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Maximal Controlled Invariant Sets of Switched Linear Systems

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1 Abstract

In this paper, controlled invariant sets of switched linear systems are studied. In particular, the problem of finding the maximal controlled invariant set (MCIS) contained in a linear space is addressed.

Control of dynamic systems by switchings have been studied intensively. Typical results include stability properties [1], sliding mode analysis [2][3] and reachability and controllability issues [4].

The study of controlled invariance in this paper is motivated by controller design paradigm based on viability theory. See, for example, [5] and [6].

The dynamics of the systems considered in this paper can be described by a family of state-space representations with shared state-space

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i u(t), \\ i &\in \{1, 2, \dots, m\}. \end{aligned}$$

The system admits two inputs, namely the continuous input $u(t)$ and the discrete input Δ_k that induces the switchings between the dynamics in the family.

There are two main cases studied in this paper. The first case deals with the situation where infinitely often switchings are not allowed. The second case deals with the situation where they are allowed.

If infinitely often switchings are not allowed, consider the following iteration.

$$V_0 = \mathcal{V}, \quad (1a)$$

$$V_{i+1} = V_i \cap \left(\bigcup_{j=1}^{n_i} \bigcup_{k=1}^m \mathcal{V}_{ij} \cap A_k^{-1} (\mathcal{V}_{ij} + \text{Im } B_k) \right). \quad (1b)$$

Here \mathcal{V}_{ij} is a linear space and $V = \bigcup_j \mathcal{V}_{ij}$. The following theorem can be proved about this iteration.

Theorem 1 *Let the iteration (1) converge to V , i.e. there is a $p > 0$ such that*

$$i \geq p \iff V_{i+1} = V_i = V.$$

Then V is the maximal controlled invariant set contained in \mathcal{V} .

It is also proved that the iteration follows some tree structure and that this tree terminates, hence guaranteeing convergence of the iteration after just a finite number of steps. The limit set of the iteration turns out to be the union of the maximal controlled invariant subspaces of each individual mode.

If infinitely often switchings are allowed, then the following iteration is used instead

$$V_0 = \mathcal{V}, \quad (2a)$$

$$V_{i+1} = \{x \in V_i \mid \text{vel}(x) \cap T_{V_i}(x) \neq \emptyset\}. \quad (2b)$$

The symbol $\text{vel}(x)$ denotes the polyhedral set of possible velocity vectors and $T_V(x)$ denotes the tangent cone of V at x respectively. As in the other case, it is proved that if the iteration converges, it will be to the MCIS.

Another result is that (2) is a generalization of (1), and by imposing certain conditions, both (1) and (2) boil down to the well known algorithm for the construction of maximal controlled invariant subspace of linear systems [7].

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