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A Generalized Poisson-Akash Distribution: Properties and Applications

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Abstract A generalized Poisson- Akash distribution which includes Poisson-Akash distribution has been proposed. Its factorial moments, raw moments and central moments have been derived and studied. Some statistical properties including generating functions, hazard rate function and unimodality have been discussed. Method of moments and the method of maximum likelihood have been discussed for estimating parameters of the distribution. Applications of the proposed distribution have been explained through two count datasets and compared with other discrete distributions.

Keywords Generalized Akash distribution, Poisson- Akash distribution, Compounding, Moments, Skewness, Kurtosis, Maximum likelihood estimation, Applications

1. Introduction

The statistical analysis and modeling of count data are crucial in almost all fields of knowledge including biological science, insurance, medical science, finance, sociology, psychology, are some among others. Count data are generated by many phenomena such as the number of insurance claimants in insurance, number of yeast cells in biological science, number of chromosomes in genetics, etc. It has been observed that, in general, count data follows under-dispersion (variance < mean), equi-dispersion (variance = mean) or over-dispersion (variance > mean). The over-dispersion of count data have been addressed using mixed Poisson distributions by different researchers including Raghavachari *et al* (1997), Karlis and Xekalaki (2005), Panjeer (2006), some among others. Mixed Poisson distributions arise when the parameter of the Poisson distribution is a random variable having some specified distributions. The distribution of the parameter of the Poisson distribution is known as mixing distribution. It has been observed that the general characteristics of the mixed Poisson distribution follow some characteristics of its mixing distributions. Various mixed Poisson distributions have been derived in statistics by selecting different mixing distribution.

The classical negative binomial distribution (NBD) derived by Greenwood and Yule (1920) is the mixed Poisson distribution where the parameter of the Poisson random variable is distributed as a gamma random variable. The NBD has been used to model over-dispersed count data. However, the NBD may not be suitable for some over-dispersed count data due to its theoretical or applied point of view. Other mixed Poisson distributions arise from alternative mixing distributions. For example, the Poisson-Lindley distribution, introduced by Sankaran (1970), is a Poisson mixture of Lindley (1958) distribution. The Poisson-Akash distribution, introduced by Shanker (2017), is a Poisson mixture of Akash distribution proposed by Shanker (2015. It has been observed by Karlis and Xekalaki (2005) that there are naturally arising situations where a good fit is not obtainable with a particular mixed Poisson distribution in case of over-dispersed count data. This shows that there is a need for new mixed Poisson distribution which gives a better fit as compared with the existing mixed Poisson distributions.

Shanker (2017) introduced the discrete Poisson- Akash distribution (PAD) to model count data and defined by its probability mass function (pmf)

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$$P_1(x;\theta)=\frac{\theta^3}{\theta^2+2}\cdot\frac{x^2+3x+\left(\theta^2+2\theta+3\right)}{(\theta+1)^{x+3}}\;;x=0,1,2,...,\theta>0 \tag{1.1}$$

Moments and moments based measures, statistical properties; estimation of parameter using both the method of moments and the method of maximum likelihood and applications of PAD has been discussed by Shanker (2017). The distribution arises from the Poisson distribution when its parameter λ follows Akash distribution introduced by Shanker (2015) and defined by its probability density function (pdf)

$$f_1(x,\theta)=\frac{\theta^3}{\theta^2+2}\left(1+x^2\right)e^{-\theta x}\;;x>0,\theta>0 \tag{1.2}$$

The pdf (1.2) is a convex combination of exponential (θ) and gamma $(3,\theta)$ distributions. Shanker (2015) discussed statistical properties including moments based coefficients, hazard rate function, mean residual life function, mean deviations, stochastic ordering, Renyi entropy measure, order statistics, Bonferroni and Lorenz curves, stress- strength reliability, along with estimation of parameter and applications to model lifetime data from biomedical science and engineering.

