Bus Allocation to Short-turning and Interlining lines

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ABSTRACT
We propose injecting flexibility into public transport service planning by introducing a demand-driven method for generating and assigning buses to short-turning and interlining services. This study formulates, solves and applies the problem of assigning vehicles to the lines of a bus network subject to the dual objective of (a) improving the passenger waiting times at stops and (b) reducing the operational costs. At first, the vehicle allocation problem is expanded with the explicit consideration of interlining and short-turning lines that provide greater operational flexibility. The paper introduces a rule-based approach for generating interlining and short-turning lines that are considered as "virtual lines" because some of them might remain inactive if their operation does not improve the vehicle allocation solution. The bus allocation problem to existing and virtual lines is modeled as a combinatorial, multi-objective optimization problem and is solved with a Genetic Algorithm (GA) meta-heuristic that can return improved solutions by avoiding the exhaustive exploration of a combinatorial solution space. The vehicle allocation to existing and virtual lines is applied to the bus network of The Hague with the use of Automated Fare Collection (AFC) data from 24 weekdays and General Transit Feed Specification (GTFS) data. Sensitivity analysis results demonstrate a significant reduction potential in passenger waiting time and operational costs without adding a large number of short-turning and interlining line options that could impede the practicality of the bus services.

Keywords: tactical planning; vehicle allocation; interlinings; bus operations; evolutionary optimization; route design

INTRODUCTION

Ideally, public transport supply will perfectly correspond and scale to passenger demand. However, this is impossible in real-world operations due to the uneven distribution of demand over time and space. This results in inefficiencies for both passengers and operators and creates the need to re-dimension the fleet and circulate vehicles between demand areas.

Planning decisions regarding public transport services in general and bus networks in particular, are typically made at the strategic, tactical and operational planning level Ibarra-Rojas et al. (*1*), Gkiotsalitis and Maslekar (*2*). At the strategic level, the network and route-design problem is addressed where the alignment of the bus lines and the location of the bus stops are determined (Mandl (*3*), Ceder and Wilson (*4*), Borndörfer et al. (*5*)). Subsequently, at the tactical planning level, the sub-problems of bus frequency settings Gkiotsalitis and Cats (*6*), timetable design (Gkiotsalitis and Maslekar (*7*), Gkiotsalitis and Kumar (*8*), Gkiotsalitis and Stathopoulos (*9*)), vehicle scheduling Ming et al. (*10*), driver scheduling Wren and Rousseau (*11*) and driver rostering Moz et al. (*12*) are typically addressed in a sequential order.

Apart from the strategic and tactical planning, bus operators can take decisions over the course of the daily operations. In the operational planning phase, near real-time control measures such as stop-skipping (Sun and Hickman (*13*), Yu et al. (*14*), Chen et al. (*15*)) or bus holdings at specific stops (Newell (*16*), Wu et al. (*17*), Gavriilidou and Cats (*18*)) can be deployed. Notwithstanding, bus holding tends to increase the inconvenience of on-board passengers who are held at stops Fu and Yang (*19*) and stop-skipping increases the inconvenience of passengers who cannot board the bus that skips their stop Liu et al. (*20*).

Typically, the strategic, tactical and operational planning problems are addressed at different levels with the exception of a number of works that solve together the strategic-level problem of route design and the tactical-level problems of frequency settings and timetable design (Yan et al. (*21*), Zhao and Zeng (*22*)). Especially, the simultaneous solution of the route design and the frequency settings problem has the potential of improving the efficiency of the operations by modifying the bus routes and the corresponding frequencies to better cater for the passenger demand imbalances.

The frequency settings problem has been studied by several works in the literature (Yu et al. (*23*), Gkiotsalitis and Cats (*24*)). Unlike frequencies, modifying bus routes on a regular basis for improving the demand matching (i.e. operating different routes on different times of the day) and reducing the operational costs is not practical because passengers rely heavily on the pre-defined routes of the bus network and frequent route changes increase significantly the passenger inconvenience even if they are properly communicated (Kepaptsoglou and Karlaftis (*25*), Daganzo (*26*)). Given the above, bus operators tend to modify the frequencies of bus lines, but they are reluctant to modify the bus routes for improving the trade-off between the passenger demand satisfaction and operational costs at specific segments of bus lines which exhibit significant demand imbalances.

Given the practical and public acceptance issues associated with bus route variants, other flexible approaches which consider the deployment of short-turning and interlining can be considered. The works of Verbas and Mahmassani (*27*) and Verbas et al. (*28*) provide a first step in this direction since they do not allocate bus frequencies at a line level, but at a segment level considering a pre-defined set of short-turning options.

This work leverages on the potential flexibility embodied in short-turning and interlining lines in catering more efficiently to the prevailing passenger demand variations. First, observed passenger demand variations are used for generating a set of potential switch points along existing bus service lines where short-turning and interlining operations are allowed. The switch points are a subset of the bus stops of the network. Short-turning and interlining options are permitted at each switch point; thus, there is an additional set of (sub-)lines which can serve a set of targeted line segments. We denote the generated candidate short-turning and interlining lines as "virtual lines" for which vehicles can be allocated if deemed desirable. With this approach, we introduce an additional flexibility in allocating buses to lines because apart from the originally planned lines, buses can also be allocated to the set of virtual lines in order to match the passenger demand variation at different segments of bus lines without serving unnecessarily all the stops of those lines.

To this end, this work contributes by (a) modeling the above-mentioned problem for the first time and introducing an automated, rule-based scheme for generating switch point stops for short-turning and interlining "virtual lines", (b) introducing an exterior point penalization scheme for penalizing the violation of constraints and approximating the constrained optimization problem with an unconstrained one that can be solved with a problem-specific genetic algorithm and (c) investigating the potential gains in operational costs and passenger waiting times at the bus network of The Hague, the Netherlands.

METHODOLOGY

Overall framework

For the generation of potential short-turning and interlining lines from the existing bus lines, one needs to establish first a set of switch point stops. Generating all possible sub-lines and inter-lines considering each bus stop as a potential switch point is a computationally complex task and may result in a service that is difficult to operate and communicate to passengers. For this reason, works such as Verbas and Mahmassani, Verbas et al. (*27, 28*) propose to pre-define a limited set of switch stops at bus stops where a significant demand variation is observed. This approach is also adopted in this study with some adaptations. Since our work focuses on generating also inter-lines (and not only sub-lines), we examine transfer stops as well because such stops can be used for interlining without inducing additional deadheading times.

