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ARTICLE

A Comment on Dehez and Tellone, "Data games: sharing public goods with exclusion"

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This comment shows that the data cost game introduced in Dehez and Tellone (*Journal of Public Economic Theory*, 2013) coincides with the nonadditive component of the library cost game studied in Driessen, Khmelnitskaya, and Sales (*TOP*, 2012) where the core, nucleolus, and Shapley value were also investigated.

Dehez and Tellone (2013) introduced and studied data games, which provide a cooperative game-theoretic model of data sharing. We show that a data game coincides with the nonadditive component of a library game, introduced in Sales (2002) and studied in Driessen, Khmelnitskaya, and Sales (2012). In Driessen *et al.* (2012), it is proved that the nonadditive component of a library game belongs to the class of 1-concave games introduced in Driessen and Tijs (1983) and Driessen (1985), and as a corollary to that and known properties of the solutions of 1-concave games it is shown that the core of this game is always nonempty and has a regular simplex structure and that the nucleolus is linear and coincides with the τ-value and the barycenter of the core. Due to Proposition 1 below, these results can be extended straightforwardly to the data games as well, which covers the results of sections 4 and 5 in Dehez and Tellone (2013).

Library games model situations in which a university library consortium has to assign prices to its members to pay for a joint subscription for electronic scientific journals. A library game is determined by a finite set N of n university libraries (players), a finite set G of m electronic journals (goods), a demand matrix $D = (d_{ij})_{\substack{i\in N\\ j\in G}}$, where $d_{ij} \geq 0$ is the number of paper copies of the jth journal previously demanded by the ith university, a cost $c_j \geq 0$ per copy of jth journal in the prior demand, and a discount parameter $\alpha \in [0, 1]$ for journals that were not previously demanded. The characteristic function of the library cost game is given by

$$c^l(S) = \sum_{j\in G}\Big[\sum_{i\in S} d_{ij}\Big]c_j + \sum_{\substack{j\in G\\ \sum_{i\in S} d_{ij}=0}} \alpha\, c_j, \qquad \text{for all } \ S \subseteq N.$$

The characteristic function c^l is a sum of an additive function and multiplied by α nonadditive function $\bar{c}^l$ given by

$$\bar{c}^l(S) = \sum_{\substack{j\in G\\ \sum_{i\in S} d_{ij}=0}} c_j, \qquad \text{for all } \ S \subseteq N.$$

Observe that if for every $j \in G$ there is $i \in N$ with $d_{ij} > 0$, then $\bar{c}^l(N) = 0$.

A data-sharing game is determined by a finite set N of n players, a finite set G of data of m types (public goods), a collection of sets $G_i \subseteq G$, $i \in N$, that specify the types of data held by each player, and a vector of costs c_j of reproducing the data of type $j \in G$. It is also assumed that $\cup_{i\in N} G_i = G$. Then the characteristic function c^d of the data cost game on the player set N is given by

$$c^d(S) = \sum_{j\in G\setminus G_S} c_j, \qquad \text{for all} \quad S \subseteq N,$$

where $G_S = \cup_{i\in S} G_i$ for all $S \subseteq N$. Notice that $c^d(N) = 0$.

Proposition 1. *Assume that in both games, a data game c^d and a library game c^l, the sets N, G, and the cost vectors c_j, $j \in G$, are the same. Assume also that for every $j \in G$ there is $i \in N$ with $d_{ij} > 0$ and $d_{ij} = 0$ iff $j \notin G_i$. Then c^d coincides with $\bar{c}^l$.*

Proof. The assumption that for every $j \in G$ there is $i \in N$ with $d_{ij} > 0$ and $d_{ij} = 0$ iff $j \notin G_i$ is equivalent to the assumption $\cup_{i\in N} G_i = G$ required for c^d by definition of a data game. To prove the coincidence of games c^d and $\bar{c}^l$, it suffices to show that for any coalition $S \subseteq N$, the set $\{j \in G \mid \sum_{i\in S} d_{ij} = 0\}$ in the library game coincides with the set $G \setminus G_S$ in the data game. Clearly, for any $j \in G$, the coalitional constraint $\sum_{i\in S} d_{ij} = 0$ is equivalent to individual constraints $d_{ij} = 0$ for all $i \in S$, which is the same as $j \notin G_i$ for all $i \in S$, i.e., $j \notin G_S$, or equivalently, $j \in G \setminus G_S$. ■

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