The first four moments about origin and the variance of PAD (1.1) obtained by Shanker (2017) are given by

$$\mu_1'=\frac{\theta^2+6}{\theta\left(\theta^2+2\right)}$$

$$\mu_2'=\frac{\theta^3+2\theta^2+6\theta+24}{\theta^2\left(\theta^2+2\right)}$$

$$\mu_3'=\frac{\theta^4+6\theta^3+12\theta^2+72\theta+120}{\theta^3\left(\theta^2+2\right)}$$

$$\mu_4'=\frac{\theta^5+14\theta^4+42\theta^3+192\theta^2+720\theta+720}{\theta^4\left(\theta^2+2\right)}$$

$$\mu_2=\sigma^2=\frac{\theta^5+\theta^4+8\theta^3+16\theta^2+12\theta+12}{\theta^2\left(\theta^2+2\right)^2}.$$

Recently Shanker *et al* (2018) proposed a generalized Akash distribution (GAD) having parameters θ and α and defined by its pdf

$$f_2(x;\theta,\alpha)=\frac{\theta^3}{\theta^2+2\alpha}\left(1+\alpha x^2\right)e^{-\theta x}\;;x>0,\ \theta>0,\ \alpha>0 \tag{1.3}$$

Its structural properties including moments, hazard rate function, mean residual life function, mean deviations, stochastic ordering, Renyi entropy measure, order statistics, Bonferroni and Lorenz curves, stress- strength reliability, estimation of parameters and applications for modeling survival time data has been discussed by Shanker *et al* (2018). It can be easily shown that at $\alpha=1$, GAD (1.3) reduces to Akash distribution (1.2).

The main purpose of this paper is to introduce a generalized Poisson- Akash distribution, a Poisson mixture of generalized Akash distribution proposed by Shanker *et al* (2018). Its moments based measures including coefficients of variation, skewness, kurtosis and index of dispersion have been derived and their nature and behavior has been discussed graphically. Its statistical properties including generating functions, hazard rate function and unimodality have been discussed. The estimation of parameters has been discussed using method of moments and the method of maximum likelihood. Applications and goodness of fit of the distribution have also been discussed through two examples of observed real count datasets and the fit has been found quite satisfactory over other discrete distributions.

2. A Generalized Poisson- Akash Distribution

Assuming that the parameter λ of the Poisson distribution follows GAD (1.3), the Poisson mixture of GAD can be obtained as

$$P_2(x;\theta,\alpha)=\int_0^\infty \frac{e^{-\lambda}\lambda^x}{\Gamma(x+1)}\frac{\theta^3}{\theta^2+2\alpha}\left(1+\alpha\lambda^2\right)e^{-\theta\lambda}d\lambda \tag{2.1}$$

$$=\frac{\theta^3}{\left(\theta^2+2\alpha\right)\Gamma(x+1)}\left[\int_0^\infty e^{-(\theta+1)\lambda}\lambda^{x+1-1}d\lambda+\alpha\int_0^\infty e^{-(\theta+1)\lambda}\lambda^{x+3-1}d\lambda\right]$$

$$=\frac{\theta^3}{\left(\theta^2+2\alpha\right)\Gamma(x+1)}\left[\frac{\Gamma(x+1)}{(\theta+1)^{x+1}}+\frac{\alpha\Gamma(x+3)}{(\theta+1)^{x+3}}\right]$$

$$=\frac{\theta^3}{\theta^2+2\alpha}\frac{\left(x^2+3x\right)\alpha+\left(\theta^2+2\theta+2\alpha+1\right)}{(\theta+1)^{x+3}};x=0,1,2,...,\theta>0,\alpha>0 \tag{2.2}$$

We would call this pmf a generalized Poisson - Akash distribution (GPAD). It can be easily verified that PAD (1.1) is a particular case of GPAD for $\alpha=1$. The nature and behavior of GPAD for varying values of the parameters θ and α have been explained graphically in figure 1.