Given the fact that some transfer stops might be very close to bus stops where a significant variation of passenger demand is observed, we prioritize first the transfer stops and we apply an ad-hoc rule which dictates that none of the two preceding $(s-2, s-1)$ or following $(s+1, s+2)$ bus stops of a switch stop, s, can be considered as switch points as well. This ad-hoc rule helps to reduce the number of switch points without affecting significantly the final outcome (i.e., short-turning lines that perform short-turns at neighboring stops are not expected to perform much differently).

In addition to the above, we establish the following assumptions for (a) the determination of the switch points and (b) the generation of potential sub/inter-lines:

(1) All transfer stops are considered as potential switch points. Bus stops where a significant ridership change is observed (i.e., bus stops at which the on-board passenger change is greater than a pre-defined percentage of $z\%$) are also considered as potential switch points;

(2) Neighboring bus stops, $s \pm 2$, of a switch stop s that belong to the same line cannot be considered as switch points;

(3) Interlining connections are required to return to the origin station after completing their trip;

(4) Interlining lines can serve segments of at most two originally planned bus lines;

(5) Any interlining line which serves segments of two originally planned lines cannot have a total trip travel time which exceeds a pre-defined limit of y minutes (which may be defined by the transit agency and prevents the generation of excessively long interlining lines);

(6) Lengthy deadheading times may not be allowed by transit agencies; thus, an upper limit of k minutes for total deadheading times is applied for each of the virtual lines.

Before proceeding further into problem formulation, the following notation is introduced:

$\{L,S\}$	is a bus network with $L=\{1,2,...,\lvert L\rvert\}$ bus lines including original and virtual lines. Virtual lines represent sub-lines and inter-lines of the originally planned ones;
$L_o=\{1,2,...,\lvert L_o\rvert\}$	is the set of the originally planned lines;
$S=\{1,2,...,\lvert S\rvert\}$	is the set of stops of the bus network;
$S_l=\{1,2,...,\lvert S_l\rvert\}$	a set denoting the bus stops of line $l\in L$ in a sequential order starting from the first stop;
$S'\subset S$	set of stops that cannot be used as switch points due to regulatory or operational constraints;
$\mathbf{T}\in\mathbb{R}_+^{\lvert S\rvert\times\lvert S\rvert}$	a $\lvert S\rvert\times\lvert S\rvert$ dimensional matrix where each $t_{i,j}\in\mathbf{T}$ denotes the planned travel time between the bus stop pair i,j including the dwell time component (boarding and alighting times);
$\mathbf{U}\in\mathbb{R}_+^{\lvert S\rvert\times\lvert S\rvert}$	a $\lvert S\rvert\times\lvert S\rvert$ dimensional matrix where each $u_{i,j}\in\mathbf{U}$ denotes the planned travel time between the bus stop pair i,j excluding the dwell times for boarding/alighting (utilized for estimating the deadheading times);
$\mathbf{f}\in\mathbb{R}_+^{\lvert L\rvert}$	vector where each $f_l\in\mathbf{f}$ denotes the frequency of bus line $l\in L$ in vehicles per hour;
$\mathbf{h}\in\mathbb{R}_+^{\lvert L\rvert}$	vector where each $h_l\in\mathbf{h}$ denotes the dispatching headway of bus line $l\in L$ (note: $h_l=\frac{60min/h}{f_l}$, $\forall l\in L$);
$\mathbf{r}\in\mathbb{R}_+^{\lvert L\rvert}$	vector where each $r_l\in\mathbf{r}$ denotes the total round-trip time required for completing the round-trip of line $l\in L$ in hours;
$\mathbf{n}\in\mathbb{R}_+^{\lvert L\rvert}$	vector where each $n_l\in\mathbf{n}$ denotes the number of buses required for operating line $l\in L$ for a given frequency f_l (note: $n_l=r_lf_l$, $\forall l\in L$);
$\mathbf{B}\in\mathbb{N}^{\lvert L_o\rvert\times\lvert S\rvert\times\lvert S\rvert}$	a matrix where each $b_{l_o,i,j}\in\mathbf{B}$ denotes the passenger demand between each pair of bus stops i,j for each originally planned line $l_o\in L_o$;
$\mathbf{D}\in\mathbb{N}^{\lvert L_o\rvert\times\lvert S\rvert}$	a matrix where each $d_{l_o,s}\in\mathbf{D}$ denotes the average on-board occupancy for the segment starting at stop s for an originally planned line $l_o\in L_o$;
$\delta_{l,l_o,i,j}$	a dummy variable where $\delta_{l,l_o,i,j}=1$ if line $l\in L$ is able to serve the passenger demand $b_{l_o,i,j}$ and $\delta_{l,l_o,i,j}=0$ if not;

Nomenclature (1/2)

γ	a constant denoting the total number of available buses (note: $\sum_{l\in L} n_l \leq \gamma$ for ensuring that the total number of buses utilized from all lines $l \in L$ is within the allowable number of buses);
$\mathbf{O} \in \mathbb{R}_+^{\lvert L_o\rvert\times\lvert S\rvert\times\lvert S\rvert}$	a matrix where each $O_{l_o,i,j} \in \mathbf{O}$ denotes the passenger-related waiting cost for every Origin-Destination (OD) pair of the originally planned line l_o;
$\mathbf{e}$	an $\lvert L\rvert$-valued vector of dummy variables where $e_l = 1$ denotes that at least one vehicle has been assigned to bus line $l \in L$ and $e_l = 0$ denotes that no vehicles are assigned to that line (in such case, $n_l = 0$);
ψ	a percentage denoting the lowest bound for the number of buses that should be allocated to the originally planned lines;
η	a constant denoting the total number of virtual lines that can be operational (i.e., operated by at least one bus);
k	maximum allowed limit of deadheading times for each virtual line;
y	maximum total trip travel time for inter-lining lines;
Q	discrete set of values from which one can select the number of buses allocated to an originally planned line;
Q'	discrete set of values from which one can select the number of buses allocated to a virtual line;
z	a percentage beyond which a change in passenger ridership (i.e., on-board occupancy) between two consecutive bus stops can justify the generation of sub/inter-lines;
β_1	unit time value associated with the passenger-related waiting time cost (€/h);
β_2	unit time value associated with the total vehicle travel time for serving all lines (€/h);
β_3	unit time value associated with the depreciation cost of using an extra bus (€/bus);
S^*	the set of the generated switch points (note: $S^* \subset S \land S^* \cap S' = \varnothing$);
τ	the planning period, a constant.

