}$ sets the relative timing of the observations with respect to the stochastic force $f(t)$. (b) The presence of a piston (indicated by the red bar) introduces collision-induced phase jumps to the particle motion, which cancel the squeezing effect in the case of under-damped motion. In the case of over-damped motion, the particle tends to 'stick' to the piston, which leads to a strong enhancement of the probability density in this region. (c) Typical trajectories $x(t)$ of the trapped particle. In the over-damped regime, the particle tends to collide several times with the piston (red bar) before being accelerated in the opposite direction.

finally accelerated in the opposite direction.

We will now focus on the case of critical damping. By recording the (elastic) collision events in our numerical simulations, we can derive the work W that is required to compress the gas to half of its initial volume. In Fig. 3 this work is shown as a function of the parameter t_0, which defines the points in time, namely t_0, $\nu^{-1}+t_0$, $2\nu^{-1}+t_0$, ..., at which the compression steps are executed. At a relative timing around $t_0 \simeq 0.3\,\nu^{-1}$ and $t_0 \simeq 0.8\,\nu^{-1}$, the work W is found to exponentially decrease with the squeezing parameter r (note the logarithmic scale in Fig. 3). Under those conditions, the squeezing effect reduces the occurrence of large positive momenta close to the piston (indicated by the red bar), which causes a reduced pressure exerted on the piston. Our numerical results, thus, give clear evidence that squeezing can be exploited to reduce the required work for the reset of a one-bit memory.

In the remainder of this work, we discuss a simplifying analytical model that captures the key aspects of the described phenomenon. The presence of a piston at position x_0 introduces a cut-off in phase space: $\rho(\hat{x} > x_0, \hat{p}) = 0$. We will consequently model a spatially compressed squeezed thermal state by the phase-space density

$$\rho(\hat{x}, \hat{p}) = Z^{-1}\, \rho_{\rm sq}(\hat{x}, \hat{p})\, \Theta(x_0 - \hat{x}) \;, \tag{3}$$

in which $\Theta(x)$ is the Heaviside step function ($\Theta(x) = 1$ for $x > 0$, $\Theta(x) = 0$ otherwise) and Z is a normalization constant such that $\iint \rho(\hat{x}, \hat{p})\, d\hat{x} d\hat{p} = 1$. We choose to perform the compression step against a purely momentum squeezed state of the gas as depicted in Fig. 2. To this end, we set $T_x = T \exp(2r)$ and $T_p = T \exp(-2r)$ [17, 19]. To derive the work required for an isothermal compression at constant squeezing, we use a common ansatz in the kinetic gas theory relating the pressure exerted on the piston to the average momentum transfer by elastic collisions: $P = \int_0^\infty 2\hbar\omega \hat{p}^2 \rho(x_0, p)\, d\hat{p}$. Using eq. (3), this results in

$$P = 2b_0 g(b_0) k_{\rm B} T_p / x_0 \;, \tag{4}$$

in which we have introduced $b_0 = \sqrt{\hbar\omega x_0^2 / 2k_{\rm B}T_x}$ and the auxiliary function $g(x) = \pi^{-1/2} \exp(-x^2)/(\mathrm{erf}(x) + 1)$. With this, the work required for the isothermal compression $W = \int_\infty^0 P\, dx_0$ follows as

$$W = \ln 2\, k_{\rm B} T_p = \ln 2\, k_{\rm B} T\, \mathrm{e}^{-2r} \;, \tag{5}$$

which is fully consistent with the exponential decrease in the work input suggested by the numerical results.

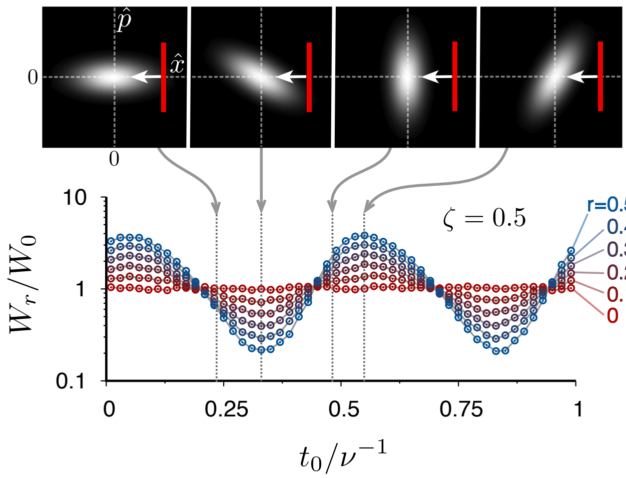

Figure 3. Required work to compress the single-particle gas to half of its initial volume as a function of the timing parameter t_0, which defines the points in time, namely t_0, $\nu^{-1} + t_0$, $2\nu^{-1} + t_0$, ..., at which the compression steps are executed. The simulations are performed in the critically damped regime ($\zeta = 0.5$) at constant temperature T. Close to $t_0 \simeq 0.3\,\nu^{-1}$ and $t_0 \simeq 0.8\,\nu^{-1}$, the work is found to exponentially decrease with the squeezing parameter r. Under these conditions, the squeezing effect reduces the occurrence of large positive momenta at the position of the piston, which leads to a reduced pressure.