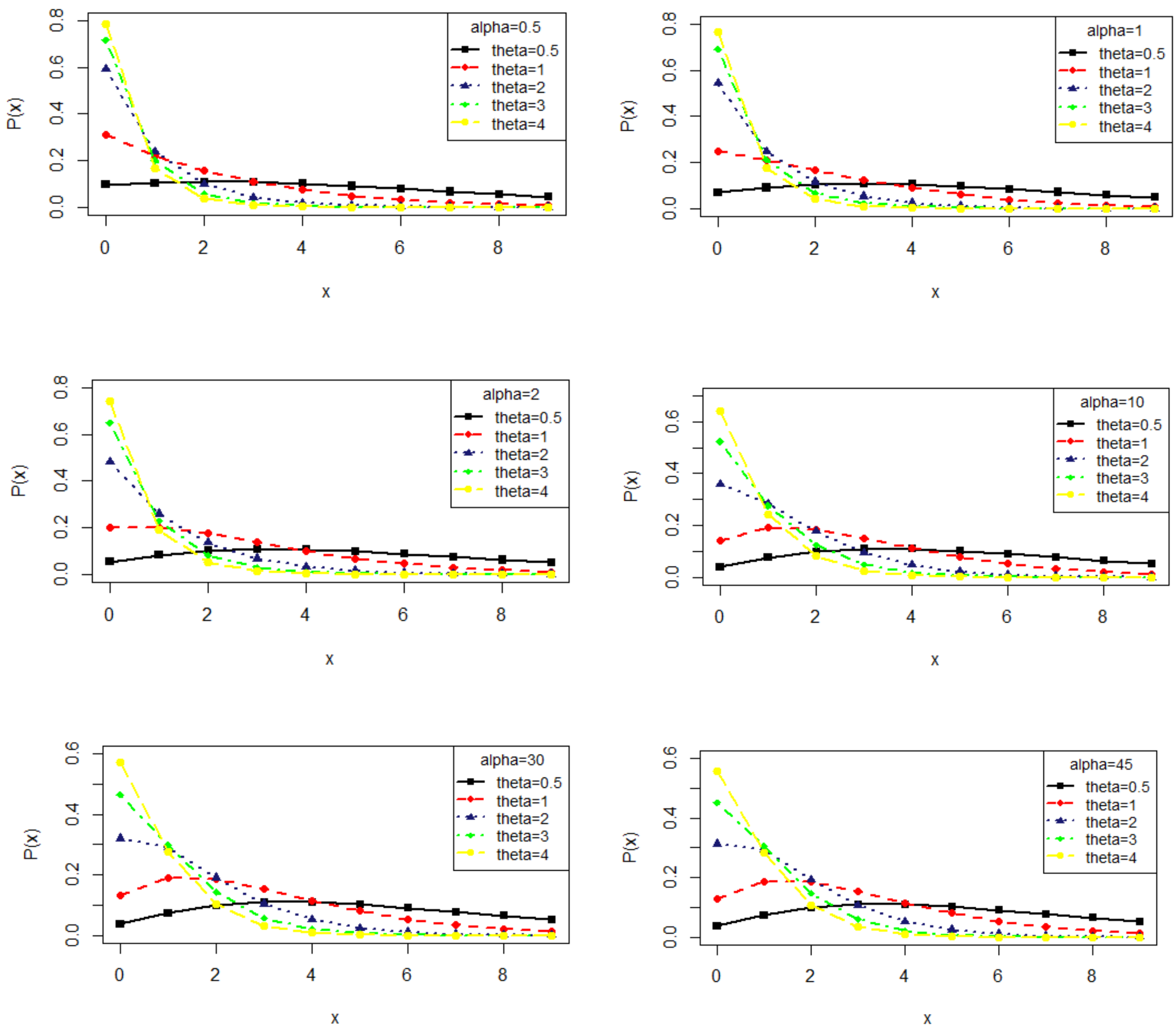

Figure 1. Probability mass function plot of GPAD for varying values of parameters θ and α

3. Moments

3.1. Factorial Moments

Using (2.1), the r th factorial moment about origin of the GPAD (2.2) can be obtained as

$$\mu_{(r)}' = E\left[E\left(X^{(r)} \mid \lambda\right)\right], \text{ where } X^{(r)} = X\left(X-1\right)\left(X-2\right)...\left(X-r+1\right)$$

$$= \int_0^\infty \left[\sum_{x=1}^\infty x^{(r)} \frac{e^{-\lambda}\lambda^x}{\Gamma(x+1)}\right] \frac{\theta^3}{\theta^2+2\alpha}\left(1+\alpha\lambda^2\right) e^{-\theta\lambda} d\lambda$$

$$= \frac{\theta^3}{\theta^2+2\alpha}\int_0^\infty \left[\lambda^r \left\{\sum_{x=r}^\infty \frac{e^{-\lambda}\lambda^{x-r}}{(x-r)!}\right\}\right]\left(1+\alpha\lambda^2\right) e^{-\theta\lambda} d\lambda$$

