Tribology Letters (2018) 66:99
https://doi.org/10.1007/s11249-018-1050-x

REPLY TO THE COMMENT

Response to Dr Greenwood's Comments on "Extending the Double-Hertz Model to Allow Modeling of an Adhesive Elliptical Contact"

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Received: 23 May 2018 / Accepted: 25 June 2018 / Published online: 4 July 2018
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We are honored that our work on the extension of the Double-Hertz (DH) model for adhesive elliptical contacts [1] was commented by Greenwood, whose paper with Johnson [2] was the source of inspiration for our work.

In a comment to our published work, Greenwood argues that rather than predicting the pull-off forces, the extended DH model for elliptical contacts is actually predicting the force at which stable, local peeling starts to occur. He also concludes that our contact analysis that is based only on the major axis underestimates the pull-off force.

In our work [1], the adhesive region is assumed to be in the shape of an annulus, bounded by a contact ellipse of semi-major axis a and semi-minor axis b on the inside, and an adhesive ellipse of semi-major axis c and semi-minor axis d on the outside, as shown in Fig. 1.

The ellipticity ratios for the contact and the adhesive ellipses are termed β_{ab} and β_{cd}, respectively, given by

$$\beta_{ab} = b/a \tag{1a}$$

$$\beta_{cd} = d/c. \tag{1b}$$

A contact problem with an annular elliptical adhesive region is difficult to solve as a, b, c, and d are unknown a priori. Hence, we assume that it is appropriate for the extended DH model to first employ the simplest assumption concerning the ellipticity ratio of the adhesive region β, that is, both β_{ab} and β_{cd} have similar values, given by

$$\beta = \beta_{ab} = \beta_{cd}. \tag{2}$$

We fully acknowledge that Eq. (2) is not correct for adhesive elliptical contacts; it was also not our intention to suggest this being the case. The original paper was intended as a reformulation of the DH model to allow incorporation of arbitrary β_{ab} and β_{cd} values. In [1], Eq. 2 has been used as a zero, very rough approximation, as a way to better understand adhesive elliptical contacts.

When the pull-off force is achieved, the expression in Eq. (2) becomes

$$\beta_{(\text{pull-off})} = \beta_{ab(\text{pull-off})} = \beta_{cd(\text{pull-off})}. \tag{3}$$

Assuming that the ellipticity ratio remains constant throughout the contact, at the pull-off force, both β_{ab} and β_{cd} become equal to the ellipticity ratio at the initial loading, β_0; thus,

$$\beta = \beta_{(\text{pull-off})} = \beta_0. \tag{4}$$

Again, similar to Eq. (2), we understand that this assumption is not correct as the ellipticity ratio clearly does not remain constant.

And indeed, the assumption in Eq. (4) is the reason why the pull-off forces predicted by the extended DH model in [1] for $\beta_0 = 0.8$ and $\beta_0 = 0.9$ are lower than the approximate Johnson–Kendall–Roberts (JKR) model [3], at the limiting case close to the JKR domain, where these ratios' change significantly due to the elastic deformation caused by the surface forces. However, it is already mentioned in our paper [1] that Eq. (4) is expected to be valid only for $\beta_0 = 0.99$, as the shapes of both contact and adhesive ellipses are similar to a circular contact. For $\beta_0 = 0.8$ and $\beta_0 = 0.9$, the contact shapes are obviously elongated in the major axis direction, and hence the application of Eq. (4) results in inaccurate pull-off force predictions, by assuming $\beta_{ab} = \beta_{cd}$ for μ values close to the JKR domain. However, it is worth noting that the assumptions in Eq. (4) are also expected to be approximately valid for elliptical contacts at relatively low μ values, where the elastic deformation due to surface forces is relatively

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Electronic supplementary material The online version of this article (https://doi.org/10.1007/s11249-018-1050-x) contains supplementary material, which is available to authorized users.

