

# Reduced turbophoresis in two-way coupled particle-laden flows

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## 1 Introduction

Direct numerical simulation of turbulent channel flow is employed to show that two-way coupling effects in particle-laden flows leads to reduced preferential clustering and turbophoresis [9] even for low values of volume and mass fraction. The effect of including two-way coupling on the phenomenon of turbophoresis and preferential clustering is studied for particles of different response times at a variety of loading ratios.

We adopt the Euler-Lagrange formalism in which a large number of discrete point particles is embedded in a continuous flow description. We consider particle-laden turbulent flows for different values of the volume fraction  $\Phi_v = N_p V_p / V_{tot}$  and mass fraction  $\Phi_m = (\rho_p / \rho_f) \Phi_v$ , defined in terms of the number of particles  $N_p$ , each having volume  $V_p$  and flowing in the flow domain of volume  $V_{tot}$ . The mass density of the particles is denoted by  $\rho_p$ , while  $\rho_f$  represents the fluid density.

For values of the volume fraction smaller than  $\Phi_v \leq O(10^{-6})$ , the interaction between the two phases is classified as a *one-way coupling* [4] since momentum exchange only occurs from the fluid to the particulate phase. For volume fractions values higher than  $\Phi_v \geq O(10^{-6})$ , the interaction is classified as a *two-way coupling interaction* where the exchange of momentum between the two phases can no longer be neglected.

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Turbulence modification due to the presence of small dispersed particles is expected to be strengthened in regions where preferential concentration effects are important. As shown by [3] from DNS simulations at  $\Phi_v = 6.8 \times 10^{-5}$  and  $\Phi_m = 0.5$ , the inclusion of two-way coupling modifies the local coherency of the flow and the particulate phase, thereby contributing to a modification of preferential clustering effects.

Several studies have been dedicated to understanding the influence of dispersed particle motion on turbulence. As pointed out by [2] in a recent review of turbulence modulation this mechanism is still purely understood in particle laden flows, and several contradictory results have been presented.

For a turbulent channel flow, the damping of fluid turbulence with increasing volume fraction for loading ratios up to  $\Phi_m = 0.8$  was reported experimentally by [10] and [13] but such an attenuation was not captured by LES simulations performed by [14]. In contrast, [11], found significant feedback effects for a vertical channel at  $Re_\tau = 125$ . Using a coarse DNS, at loading ratios in the range of  $\Phi_m = 0.2 - 2$ , they reported that the phase coupling reduces the near-wall concentration and increases the anisotropy of turbulence in the carrier phase. Apparently, the capturing of small-scale turbulence structures in a flow is essential to represent the dynamics of small particles as emphasized in [8].

To the Authors knowledge, the effect of two-way coupling and modified particle clustering in the lower range of two-way coupling interactions has not been studied before; it forms the objective of the present paper. We report DNS simulations of two-way coupled suspended particles at volume and mass ratios of  $10^{-6} < \Phi_v < 10^{-4}$  and  $0.0007 < \Phi_m < 0.077$  respectively. This range of loading for a two-way coupled suspension is much lower than previously reported in two-way coupling studies [11, 3] because so far it was considered to have a negligible effect on preferential clustering and turbophoresis.

## 2 Mathematical modeling of particle-laden flow

### 2.1 Fluid phase

The fluid motion is governed by the continuity equation  $\nabla \cdot u_i = 0$  and the Navier-Stokes equations for an incompressible fluid formulated in a dimensionless form as,

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re_\tau} \frac{\partial^2 u_i}{\partial x_j^2} - f_{i,R}, \quad (1)$$

where  $Re_\tau = (u_\tau h)/\nu_f$  is the characteristic Reynolds number based on friction velocity  $u_\tau$ , the kinematic viscosity  $\nu_f$  and the half width of the channel  $h$ . The last term  $f_{i,R}$  in the *rhs* of Equation (1) accounts for the two-way coupling between the two phases. It represents the feedback effect of the particles on the fluid carrier, i.e. the reactive force density exerted by the motion of particles on the fluid.

A cartesian flow solver was used for the present study to numerically solve the Navier-Stokes equations using the fractional time-step approach and a fully explicit projection scheme with pressure correction. The spatial discretisation was based on a second-order finite-difference scheme on orthogonal grids with a staggered variable arrangement [6].