Taking $x-r=y$, we get

$$\mu_{(r)}' = \frac{\theta^3}{\theta^2+2\alpha}\int_0^\infty \left[\lambda^r \left\{\sum_{y=0}^\infty \frac{e^{-\lambda}\lambda^{y}}{y!}\right\}\right]\left(1+\alpha\lambda^2\right) e^{-\theta\lambda} d\lambda$$

$$= \frac{\theta^3}{\theta^2+2\alpha}\int_0^\infty \lambda^r \left(1+\alpha\lambda^2\right) e^{-\theta\lambda} d\lambda$$

$$= \frac{r!\left\{\theta^2+(r+1)(r+2)\alpha\right\}}{\theta^r\left(\theta^2+2\alpha\right)} \quad ; r=1,2,3,.... \tag{3.1.1}$$

Taking $r = 1,2,3,$ and 4 in (3.1.1), the first four factorial moments about origin of GPAD (2.2) can be obtained as

$$\mu_{(1)}' = \frac{\theta^2+6\alpha}{\theta\left(\theta^2+2\alpha\right)}$$

$$\mu_{(2)}' = \frac{2\left(\theta^2+12\alpha\right)}{\theta^2\left(\theta^2+2\alpha\right)}$$

$$\mu_{(3)}' = \frac{6\left(\theta^2+20\alpha\right)}{\theta^3\left(\theta^2+2\alpha\right)}$$

$$\mu_{(4)}' = \frac{24\left(\theta^2+30\alpha\right)}{\theta^4\left(\theta^2+2\alpha\right)}.$$

3.2. Moments about Origin (Raw moments)

The first four moments about origin, using the relationship between factorial moments about origin and the moments about origin, of GPAD (2.2) can be obtained as

$$\mu_1' = \frac{\theta^2+6\alpha}{\theta\left(\theta^2+2\alpha\right)}$$

$$\mu_2' = \frac{(\theta+2)\theta^2+6(\theta+4)\alpha}{\theta^2\left(\theta^2+2\alpha\right)}$$

$$\mu_3' = \frac{\left(\theta^2+6\theta+6\right)\theta^2+6\left(\theta^2+12\theta+20\right)\alpha}{\theta^3\left(\theta^2+2\alpha\right)}$$

$$\mu_4' = \frac{\left(\theta^3+14\theta^2+36\theta+24\right)\theta^2+6\left(\theta^3+28\theta^2+120\theta+120\right)\alpha}{\theta^4\left(\theta^2+2\alpha\right)}.$$

3.3. Moments about the Mean (Central moments)

Using the relationship $\mu_r = E\left(Y-\mu_1'\right)^r = \sum_{k=0}^{r}\binom{r}{k}\mu_k'\left(-\mu_1'\right)^{r-k}$ between moments about the mean and the moments about origin, the moments about the mean of the GPAD (2.2) can be obtained as

$$\mu_2 = \frac{(\theta+1)\theta^4+8(\theta+2)\theta^2\alpha+12(\theta+1)\alpha^2}{\theta^2\left(\theta^2+2\alpha\right)^2}$$

$$\mu_3 = \frac{\left(\theta^2+3\theta+2\right)\theta^6+2\left(5\theta^2+27\theta+30\right)\theta^4\alpha+4\left(7\theta^2+33\theta+18\right)\theta^2\alpha^2+24\left(\theta^2+3\theta+2\right)\alpha^3}{\theta^3\left(\theta^2+2\alpha\right)^3}$$

$$\mu_4 = \frac{\begin{gathered}\left(\theta^3+10\theta^2+18\theta+9\right)\theta^8+4\left(3\theta^3+47\theta^2+132\theta+96\right)\theta^6\alpha+8\left(6\theta^3+103\theta^2+258\theta+153\right)\theta^4\alpha^2\\+10\left(5\theta^3+85\theta^2+180\theta+108\right)\theta^2\alpha^3+48\left(\theta^3+16\theta^2+30\theta+15\right)\alpha^4\end{gathered}}{\theta^4\left(\theta^2+2\alpha\right)^4}$$