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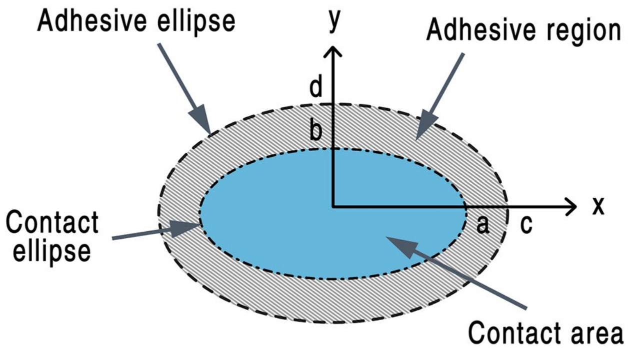

Fig. 1 Adhesive region of a DH-based elliptical contact

low. Although for increasing μ values, it is expected that the geometry of the adhesive contact will evolve from an elliptical geometry to a JKR-like geometry.

To show that the extended DH model framework [1] can predict accurately the pull-off force for various contact cases with the correct β_{ab} and β_{cd} assumptions, we simulate the adhesive elliptical contact using a Boundary Element Model (BEM) that employs the Conjugate Gradient Method (CGM) [4], which adhesive stresses are described using the Maugis-Dugdale model in [5]. The numerical simulation can be done for various contact cases; here the contact for β_0 values of 0.99, 0.8, and 0.5 is simulated, similar to the cases discussed in Greenwood's comment. The model is further explained in Supplementary Material. The obtained β_{ab}, β_{cd}, and a values from the numerical simulations are then applied in the extended DH model to predict the pull-off forces.

Pull-off force predictions from various models are compared in Table 1, for $\mu = 1$. The table includes pull-off force predictions by the extended DH model and the numerical simulations, Greenwood's result which he applied a direct JKR solution for the extended DH model, and also results using the approximate JKR model in [3]. When the pull-off force is achieved, the numerical simulations predict β_{ab} and β_{cd} to have different values from each other, higher than β_0 at the initial loading. It has to be noted that unlike the results in Greenwood's comment, here the pull-off forces are transformed into non-dimensional forms similar to [1], following the Derjaguin approximation for the case of an adhesive contact between two cylinders of radii R_1 and R_2, crossed at an angle, θ, to each other [6]. Then, the applied force, W, can be expressed in a non-dimensional form as W^*, which is given as

$$W^* = \frac{W \sin\theta}{2\pi R \Delta\gamma}, \tag{5}$$

where R is the relative radius and $\Delta\gamma$ is the surface energy. From Table 1, it is shown that the extended DH model predicts higher pull-off forces compared to the approximate JKR model [3] and the calculations by Greenwood. This situation is expected as the results from the extended DH model are obtained for low μ value of 1, which is outside the JKR domain. Results in Table 1 show that with a proper assumption for the adhesive region (varying β_{ab} and β_{cd} throughout the contact), the extended DH model can indeed predict accurately the pull-off force for various cases of adhesive elliptical contacts.

Finally, it is worth to note that the purpose of our paper [1] is to present the development of an adhesive elliptical model, achieved by extending the DH theory. The model development is completed, and validated for a high β_0 value of 0.99. For lower β_0 values, assuming constant ellipticity ratios for the adhesive region is indeed inaccurate, as also discussed in our paper [1]. We have shown through numerical simulations that both β_{ab} and β_{cd} do change significantly as the contact progresses. With this knowledge, we have continued the work on the extended DH model by finding the solutions for β_{ab} and β_{cd} that are suitable for a wide range of contact conditions. These solutions are already obtained and are planned to be published in a follow-up paper. By employing these solutions in the extended DH model, accurate prediction of adhesive elliptical contacts can be made for various contact conditions.

We again would like to thank Greenwood for his comments, which provide us an opportunity to elaborate more on our work.

Table 1 Comparison between pull-off force predictions for various elliptical contacts at $\mu = 1$

$\beta_0 = b/a$ (Hertzian)	Direct K_I (Greenwood) W^*	Approx. JKR model W^*	Numerical model			Extended DH model W^*
			$\beta_{ab(pull-off)}$	$\beta_{cd(pull-off)}$	W^*	
0.99	0.7462	0.7500	0.9927	0.9943	0.7901	0.7941
0.8	0.6584	0.7370	0.8232	0.8489	0.7785	0.7864
0.5	0.4453	0.6366	0.5486	0.5982	0.6926	0.692

Acknowledgements The authors gratefully acknowledge support from the Ministry of Education Malaysia, Universiti Teknikal Malaysia Melaka, and Green Tribology and Engine Performance (G-TriboE) research group.

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