Nomenclature (2/2)

Generating the set of switch stops

Using the above notation and the rules described in assumptions (1)-(2), an exhaustive, rule-based graph search is devised for determining the switch points of the bus network. The rule-based graph search for determining the switch points is presented in alg.1.

The 5-th line in algorithm 1 states that if a stop s is a transfer stop, it does not belong already to the set of switch points and does not belong to the set of stops that cannot be used as switch points due to regulatory constraints; then, it can be added to the set of switch points. After doing this, it is checked whether there are any neighboring stops of the examined bus stop, s, that are already allotted to the switch points' set and, if this is the case, bus stop s is excluded from the set of switch stops (lines 7-11 of alg.1).

A bus stop s can also be a switch point even if it is not a transfer stop as described in lines 13-17 of alg.1. In more detail, if bus stop s is not yet a switch point and the ridership change between stop s and $s+1$ is more than $z\%$; then, this bus stop can be added to the switch points' set. Before adding bus stop s to the switch points' set, the algorithm checks whether (a) any neighboring stop belongs already to the set of switch points; (b) bus stop s is not already in the set S^* and (c) bus

Algorithm 1 Rule-based graph search for determining the switch points

```
1:  function RULE-BASED GRAPH SEARCH
2:      Initialize a set of switch points $S^* \leftarrow \varnothing$;
3:      for each originally planned line $l \in L_o$ do
4:          for each bus stop $s \in S_l \setminus \{1, |S_l|\}$ do
5:              if bus stop $s$ is a transfer stop and $s \notin S^* \wedge s \notin S'$ then
6:                  Set $S^* \leftarrow S^* \cup \{s\}$;
7:                  for each neighboring stop $s' \in (s-2, s-1, s+1, s+2)$ do
8:                      if $s' \in S^*$ then
9:                          $S^* \leftarrow S^* \setminus \{s\}$;
10:                     end if
11:                 end for
12:             end if
13:             if the on-board occupancy $r_{l,s}$ varies by more than $z\%$ from $r_{l,s-1}$ then
14:                 if $\{s+1, s+2\} \cap S^* = \varnothing \wedge \{s-2, s-1\} \cap S^* = \varnothing \wedge s \notin S^* \wedge s \notin S'$ then
15:                     Set $S^* \leftarrow S^* \cup \{s\}$
16:                 end if
17:             end if
18:         end for
19:     end for
20: end function
```

stop s does not belong to the set of stops, S', which cannot be switch points (these requirements are expressed in line 14 of alg.1).

Note that the number of switch points that are generated through this process is not fixed a priori and it can vary based on the value of $z\%$. This flexible formulation allows transit agencies to control the generation of sub-lines and inter-lines by reducing or increasing the number of potential switch point stops according to their preferences.

Generating candidate short-turning and interlining lines

Given the switch points determined by algorithm 1, short-turning and interlining lines are generated using an exhaustive graph search. For generating short-turning lines, for each originally planned line, $l_o \in L_o$, we define a set V_{l_o} that contains the first and last stop of line l_o and all switch point stops that are served by line l_o. Each short-turning line is generated by considering a pair of stops that belong to the set V_{l_o} as the origin and destination of that short-turning line. In case that the origin and destination bus stops of a short-turning line are neither the first nor the last stop of the corresponding originally planned line, then a deadhead is needed after the completion of each trip to allow bus drivers to rest at one of the two terminals of the originally planned line before starting their next trip. The automated procedure for generating short-turning lines based on the switch point stops is detailed in the flow diagram of figure 1a.