4. Coefficient of Variation, Skewness, Kurtosis and Index of Dispersion

The coefficient of variation $(C.V)$, coefficient of Skewness $\left(\sqrt{\beta_1}\right)$, coefficient of Kurtosis (β_2) and index of dispersion (γ) of the GPAD (2.2) are thus obtained as

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{(\theta+1)\theta^4+8(\theta+2)\theta^2\alpha+12(\theta+1)\alpha^2}}{\theta^2+6\alpha}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{\left\{\begin{gathered}\left(\theta^2+3\theta+2\right)\theta^6+2\left(5\theta^2+27\theta+30\right)\theta^4\alpha+4\left(7\theta^2+33\theta+18\right)\theta^2\alpha^2\\+24\left(\theta^2+3\theta+2\right)\alpha\end{gathered}\right\}}{\left\{(\theta+1)\theta^4+8(\theta+2)\theta^2\alpha+12(\theta+1)\alpha^2\right\}^{3/2}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{\left\{\begin{gathered}\left(\theta^3+10\theta^2+18\theta+9\right)\theta^8+4\left(3\theta^3+47\theta^2+132\theta+96\right)\theta^6\alpha\\+8\left(6\theta^3+103\theta^2+258\theta+153\right)\theta^4\alpha^2+10\left(5\theta^3+85\theta^2+180\theta+108\right)\theta^2\alpha^3\\+48\left(\theta^3+16\theta^2+30\theta+15\right)\alpha^4\end{gathered}\right\}}{\left\{(\theta+1)\theta^4+8(\theta+2)\theta^2\alpha+12(\theta+1)\alpha^2\right\}^2}$$

$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{(\theta+1)\theta^4+8(\theta+2)\theta^2\alpha+12(\theta+1)\alpha^2}{\theta\left(\theta^2+2\alpha\right)\left(\theta^2+6\alpha\right)} = \left[1+\frac{\theta^4+16\alpha\theta^2+12\alpha^2}{\theta^5+8\alpha\theta^3+12\alpha^2\theta}\right].$$

Now from the index of dispersion it is obvious that if $\alpha \to \infty$ and $\theta > 0$, then $\sigma^2 > \mu_1'$ (over dispersion) and hence GPAD is a suitable model for over dispersed data. Nature and behavior of coefficient of variation, coefficient of skewness, coefficient of kurtosis and index of dispersion of GPAD for varying values of parameters θ and α have been shown graphically in figure 2.