Since the probability density function in eq. (3) factorizes as $\rho(\hat{x},\hat{p}) = \rho(\hat{x})\,\rho(\hat{p})$ with $\rho(\hat{x}) = \int_{-\infty}^{+\infty} \rho(\hat{x},\hat{p})\,d\hat{p}$ and $\rho(\hat{p}) = \int_{-\infty}^{+\infty} \rho(\hat{x},\hat{p})\,d\hat{x}$, the entropy associated to this state results additively from the contributions of the two quadratures: $S = S_x + S_p$. These contributions can be determined using the Shannon entropy, which coincides with the physical entropy in the case of Gibbs ensembles. From $S_p = -k_B \int_{-\infty}^{+\infty} \rho(\hat{p})\,\ln(\rho(\hat{p}))\,d\hat{p}$, one concludes that the entropy in the momentum quadrature follows as

$$S_p/k_B = \ln(k_B T_p/\hbar\omega)/2 + C\,. \tag{6}$$

with an additive constant C. Note that this result does not reflect the correct low-temperature behavior of the entropy, which is an artifact of the purely classical calculation. This is, however, not crucial for the purpose of this work. In the same way, we derive a corresponding expression for the entropy in the position quadrature

$$S_x/k_B = \ln\left[\frac{x_0}{b_0 g(b_0)}\right] - b_0 g(b_0) - b_0^2 + C' \tag{7}$$

During the free expansion (step i) no work is performed. The internal energy $U = \iint d\hat{x}\,d\hat{p}\,\rho(\hat{x},\hat{p})\,(\hbar\omega/2)\,(\hat{x}^2 + \hat{p}^2)$, which using eq. (3) evaluates to

$$U = \frac{k_B}{2}\left[(1 - 2b_0 g(b_0))\,T_x + T_p\right]\,, \tag{8}$$

remains constant: $U(T_x, T_p, x_0 = \infty) = U(T_x, T_p, x_0 = 0)$. Consequently, there is no net heat flow between system and environment and the entropy of the environment remains constant: $(\Delta S)_{\text{env}} = 0$. The total entropy change $\Delta S = (\Delta S)_{\text{env}} + (\Delta S)_{\text{sys}}$ is solely determined by the entropy change of the system $(\Delta S)_{\text{sys}} = \Delta S_x + \Delta S_p$, which here is given by $(\Delta S)_{\text{sys}} = S_x(x_0 = \infty) - S_x(x_0 = 0)$. With eq. (7), this leads to a total entropy change of

$$\Delta S = k_B \ln 2\,. \tag{9}$$

This is the expected result for an irreversible doubling of the phase space volume. During the isothermal compression (step ii) the invested work W is dissipated as heat, which leads to an entropy increase in the environment of $(\Delta S)_{\text{env}} = W/T_p = k_B \ln 2$ that exactly cancels the entropy decrease in the system $(\Delta S)_{\text{sys}} = -k_B \ln 2$. Thus, no net change in the total entropy occurs during this step. The same is obviously true for the third and last step, the insertion of the partition. This means that the total entropy change of the universe during the erasure process solely results from the entropy increase during the free expansion and is consequently given by eq. (9). In summary, we find that the reset of one bit of classical information in a squeezed thermal memory leads to the same entropy increase of $k_B \ln 2$ as in a standard thermal memory, while the required work can be exponentially lowered with the squeezing factor. We expect that an experimental verification of the predicted effect using well-established experimental platforms such as optically trapped nano-particles [7, 8] and nanomechanical devices [28] is within reach.

Squeezed thermal environments distinguish themselves from standard thermal reservoirs by the presence of statistically correlated fluctuations. The significance of such non-equilibrium reservoirs stems from the fact that they may naturally arise in systems operating in a pulse-driven fashion as is common, for example, in digital electronics. The dissipated power in today's microprocessors is due to both static leakage and dynamic switching, in approximately equal parts [2]. The dynamic power dissipation arising, for example, from charging or discharging processes can lead to complicated energy transport phenomena and temperature transients, which depend on geometry, thermal conductivity of materials and thermal resistance of interfaces. It seems natural that such an environment may be characterized by statistically correlated thermal fluctuations, which cannot be described by an equilibrium framework anymore. An example for this may be periodic correlations at the frequency of the CPU clock. Using advanced design approaches, such as thermal rectification [30], thermal flows can even be decoupled from electronic currents expanding the possibilities to deliberately engineer thermal environments. In future, combining concepts of electronics and non-equilibrium thermodynamics will open up new routes for more energy efficient electronics.

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