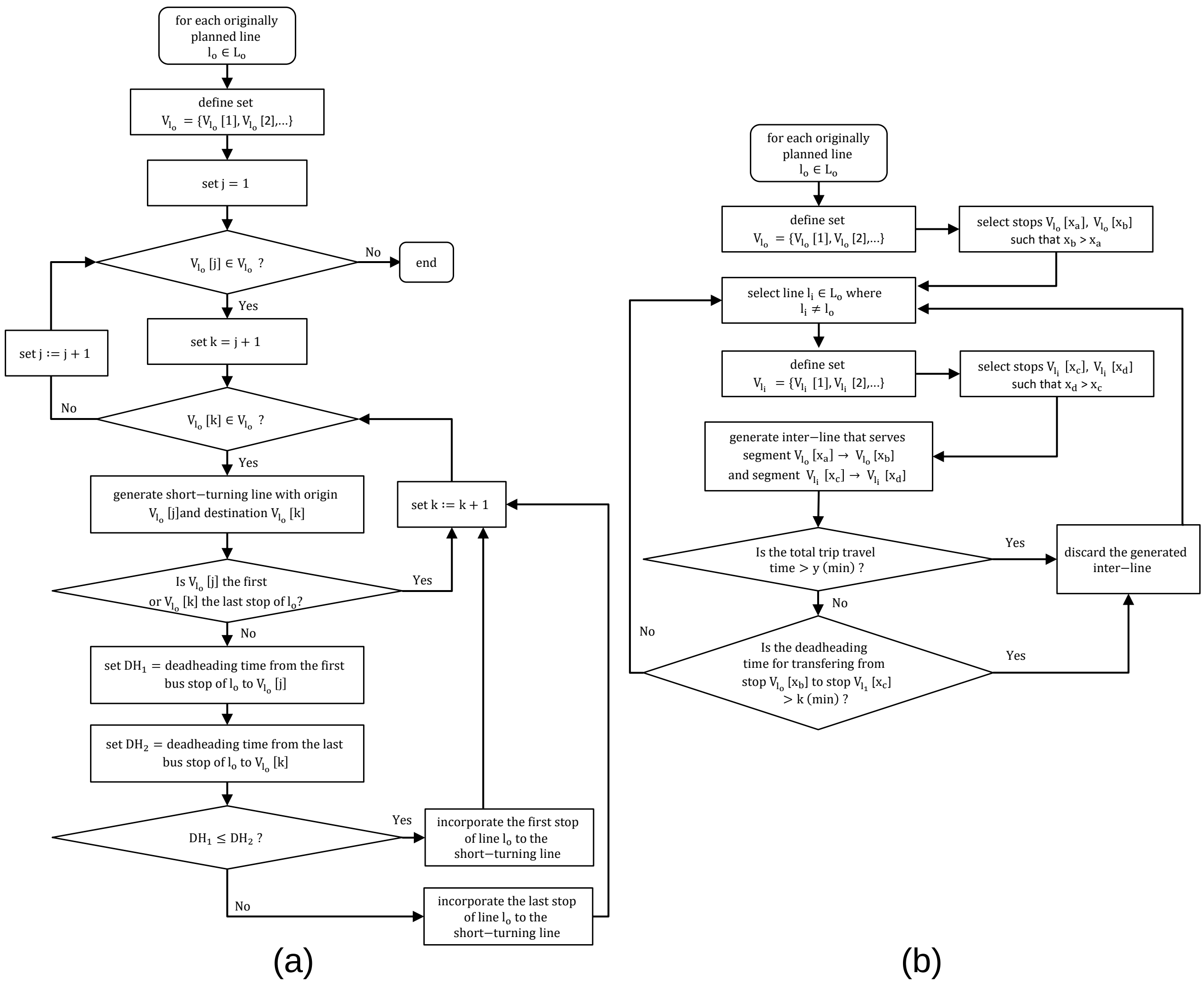

FIGURE 1 : Process of generating (a) short-turning lines at specific switch points; (b) inter-lining lines at specific switch points

From the flow diagram of fig.1a, one can note that the process starts from the first stop of each originally planned line and new short-turning lines are generated by using as destination stop each switch point stop which belongs to that originally planned line. The procedure continues until all stops that belong to the set V_{l_o} are used as destination stops for generating new short-turning lines. After that, a new stop from the set V_{l_o} is used as a first stop from which we generate short-turning lines and the procedure continues until exhausting the set of stops that belong to V_{l_o}.

The process of generating inter-lining lines involves further steps for finding routes that serve segments of two originally planned lines. If an inter-line serves segments of two originally-planned lines and the transfer occurs at a transfer stop between those lines, then the inter-line does not incur any deadheading costs. In any other case, an inter-line induces a deadheading cost for transferring from one originally planned line to another. Following assumptions (4-6), the potential inter-lines of a bus network are generated via a rule-based enumeration as presented in the flow diagram of figure 1b.

Vehicle allocation and frequency determination

The vehicle allocation problem to originally planned and virtual lines is formulated considering the inherently contradictory objectives of reducing the waiting cost of passengers at bus stops and reducing the operational costs. The operational costs are expressed in the form of (a) vehicle running times and (b) depreciation costs for each extra vehicle allocated to the bus network. In this work, we formulate a single, compensatory objective function by introducing the weight factors, $\beta_1, \beta_2, \beta_3$ that convert the passengers' waiting costs and the operational costs into monetary values.

Given that the dummy variable $\delta_{l,l_o,i,j}$ denotes whether a bus line $l \in L$ serves the passenger demand $b_{l_o,i,j}$ or not, the joint headway of all lines serving the i,j demand pair of the originally planned line $l_o \in L_o$ is:

$$[\sum_{l \in L} \delta_{l,l_o,i,j} \frac{n_\rho}{r_\rho}]^{-1} \tag{1}$$

In addition, if each $O_{l_o,i,j} \in \mathbf{O}$ denotes the passenger-related waiting cost for each OD pair of the originally planned line l_o and passenger arrivals at stops are random (an assumption that is commonly used for high-frequency services Osuna and Newell (*29*)); then,

$$O_{l_o,i,j} = \frac{b_{l_o,i,j}}{2} [\sum_{l \in L} \delta_{l,l_o,i,j} \frac{n_\rho}{r_\rho}]^{-1} \tag{2}$$

The decision variables of the optimization problem are the number of buses $\mathbf{n} = (n_1, n_2, ..., n_L)$ that can be allocated to each line $l \in L$. In addition, bus operators have to conform to a set of constraints. First, the total number of allocated buses to all lines, $\sum_{l \in L} n_l$, should not exceed the number of available buses γ:

$$\sum_{l \in L} n_l \leq \gamma \tag{3}$$

Furthermore, a minimum percentage $\psi\%$ of the total number of available buses should be allocated to the originally planned lines to ensure a minimum level of service for the originally planned lines. This constraint is introduced because in many cases the bus operators have a contractual commitment for operating at least a number of buses at the original lines:

$$\sum_{l \in L_o} n_l \geq \psi\gamma \tag{4}$$

In addition, in this study the average waiting of passengers is constrained by an upper threshold value Θ to ensure that the bus operator does not reduce the operational costs to such an extent that the quality of service for passengers is significantly compromised:

$$\sum_{l_o \in L_o} \sum_{i \in S} \sum_{j \in S} \frac{b_{l_o,i,j}}{2} \left(\sum_{l \in L} \delta_{l,l_o,i,j} \frac{n_\rho}{r_\rho} \right)^{-1} / \sum_{l_o \in L_o} \sum_{i \in S} \sum_{j \in S} (b_{l_o,i,j}) \leq \Theta \tag{5}$$

Finally, it is possible to set the lowest and highest bounds for the number of buses that can be allocated to the original and virtual lines. The number of buses n_l that are allocated to each original line L_o can take values from an admissible set Q and the buses that are allocated to virtual lines $L - L_o$ can take values from another set Q' since the original and virtual lines can have different distinct core requirements. For instance, all originally planned lines should be operational and a minimum number of buses should be allocated to them. In contrast, virtual lines that do not improve the service might not be used; thus, the set Q' permit refraining from assigning any vehicles to a virtual line.

The sets Q and Q' can be defined by the bus operator according to the lowest and highest frequency that is permitted for each virtual and original line. For instance, some virtual lines might be set to have a frequency value equal to zero (inactive virtual lines) whereas all originally planned lines might need to have a frequency of at least three vehicles per hour to satisfy service requirements.