## 2.2 Particulate phase

Following a Lagrangian approach for the particulate phase, an equation of motion is solved for each of the embedded particles. These are treated as point-particles assuming that they are solid, rigid and spherical, colliding fully elastically with the wall boundaries. The importance of particle inertia is expressed by the particle relaxation time  $\tau_p$  is defined as,

$$\tau_p = \frac{d_p^2 \rho_p}{18\nu_f \rho_f}, \tag{2}$$

where the subscripts  $f$  and  $p$  denote the fluid and particulate phase respectively and  $d_p$  denotes the particle diameter. Using  $\tau_f = h/u_\tau$  as the characteristic time scale of the flow, the Stokes number  $St_\tau$  which is defined as the ratio between the particulate time scale  $\tau_p$  and the fluid time scale  $\tau_f$  becomes,

$$St_\tau = \frac{\rho_p}{\rho_f} \left(\frac{d_p}{h}\right)^2 \frac{Re_\tau}{18} = \frac{St^+}{Re_\tau}, \tag{3}$$

where  $St^+$  is the Stokes number defined with respect to the fluid time scale  $\tau_f^+ = \nu/u_\tau^2$ .

Due to the density ratio ( $\rho_p/\rho_f = 769.23$ ) and the size of the released particles ( $d_p/h < 3.3 \times 10^{-3}$ ) used here, only the drag force was considered to contribute to the equation of motion [1, 12]. Particle lift, buoyancy, Basset force, added mass and rotation effects were thus neglected for the present analysis. Under these assumptions, the motion of a dispersed particle located at  $\mathbf{x}_p$  and travelling at velocity  $\mathbf{u}_p$  is governed by the dimensionless equation,

$$\frac{d\mathbf{u}_p}{dt} = \frac{\mathbf{u}_f(\mathbf{x}_p) - \mathbf{u}_p}{St_\tau} (1 + 0.15Re_p^{0.687}) \equiv \mathbf{H}_p, \tag{4}$$

where we introduced the short-hand notation  $\mathbf{H}_p$  for the right hand side and  $Re_p = \frac{d_p|\mathbf{u}_p - \mathbf{u}_f|}{\nu_f}$  is the particle Reynolds number. The local fluid velocity  $\mathbf{u}_f(\mathbf{x}_p)$  at the location of the particle  $\mathbf{x}_p$  is computed based on the interpolation scheme presented in [7]. The location of each particle  $\mathbf{x}_p$  is updated by integrating  $\frac{d\mathbf{x}_p}{dt} = \mathbf{u}_p$ , using a simple Euler scheme.

Case	$St^+$ $\frac{\tau_p}{(v/u_\tau^2)} \times 10^6$	$\Phi_v$ $\times 10^6$	$d_p/h$ $\times 10^3$	$\Phi_m$ (%)	$N_p$
A1	2.5	1	1.612	0.077	71938
A2	2.5	10	1.612	0.769	719383
A3	2.5	100	1.612	7.692	7193834
B1	5	1	2.280	0.077	25434
B2	5	10	2.280	0.769	254340
B3	5	100	2.280	7.692	2543405
C1	10	1	3.225	0.077	8992
C2	10	10	3.225	0.769	89922
C3	10	100	3.225	7.692	899229

**Table 1** Test cases considered for the case of two-way coupled particle-laden channel flow at  $Re_\tau = 150$ .

### 2.3 Force coupling

The ‘point force approximation’ is used to account for the coupling between the two phases. The point force exerted by each particle on the surrounding fluid is entirely allocated to the finite fluid element that contains the particle [5]. Thus, the reactive force per unit mass in the  $N - S$  Equation (1) contributing to the total flux at location  $\mathbf{x}$ , is given by,