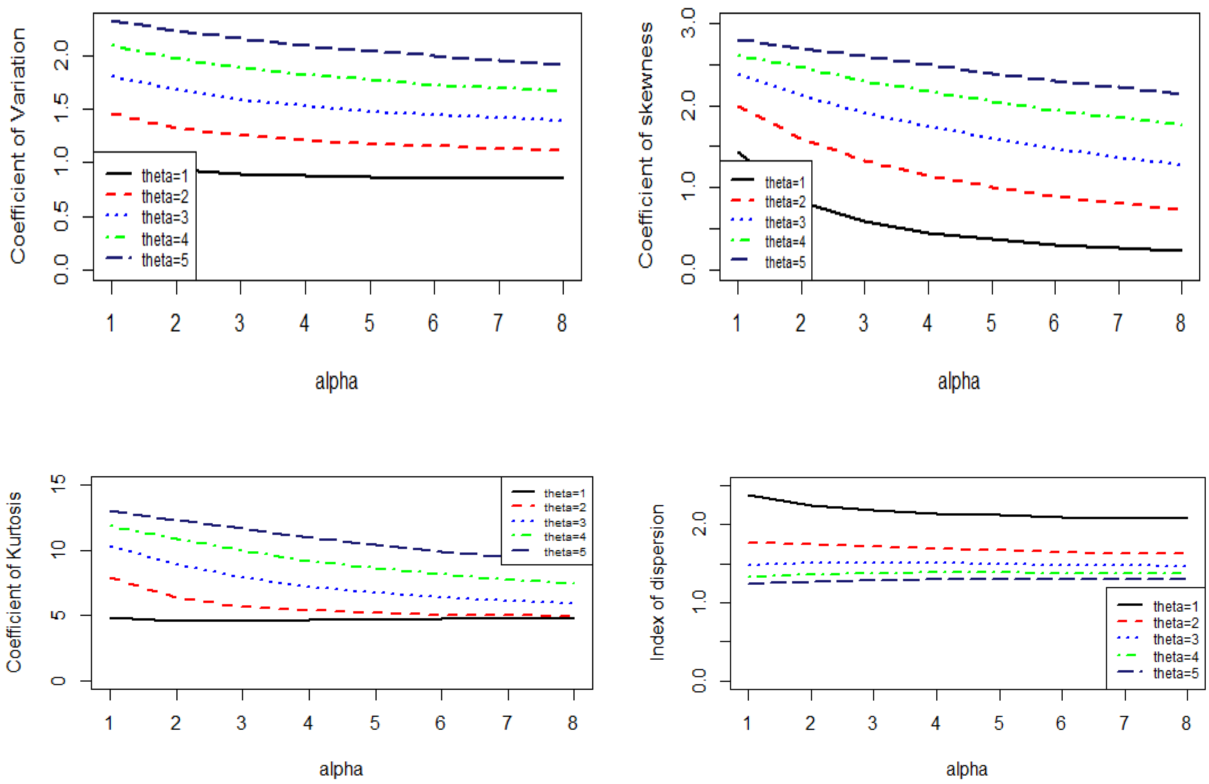

Figure 2. Nature and behavior of coefficient of variation, coefficient of skewness, coefficient of kurtosis and index of dispersion of GPAD for varying values of parameters θ and α

5. Statistical Properties

5.1. Generating Functions

The probability generating function of GPAD can be obtained as

$$P_X(t)=\frac{\theta^3}{(\theta^2+2\alpha)(\theta+1)^3}\left[\alpha\sum_{x=0}^{\infty}x^2\left(\frac{t}{\theta+1}\right)^x+3\alpha\sum_{x=0}^{\infty}x\left(\frac{t}{\theta+1}\right)^x+\left(\theta^2+2\theta+2\alpha+1\right)\sum_{x=0}^{\infty}\left(\frac{t}{\theta+1}\right)^x\right]$$

$$=\frac{\theta^3}{(\theta^2+2\alpha)(\theta+1)^3}\left[\frac{\alpha\left\{t(\theta+1-t)+2t^2\right\}(\theta+1)}{(\theta+1-t)^3}+\frac{3\alpha t(\theta+1)}{(\theta+1-t)^2}+\frac{(\theta^2+2\theta+2\alpha+1)(\theta+1)}{(\theta+1-t)}\right]$$

$$=\frac{\theta^3}{(\theta^2+2\alpha)(\theta+1)^2}\left[\frac{\alpha\left\{t(\theta+1-t)+2t^2\right\}}{(\theta+1-t)^3}+\frac{3\alpha t}{(\theta+1-t)^2}+\frac{(\theta^2+2\theta+2\alpha+1)}{(\theta+1-t)}\right]$$

$$=\frac{\theta^3}{(\theta^2+2\alpha)(\theta+1)^2}\left[\frac{2\alpha t^2}{(\theta+1-t)^3}+\frac{4\alpha t}{(\theta+1-t)^2}+\frac{(\theta^2+2\theta+2\alpha+1)}{(\theta+1-t)}\right]$$

The moment generating function of GPAD is thus given by

$$M_X(t)=\frac{\theta^3}{(\theta^2+2\alpha)(\theta+1)^2}\left[\frac{2\alpha e^{2t}}{(\theta+1-e^t)^3}+\frac{4\alpha e^t}{(\theta+1-e^t)^2}+\frac{(\theta^2+2\theta+2\alpha+1)}{(\theta+1-e^t)}\right]$$

5.2. Increasing Hazard Rate and Unimodality

We have

$$\frac{P_2\left(x+1;\theta,\alpha\right)}{P_2\left(x;\theta,\alpha\right)}=\left(\frac{1}{\theta+1}\right)\left[1+\frac{2\left(x+2\right)\alpha}{\left(x^2+3x\right)\alpha+\left(\theta^2+2\theta+2\alpha+1\right)}\right].$$

It can be easily verified that this is a decreasing function in x, and hence $P_2\left(x;\theta,\alpha\right)$ is log-concave. Now using the results of relationship between log-concavity, unimodality and increasing hazard rate (IHR) of discrete distributions available in Grandell (1997), it can concluded that GPAD (2.2) has an increasing hazard rate and is unimodal.