The resulting optimization program considering the passengers' waiting times and the operational costs is:

$$\underset{\mathbf{n}}{\text{argmin}} f(\mathbf{n}) := \beta_1 \left(\sum_{l_o \in L_o} \sum_{i \in S} \sum_{j \in S} \frac{b_{l_o,i,j}}{2} \left(\sum_{l \in L} \delta_{l,l_o,i,j} \frac{n_\rho}{r_\rho} \right)^{-1} \right) + \beta_2 \left(\sum_{l \in L} n_l r_l \left\lfloor \frac{\tau}{r_l} \right\rfloor \right) + \beta_3 \left(\sum_{l \in L} n_l \right) \tag{6}$$

subject to:

$$c_1(\mathbf{n}) := \left(\sum_{l=1}^{L} n_l \right) - \gamma \leq 0 \tag{7}$$

$$c_2(\mathbf{n}) := \psi\gamma - \sum_{l \in L_o} n_l \leq 0 \tag{8}$$

$$c_3(\mathbf{n}) := \frac{\sum\limits_{l_o \in L_o} \sum\limits_{i \in S} \sum\limits_{j \in S} \frac{b_{l_o,i,j}}{2} \left(\sum_{l \in L} \delta_{l,l_o,i,j} \frac{n_\rho}{r_\rho} \right)^{-1}}{\sum\limits_{l_o \in L_o} \sum\limits_{i \in S} \sum\limits_{j \in S} (b_{l_o,i,j})} - \Theta \leq 0 \tag{9}$$

$$n_l \in Q, \forall l \in L_o \tag{10}$$

$$n_l \in Q', \forall l \in L - L_o \tag{11}$$

$$\eta \geq \sum_{l \in L - L_o} e_l \tag{12}$$

The first term of the objective function computes the waiting times of passengers at all stops for a given allocation of $\mathbf{n}$ vehicles to originally planned and virtual lines. The second term computes the total vehicle running times for serving all bus lines within a planning period τ where the round-trip travel time r_l of any line $l \in L$ contains the required layover times (i.e., deadheading and resting times of drivers). Finally, the third term corresponds to the depreciation costs when using $\sum_{l \in L} n_l$ vehicles.

The inequality constraint of eq.7 ensures that the total number of allocated vehicles to originally planned and virtual lines, $\sum_{l \in L} n_l$, should not exceed the number of available buses, γ. The inequality constraint of eq.8 denotes that at least a percentage $\psi\%$ of the total number of available vehicles, γ, should be allocated to the originally planned lines $l \in L_o$.

The inequality constraint of eq.9 introduces an upper limit, Θ, to the average waiting time per passenger ensuring that solutions which yield significantly longer passengers' waiting times are not considered even if they reduce the operational costs. Eq.10 and 11 ensure that the number of buses allocated to each line is selected from a discrete set of values determined by the transit agency. Finally, the inequality constraint of eq.12 ensures that the number of operational virtual lines, $\sum_{l \in L-L_o} e_l$, does not surpass the maximum allowed number of operational virtual lines, η.

The above constrained optimization problem of allocating buses to originally planned and virtual lines has a fractional, nonlinear objective function and one fractional constraint together with other linear constraints. The dimensions of this problem are equal to the number of lines L and the required number of computations for computing a globally optimal solution with simple enumeration (brute-force method) is $|Q|^L$ if we assume that $|Q| \simeq |Q'|$. Hence, the problem is computationally intractable given the exponential computational complexity even for small-scale networks. We therefore develop an approximation of the combinatorial, constrained optimization problem as detailed in the following section.

SOLUTION METHOD

Approximating the constrained vehicle allocation problem using exterior point penalties

The constrained bus allocation optimization problem of eq.6-12 can be simplified by using a penalty method which yields an unconstrained formulation. Given the highly constrained environment within which service providers operate, we introduce exterior penalties so that the satisfaction of constraints is prioritized.

By introducing a penalty function, $\wp(\mathbf{n})$, which approximates the constrained optimization problem of eq.6-12, the following unconstrained one is obtained:

$$
\begin{aligned}
\underset{\mathbf{n}}{\text{argmin}}\,\wp(\mathbf{n}) := \quad & f(\mathbf{n}) + w_1(\min[-c_1(\mathbf{n}),0])^2 + w_2(\min[-c_2(\mathbf{n}),0])^2 + w_3(\min[-c_3(\mathbf{n}),0])^2 \\
\text{subject to:} \quad & n_l \in Q, \forall l \in L_o \\
& n_l \in Q', \forall l \in L - L_o \\
& \eta \geq \sum_{l \in L-L_o} e_l
\end{aligned}
\tag{13}
$$

where w_1, w_2 and w_3 are used to penalize the violation of constraints and are positive real numbers with sufficiently high values to ensure that priority is given to the satisfaction of constraints. The penalty function $\wp(\mathbf{n})$ is equal to the score of the objective function $f(\mathbf{n})$ if at some point we reach a solution $\mathbf{n}$ for which $w_1(\min[-c_1(\mathbf{n}),0])^2 + w_2(\min[-c_2(\mathbf{n}),0])^2 + w_3(\min[-c_3(\mathbf{n}),0])^2 = 0$, indicating that all constraints are satisfied for such solution. The penalty terms are added to the objective function of the constrained optimization problem and dictate that if a constraint $c_i(\mathbf{n})$ has a negative score, then $\min[-c_i(\mathbf{n}),0] = -c_i(\mathbf{n})$ and the constraint is violated for the current set of variables $\mathbf{n}$. In that case, the objective function $f(\mathbf{n})$ is penalized by the term $w_i(-c_i(\mathbf{n}))^2$ where the weight factor w_i expresses the violation importance of this constraint in relation to all others.

Solving the unconstrained problem with a problem-specific Genetic Algorithm

Encoding

For solving the unconstrained optimization problem of eq.13, an initial population P with $\{1,2,...,|P|\}$ members is introduced. Each population member, $\mathbf{m} \in P$, is a vector $\mathbf{m} = (m_1,...,m_l,...,m_{|L|})$ with $|L|$ elements (known as genes) where each element $m_l \in \mathbf{m}$ represents the number of buses allocated to the corresponding line $l \in L$ in case this solution is adopted. Each gene $m_l \in \mathbf{m}$ of an individual **m** is allowed to take an integer value from the set Q (when line l is an originally planned line) or set Q' (when line l is a sub-line or inter-line).

Therefore, a random initial population P can be generated as follows:

For $m = 1$ to $|P|$
    Introduce the m^{th} population member $\mathbf{m} = (m_1, ..., m_l, ..., m_{|L|})$
    For $l = 1$ to $|L|$
        If $l \in L_o$: $m_l \leftarrow random.choice(Q)$
        If $l \in L - L_o$: $m_l \leftarrow random.choice(Q')$
    Next l
Next i

where $m_l \leftarrow random.choice(Q)$ denotes that m_l can take any value from the discrete set Q and $m_l \leftarrow random.choice(Q')$ denotes that m_l can take any value from the set Q'.

Evaluating the fitness of individuals and selecting individuals for reproduction

In the parent selection stage the fittest population members (individuals) are selected for reproduction and they pass their genes to the next generation. This can be achieved by using the well-known roulette-wheel selection method Goldberg and Deb (*30*). In the roulette-wheel selection method, each individual **m** has a probability of being selected which is proportional to its fitness value divided by the fitness values of all other population members.