$$f_{i,R} = \frac{1}{m_f} \sum_{p=1}^M F_{i,p}, \quad (5)$$

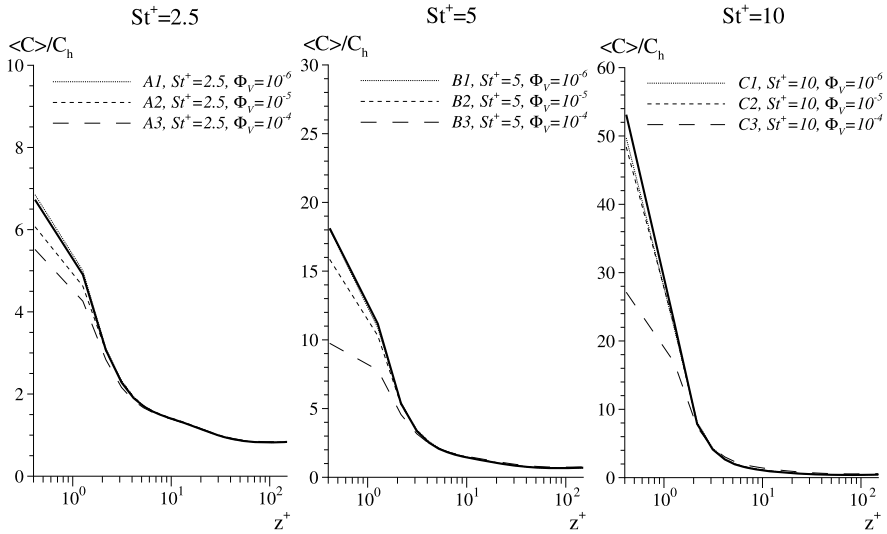
where  $m_f$  is the fluid mass contained in a grid cell and  $F_{i,p}$  is the actual force exerted by particle  $p$  along direction  $i$ . The summation is over the  $M$  particles contained in the control volume around  $\mathbf{x}$ . With the adopted definition of the particle’s acceleration (equation 4), and assuming monodisperse particles of equal mass  $m_p$ , the total force exerted by the particles on each control volume becomes,

$$f_{i,R} = \frac{m_p}{m_f} \sum_{p=1}^M H_{i,p} = \frac{\phi_m}{M} \sum_{p=1}^M H_{i,p}, \quad (6)$$

where  $\phi_m = \frac{Mm_p}{m_f}$  is the local mass fraction.

## 3 Results

DNS of particle-laden fully developed turbulent channel flows were conducted at a constant flow rate for a Reynolds number of  $Re_\tau = 150$  [8] as shown in Table 1. The computational domain used here has dimensions  $4\pi h \times 2\pi h \times 2h$  using a numerical resolution of  $128 \times 96 \times 90$  cells.



**Fig. 1** Time-averaged particle concentration normalized with the homogeneous concentration  $C_h$  for various  $St$  numbers. (Solid lines): one-way coupling, (dashed lines): two-way coupling.

After a fully developed turbulent state was reached for a particle-free flow, the particles were introduced at randomly selected locations within the domain. The simulation was performed for a total time of 72 flow-through times or 60 time units  $h/u_\tau$ . Statistics were collected and presented for the second half of this time period, i.e., for  $30 < t < 60$ .

All the present results indicate that the degree of particle clustering in the near-wall region and the strength of turbophoretic effects, are significantly modified by adding two-way momentum exchange between the dispersed particulate phase and the carrying fluid flow. The overall importance of including a two-way coupling approach was a strong function of the  $St^+$  number. For light particles, even for the highest loading ratio of  $\Phi_v = 10^{-4}$  where a two-way coupling regime would be justified [4], two-way coupling had a much smaller effect on turbophoresis and the centerline concentration when compared to those obtained for larger particles at the same volume and mass fractions (Cases B3 and C3).

For the highest particle loading at a mass fraction  $\Phi_m = 7.7\%$  considered here, the inclusion of two-way coupling effects was found to reduce turbophoresis by 18%, 46% and 50% for particles with dynamic response characterized by Stokes numbers  $St^+ = 2.5, 5$  and  $10$  respectively (Figure 1).

## 4 Conclusions

Direct numerical simulations of particle laden turbulent channel flow is employed to show that two-way coupling in particle-laden flows leads to reduced preferential clustering. It was demonstrated that with the inclusion of long-range two-way coupling effects, the average volume and mass loading fractions  $\Phi_v$  and  $\Phi_m$  only offer a rough indication on the degree of turbulence modulation and the associated preferential clustering and turbophoresis effects.

By demonstrating the relevance of two-way coupling in relation to preferential clustering, we can conclude that not only this coupling can significantly attenuate clustering and turbophoretic effects for heavier particles, but also that because of these preferential clustering mechanisms, two-way coupling expresses itself at rather lower average mass fractions than previously reported.

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