6. Parameter Estimation

6.1. Method of Moments

Since GPAD has two parameters to be estimated, taking the first two moments about origin, we get

$$\frac{\mu_2'-\mu_1'}{\left(\mu_1'\right)^2}=\frac{2\left(\theta^2+12\alpha\right)\left(\theta^2+2\alpha\right)}{\left(\theta^2+6\alpha\right)^2}=k(\text{say}).$$

Assuming $\theta^2=b\alpha$, we get

$$\frac{2\left(b+12\right)\left(b+2\right)}{\left(b+6\right)^2}=k.$$

This gives a quadratic equation in b as

$$\left(2-k\right)b^2+\left(28-12k\right)b+\left(48-36k\right)=0.$$

Replacing the first population moment about origin and the second population moment about origin with their respective sample moments, an estimate of k can be obtained and substituting the value of k in the above equation, an estimate of b can be obtained. Again, replacing the population mean with the corresponding sample mean and taking $\theta^2=b\alpha$ in $\mu_1'=\dfrac{\theta^2+6\alpha}{\theta\left(\theta^2+2\alpha\right)}$, we get $\overline{x}=\dfrac{b+6}{\theta\left(b+2\right)}$, which gives MOME $\tilde{\theta}$ of θ as $\tilde{\theta}=\dfrac{b+6}{\left(b+2\right)\overline{x}}$.

Thus MOME $\tilde{\alpha}$ of α can be expressed as $\tilde{\alpha}=\dfrac{\left(b+6\right)^2}{b\left(b+2\right)^2\left(\overline{x}\right)^2}$.

6.2. Maximum Likelihood Estimation

Let $\left(x_1,x_2,...,x_n\right)$ be a random sample of size n from the GPAD (2.2) and let f_x be the observed frequency in the sample corresponding to $X=x$ $(x=1,2,3,...,k)$ such that $\sum_{x=1}^{k}f_x=n$, where k is the largest observed value having non-zero frequency. The log- likelihood function of GPAD (2.2) can be given by

$$\log L=n\left[3\log\theta-\log\left(\theta^2+2\alpha\right)\right]-\sum_{x=1}^{k}f_x\left(x+3\right)\log\left(\theta+1\right)$$
$$+\sum_{x=1}^{k}f_x\log\left[\left(x^2+3x\right)\alpha+\left(\theta^2+2\theta+2\alpha+1\right)\right]$$

The maximum likelihood estimates $\left(\hat{\theta},\hat{\alpha}\right)$ of parameters $\left(\theta,\alpha\right)$ of GPAD (2.2) is the solutions of the following log-likelihood equations

$$\frac{\partial \log L}{\partial \theta}=\frac{3n}{\theta}-\frac{2n\theta}{\theta^2+2\alpha}-\frac{n(\bar{x}+3)}{\theta+1}+\sum_{x=1}^{k}\frac{2(\theta+1)f_x}{(x^2+3x)\alpha+(\theta^2+2\theta+2\alpha+1)}=0$$

$$\frac{\partial \log L}{\partial \alpha}=-\frac{2n}{\theta^2+2\alpha}+\sum_{x=1}^{k}\frac{(x^2+3x+2)f_x}{(x^2+3x)\alpha+(\theta^2+2\theta+2\alpha+1)}=0 .$$

where $\bar{x}$ is the sample mean. These two log- likelihood equations do not seem to be solved directly because they are not in closed forms. These two log-likelihood equations can be solved iteratively using R-software till sufficiently close values of $\hat{\theta}$ and $\hat{\alpha}$ are obtained. The initial values of parameters are taken as $\hat{\theta}=0.5$ and $\hat{\alpha}=0.5$.

7. Applications

The GPAD has been fitted to two count datasets from biological sciences to test its goodness of fit over Poisson distribution (PD), Poisson-Lindley distribution (PLD) and Poisson-Akash distribution (PAD). The maximum likelihood estimate (MLE) has been used to fit the GPAD. The first dataset is the number of Student's historic data on Haemocytometer counts of yeast cells available in Gosset (1908) and the second data set is the number of European corn- borer of Mc. Guire *et al* (1957). The fitted plots of distributions for datasets in tables 1 and 2 are presented in figure 3. It is clear from the goodness of fit of GPAD and from the fitted plots of distributions that GPAD gives much closer fit than PD, PLD, and PAD and hence it can be considered as an important distribution in ecology.