After selecting one parent using the roulette-wheel selection method, another parent is selected with the same method and the two parents cross over to produce two offsprings. The same process is repeated until the number of parents which are selected for reproduction is the same as the population size $|P|$.

Crossover and mutation

At the crossover stage, two parents exchange their genes at a randomly selected crossover point selected from the set $\{1, 2, ..., |L|\}$ for generating two offsprings. For instance, if the crossover point of two parents $\mathbf{m} = (m_1, ..., m_l, m_{l+1}, ..., m_{|L|})$ and $\mathbf{m}' = (m'_1, ..., m'_l, m'_{l+1}, ..., m'_{|L|})$ which are selected for reproduction is $l \in L$; then, the two generated offsprings will have the set of genes $(m_1, ..., m_l, m'_{l+1}, ..., m'_{|L|})$ and $(m'_1, ..., m'_l, m_{l+1}, ..., m_{|L|})$.

After the crossover stage follows the mutation stage. In our case, we specify a small probability, p_c, for replacing each gene of the generated offspring with a random value from the set Q if that gene corresponds to an originally planned line and set Q' if it corresponds to a virtual one.

The procedure described above continues iteratively until a pre-determined number of population generations, μ^{max}, is reached. The population member with the best performance is then selected as the final solution and its genes represent the number of buses that should be allocated to each original or virtual line, where, for many virtual lines, this number can be equal to zero (resulting in inactive virtual lines).

APPLICATION

Case Study Description

The proposed methodology for the allocation of buses to originally planned and virtual lines is tested for the bus network of The Hague. The Hague is a mid-sized European city and its bus network consists of $|L_o| = 8$ originally planned urban bus lines, complementing and interfacing with the tram network. The originally planned bus lines cover a compact geographical area that enables the generation of several interlining lines without requiring long deadheading times. Seven of the bus lines[1] are bi-directional and one is circular (bus line 8).

In our case study, we analyze a 6-hour period of the day that was empirically found to exhibit a relatively stable ridership pattern (from 07:00 to 13:00). The total number of available buses for operating the service trips from 07:00 to 13:00 is γ=220. The average round-trip times for each one of the 8 bus lines and the optimal allocation of buses during the 6-hour period are:

	r_l: Round-trip time in minutes	Allocated Buses		r_l: Round-trip time in minutes	Allocated Buses
Line 1	108	29	Line 5	110	31
Line 2	107	22	Line 6	50	22
Line 3	112	21	Line 7	79	25
Line 4	172	39	Line 8	138	10

For the optimal allocation of buses to the eight originally planned bus lines, we used the parameter values $\gamma, z, \beta_1, \beta_2, \beta_3, Q$ from table 1 resulting in a total bus travel time of 21,616 minutes (360.26 hours) and an average waiting time of $\simeq$ 1.78 minutes per passenger.

From the above bus allocation to originally planned lines only, one can notice that only 199 out of the 220 available buses are allocated to lines because of the vehicle running time and the depreciation costs that favor the use of less resources.

Allocating buses to short-turning and interlining lines

In this study, we used detailed smartcard data logs from 24 weekdays in order to analyze the spatio-temporal passenger demand variation from 07:00 until 13:00. The smartcard logs contain information about the origin and destination station of each passenger that used one of the eight originally planned lines in The Hague during the analysis period (2nd of March 2015 - 2nd of April 2015). The smartcard logs are used for constructing passenger OD matrices per bus line.

The deployment of algorithm 1 for generating the switch stops for all bus lines and the algorithms presented in figures 1a, 1b for generating the short-turning and interlining lines yielded 29 short-turning lines and 323 interlining lines out of 4344 possible combinations.

By allocating buses to originally planned and short-turning/interlining lines, this study investigates the potential of improving the weighted sum of equation 6 which consists of the (a) passenger waiting times, (b) total vehicle running times and (c) depreciation costs from the use of additional vehicles. The allocation of buses to short-turning and interlining lines is performed by using the GA presented in the previous section.

When performing an optimal vehicle allocation to originally planned and virtual lines, the bus operator can determine several parameter values. In particular, the minimum percentage of buses that should be allocated to originally planned lines, ψ, and the total trip travel time limit for interlining lines, y, among others. This provides an extra flexibility to the bus operator that can tailor the use of the interlining and short-turning lines to its operational needs by adjusting the problem parameters accordingly.

Initially, we allocate buses to originally planned and short-turning lines following the scenario of table 1 which depicts the values of the problem parameters.

[1]for ease of reference, the eight bus lines in the Hague are named 1,2,...,8. The actual identification numbers of the eight bus lines can be found at https://www.htm.nl/media/498240/17066htm_a4haltekrttrambus_va01juli17_web.pdf

TABLE 1 : Parameter Values

γ (total number of available buses)	220
ψ (minimum percentage of buses that should be allocated to the originally planned lines)	60%
η (total number of virtual lines that can be operational)	20
k (maximum allowed limit of deadheading times for each virtual line)	20 min
y (maximum total trip travel time for inter-lining lines)	1 h 30 min
z (percentage of passenger ridership change that justifies the generation of a switch stop)	20%
Θ (upper limit of the average waiting time of passengers)	3 min
β_1 (unit time value associated with the passenger-related waiting time cost)	4 (€/h)
β_2 (unit time value associated with the total vehicle travel time for serving all lines)	60 (€/h)
β_3 (unit time value associated with the depreciation cost of using an extra bus)	20 (€/bus)
Q (number of buses that can be allocated to an original line from 07:00 to 13:00)	$\{6,7,8,...,41\}$
Q' (number of buses that can be allocated to a virtual line from 07:00 to 13:00)	$\{0,3,4,...,15\}$

Using the existing service provision as the starting point, we allow the re-allocation of buses to the 8 original, L_o, and (29+323)=352 virtual lines, $L - L_o$. Given the large number of decision variables and the combinatorial nature of the bus allocation problem, we employ the GA proposed in this study. For the implementation of the GA, we use the *Distributed Evolutionary Algorithms in Python* (Deap) package Fortin et al. (*31*). From this package, we use the eaSimple() algorithm with the hyperparameter values of $|P| = 200$ population members; $p_c = 0.2$ mutation probability; and $\mu_{max} = 40$ maximum population generations. For the evaluation of the fitness of each population member, the penalty function of Eq.13 is programmed in Python 2.7 and the tests are implemented in a general-purpose computer with 2.40 GHz CPU and 16 GB RAM.

Results

The GA algorithm is applied for the re-allocation of buses to originally planned and virtual lines and the convergence results are presented in figure 2. The goal of the convergence is the minimization of the penalty function score of Eq.13 which is the weighted sum of the objective function and the constraint violation penalties.

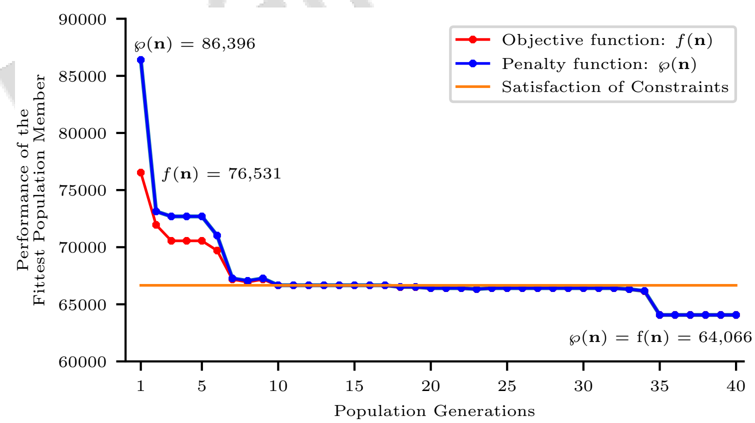

FIGURE 2 : Improvement of the exterior point penalty function score after a number of μ^{max} population generations. The horizontal line represents the area below which all constraints are satisfied (feasible solution space).

The fittest population member (solution) in the initial population has a penalty function value of $86,396$€ and an objective function value of $76,531$€. The initial $9,865$€ gap between the objective and the penalty function values indicates that the solution of the fittest population member of the initial population violates some of the constraints of the bus allocation problem.

After six iterations, we reach a point where all constraints are satisfied (at this point, the penalty function value is equal to the objective function value). At this stage, the first feasible solution is obtained. Then, the iterations continue until we reach the pre-defined maximum number, $\mu^{max} = 40$, of allowed population generations. The fittest solution at the 40^{th} population generation has a penalty function value of $64,066$€ and satisfies all constraints.

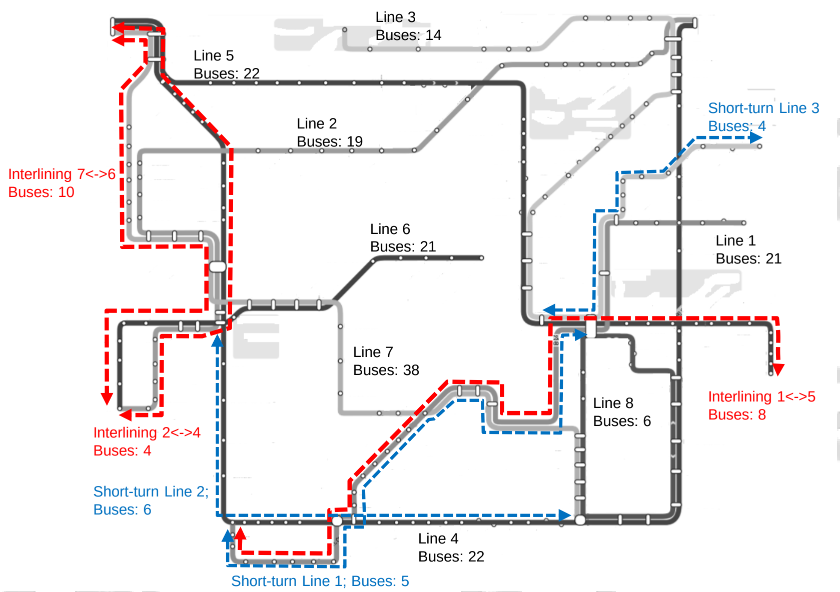

FIGURE 3 : Bus allocation to originally planned and active virtual lines

The resulting bus allocation to originally planned and virtual lines using the GA is presented in figure 3. As expected, the lion share of the 352 virtual lines remain inactive in the solution attained as GA solution filtered out 346 out of the 352 virtual lines. This solution involves 3 interlining and 3 short-turning operations. The interlining involves a relatively small number of buses and is used to circulate buses between busy lines that have an asymmetric passenger demand. Short-turning is deployed for lines that have to be partitioned due to a noticeably uneven demand pattern.

To provide more details on the performance improvement after the introduction of short-turning and interlining lines, figure 4 presents the overall waiting time costs, the vehicle running costs and depreciation costs when (a) only originally planned lines are considered; and (b) when short-turning/interlining lines are also considered. In the latter case, the overall waiting time costs are reduced from $43,436$ € to $41,445$ € and the vehicle running costs from $21,616$ € to $18,621$ €. The optimal bus allocation to both originally planned and virtual (short-turn and interlining) lines to the bus network of The Hague yields a potential reduction of 13.85% in operational costs and 4.85% in the average waiting time per passenger.

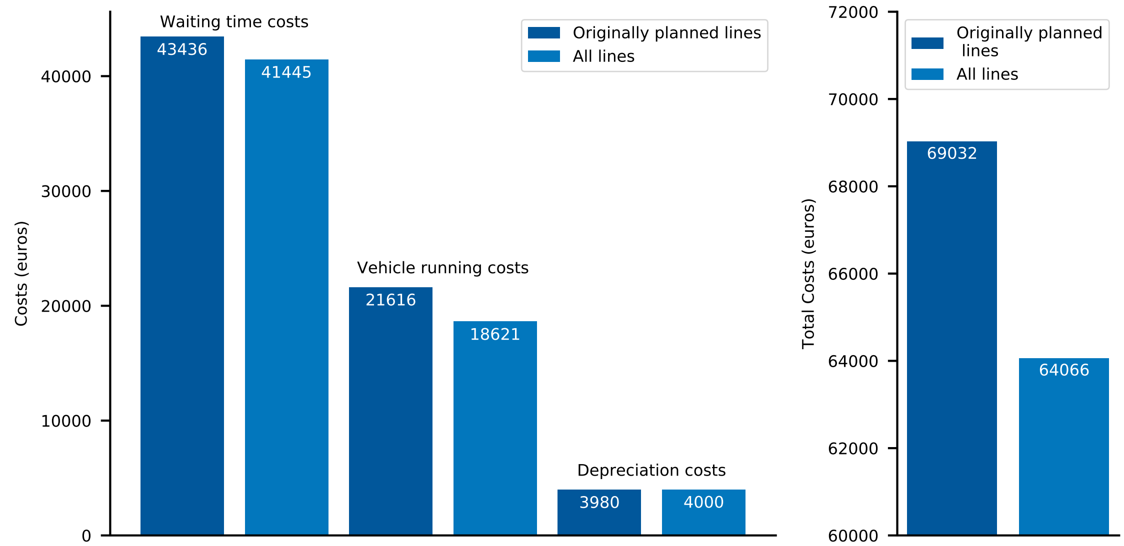

FIGURE 4 : Costs when using (a) originally planned lines only and (b) originally planned lines along with interlining and short-turning lines