Table 1. Observed and expected number of Haemocytometer yeast cell counts per square observed by Gosset (1908)

Number of yeast cells per square	Observed frequency	Expected frequency			
		PD	PLD	PAD	GPAD
0	213	202.1	234.0	236.8	220.2
1	128	138.0	99.4	95.6	114.8
2	37	47.1	40.5	39.9	44.3
3	18	10.7	16.0	16.6	14.6
4	3	1.8	6.2	6.7	4.4
5	1	0.2	2.4	2.7	1.2
6	0	0.1	1.5	1.7	0.5
Total		400.0	400.0	400.0	400.0
ML Estimates		$\hat{\theta}=0.6825$	$\hat{\theta}=1.9502$	$\hat{\theta}=2.2603$	$\hat{\theta}=3.97589$ $\hat{\alpha}=48.41345$
χ^2		10.08	11.04	14.68	3.03
d.f.		2	2	2	1
p-value		0.0065	0.0040	0.0006	0.0817
$-2\log L$		899.00	905.23	909.34	894.41
AIC		901.00	907.23	911.34	898.41

Table 2. Observed and expected number of European corn- borer of Mc. Guire *et al* (1957)

Number of corn- borer per plant	Observed frequency	Expected frequency			
		PD	PLD	PAD	GPAD
0	188	169.4	194.0	196.3	186.7
1	83	109.8	79.5	76.5	87.6
2	36	35.6	31.3	30.8	33.6
3	14	7.8	12.0	12.4	11.3
4	2	1.2	4.5	4.9	3.5
5	1	0.2	2.7	3.1	1.3
Total	324	324.0	324.0	324.0	324.0
ML Estimates		$\hat{\theta}=0.6481$	$\hat{\theta}=2.0432$	$\hat{\theta}=2.3451$	$\hat{\theta}=3.78371$ $\hat{\alpha}=18.8827$

χ^2		15.19	1.29	2.33	0.47
d.f.		2	2	2	1
p-value		0.0005	0.5247	0.3119	0.4930
$-2\log L$		724.49	714.09	715.69	711.34
AIC		726.49	716.09	719.69	715.34

The fitted plot of distributions for datasets in table 1 and 2 are shown in figure 3. It is obvious from the goodness of fit in tables 1 and 2 and the fitted plot of distributions in figure 3 that GPAD gives better fit.

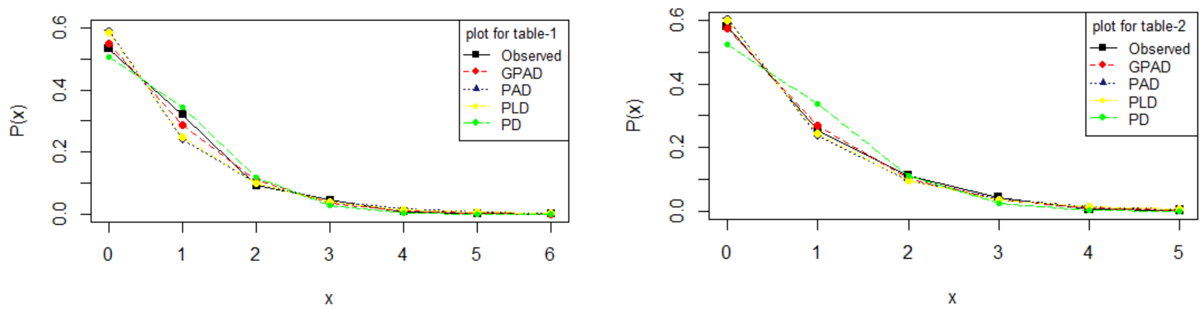

Figure 3. Fitted plots of distributions for datasets 1 and 2

8. Concluding Remarks

This paper proposes a generalized Poisson- Akash distribution which includes Poisson-Akash distribution as a particular case. Its moments and moments based measures have been derived and studied. Some statistical properties have been discussed. Method of moments and the method of maximum likelihood have been discussed for estimating parameters of the distribution. Finally, applications of the proposed distribution have been explained through two count datasets from biological sciences and the goodness of fit has been found quite satisfactory over PD, PLD and PAD.

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