Sensitivity Analysis of the parameters related to the generation of virtual lines

In this sensitivity analysis we investigate the performance changes for different values of the parameters which control the generation of short-turning and interlining lines. For example, the parameter ψ determines the minimum percentage of buses that should be allocated to the originally planned lines and its value was initially set to 60% (in the scenario of table 1). Some bus operators might, however, be more conservative and wish to ensure that at least 80 or 90% of the deployed buses are allocated to originally planned lines.

Similar to the above, some bus operators might not be willing to generate switch stop candidates at bus stops with slight ridership changes. Instead, they might consider a bus stop as switch stop candidate only when a significant ridership change is observed (i.e., $z > 50\%$). The results from this analysis are presented in figure 5 where the performances of the optimal bus allocation solutions for different values of ψ and z are presented. It should be noted here that apart from the values of ψ and z, all other parameters remain unchanged (see table 1).

In figure 5 the total cost of the operations for $\psi = 60\%$ and $z = 20\%$ is 64,066€. The total cost of the operations is the lowest (64,019€) for the most flexible scenario where the minimum number of buses that must be allocated to originally planned lines is $\psi = 40\%$ of the total number of deployed buses and $z = 10\%$.

From figure 5 one can observe that there is a broad range of values, i.e. $\psi = 60-80\%$ and $z = 10-30\%$, for which the total cost of the optimal bus allocations is relatively stable and hovers around 64,100€. This is an important finding because a more conservative (and practical) bus allocation where at least 80% of the deployed buses are allocated to the originally planned lines can be adopted without significantly increasing the total cost of operations.

Another important finding is that the solution is more sensitive to changes in ψ than in z. For instance, when $\psi = 90\%$ and $z = 20\%$ the total cost of the bus allocation is 67,923€ which is very close to the total cost of the optimal bus allocation when considering only originally planned lines (this cost was 69,032€). Notwithstanding, a comparable performance was observed when

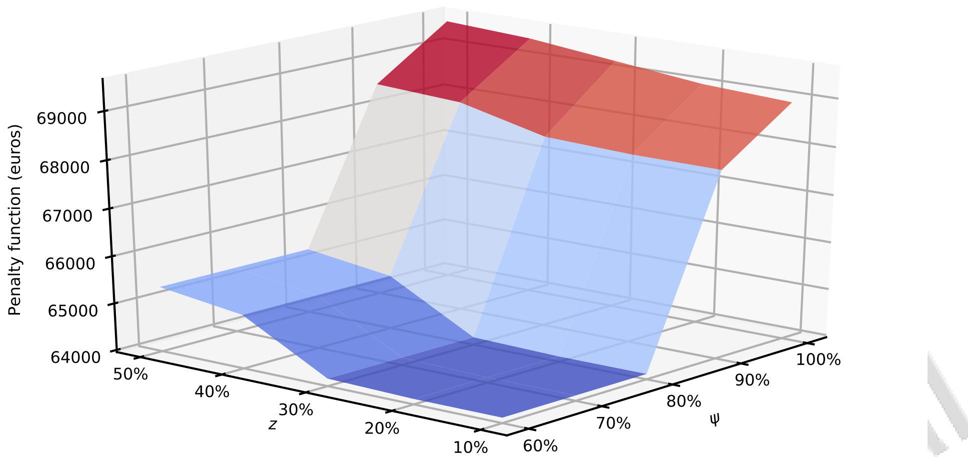

FIGURE 5 : Total cost of the optimal bus allocation for different values of the parameter ψ which controls the minimum percentage of buses that should be allocated to originally planned lines and z which affects the set of switch stop candidates

at least 60%, 70% or 80% of the buses are allocated to originally planned lines. This provides a strong advantage to the bus operator that can yield the maximum benefit by allocating the vast majority of its buses to originally planned lines and still benefit from a significant improvement of passenger/operational-related costs.

CONCLUDING REMARKS

This work develops a framework for allocating buses to originally planned and short-turning/interlining bus lines in order to reduce the passenger-related and the operational-related costs while satisfying a set of operational constraints. Following the problem formulation, a meta-heuristic solution approach is developed and applied to a case study network.

Model application demonstrates that the partial replacement of current services with virtual lines can significantly reduce (i.e. 13.85% for the real-world case study network) the vehicle running times while reducing also the average waiting time per passenger by $\simeq 5\%$. In the proposed approach, the operational short-turning and interlining lines are endogenously generated (in contrast to the works of Verbas and Mahmassani, Verbas et al., Delle Site and Filippi (*27*, *28*, *32*)), by considering a pool of virtual lines as part of the optimization process. The results indicate that the plurality of bus allocation options when considering a broader set of virtual lines can return a range of bus allocation combinations that offer almost equally large benefits. This provides a strong decision-support tool to bus service planners and operators who might have latent preferences or requirements (e.g. familiarity of bus drivers with certain lines, preference towards serving originally planned lines).

The sensitivity analysis of the model application demonstrated that re-allocating even a small share of vehicles to virtual lines can have a significant impact on the total costs of the operations (i.e., significant improvements are observed even if 80% of the deployed buses are allocated to originally planned lines). This finding demonstrates that bus operators do not need to change significantly the deployment of their buses in order to attain a reduction in the passenger/operational-related costs.

Future research direction may consider the demand elasticity to changes in service frequency. Moreover, the development of tactical planning tools that incorporate transit assignment models will potentially allow capturing the impacts of such interactions on passenger flow re-distribution.

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Author Contribution Statement

The authors confirm contribution to the paper as follows: study conception and design: K. Gkiotsalitis, O. Cats; data collection: O. Cats; analysis: Z. Wu, K. Gkiotsalitis; draft manuscript preparation and interpretation of results: K. Gkiotsalitis, O. Cats., Z. Wu
All authors reviewed the results and approved the final version of the manuscript.

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