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Last Time Buy and repair decisions for fast moving parts

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ABSTRACT

Spare part availability is essential for advanced capital goods with a long service period. Sourcing becomes challenging once the production of spare parts ceases, while the remaining service period is still long. In this paper, we focus on fast moving parts with repair of failed parts as an alternative supply option. We proceed from the methodology of Behfard et al. (2015) for slow movers, which assumes discrete demand distributions and therefore leads to excessive computation times for fast movers. We find that the use of continuous demand distributions requires significant modifications, both for the approximation of the performance indicators and for the optimization of the repair policy. We develop accurate heuristics to find the near-optimal Last Time Buy (LTB) quantity and the repair policy that we apply for two control policies: pull return - push repair, and push return - pull repair. We show that pull return - push repair is better to follow if return lead times are short and return costs are low. For long return lead times, we find that when the return cost exceeds 35%–40% of the part's value, push return - pull repair becomes more cost efficient. We also show that for relatively high demand of spare parts over the planning period (>300 for a 10 years planning period) the continuous model is a good approximation for the discrete model of Behfard et al. (2015). In addition, the computation time of our method is much lower then.

1. Introduction

Downtime of advanced capital goods may have serious consequences in terms of costs, quality of service, and safety risks. Spare parts are needed for keeping up the system during the life cycle of typically several decades. At some point in time within the life cycle, the supplier of a spare part may decide to stop the production because of new technological developments and/or decreasing demand volumes. Then, the service provider responsible for keeping up the system faces the challenge how to cover future demand until the end of the *remaining* service period (after supply disruption) that may last up to 15 years based on the industry observations (Koopman (2011)). One option is to place a so-called *Last Time Buy* (LTB) at the supplier just before supply has ceased. Making LTB decisions is inevitably hard because of the high demand uncertainty. Key causes are uncertainty in: (i) the size of the installed base and its evolvement, and (ii) the parts failure rate over time, which may be affected by usage patterns and wear-out. Though a proper demand forecast for this long period is still point of interest for further research, there are some methods that are being used in practice such as: curve fitting based on demand history, or point forecasts based on the size of the installed base and its projected evolution over the planning period (Koopman, 2011). To avoid a stock-out, a high safety inventory is required. This typically causes high excess stock at the end of the service period, which has to be disposed at a low salvage value. To mitigate stock-out risks, companies consider alternative sourcing options, e.g., repair of failed parts, retrieving parts from phased-out systems, buying parts from a secondary market, or part redesign. This (partly) postpones the decision on the number of required parts to source, and reduces the inventory levels required. In this paper, we consider the repair of failed parts that are returned from the field as the alternative sourcing option. Our collaboration with industrial partners (computer machinery, printing machines, and lithography systems) shows that repair of failed parts is one of the most accessible alternatives in terms of quantity and controllability of the process. The investigation of the other options is a point of interest for future research.

This problem is related to inventory management and reusing parts sent back from the field. The structure of the replenishment policy and computation of the related parameters are a.o. discussed in , Inderfurth (1997), Van Houtum and Zijm (1991), Zipkin (2000), Kiesmuller and Minner (2003). Still, optimizing the LTB order quantity and evaluating relevant performance indicators is not straightforward, since all the parameters are interdependent (e.g., dependency of the LTB quantity on the ordering policy and the sourcing options).

Behfard et al. (2015) develop an approximate method for slow

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moving parts with a non-stationary, discrete demand distribution. They find a near-optimal LTB quantity and base stock repair policy. When the demand level increases (e.g. larger than 300 over the remaining life cycle), the computation time quickly explodes, in particular when repair lead times are long. We will show in Section 5 (Fig. 9) how the computation time explodes for high demand levels in the model Behfard et al. (2015). Therefore, their method is not suitable for fast movers. We have observed in industry (computer hardware, printing systems) that some items may still have demand volumes up to several hundreds or even more than thousand after regular supply has been discontinued. First, this means that the method of Behfard et al. (2015) is not suitable for such volumes. Second, it shows that it is reasonable to approximate the discrete demand distribution by a continuous distribution. Silver et al. (2017) advice to use a continuous distribution in inventory systems when the mean during lead time exceeds 10. There are plenty of cases in the industrial examples above where these conditions are satisfied. For instance, we have observed the following real cases in Table 1 for the parts used in printing systems:

Table 1
Total expected demand for fast moving parts (real industry cases).

Remaining service period after LTB decision (years)	Total expected demand until the end of the service period
5	4327
10	2502

A straightforward approach is to apply optimization by simulation over all the possible scenarios (cf., Law (2015)). However, due to the number of decision variables (i.e., the time-dependent base stock levels, and the LTB quantity), this will typically require a very long computation time. For example, the simulation time of 100.000 replications for a scenario with a given LTB quantity and base stock levels takes 10 min Therefore, as an alternative we propose to modify the model of Behfard et al. (2015) by replacing discrete demand distributions by continuous distributions. It will turn out that this requires substantial modifications in the approximate performance analysis and the optimization procedure.

In the next section, we discuss the related literature. We present our model, assumptions, and the notation in Section 3. Section 4 describes the basic approach and discusses where complications arise due to the use of continuous demand distributions, followed by the details on our optimization method. We validate the accuracy of our approximations as well as our optimization in Section 5 and derive some insights from the numerical experiments. Finally, we summarize our main conclusions, and give directions for future research in Section 6.

2. Literature review

There is extensive literature about spare part management. A key method is METRIC for initial spare parts stocking and restocking in the steady state situation, c.f., Sherbrooke (2004), and Van Houtum and Kranenburg (2015). This method is not suitable when supply is discontinued within a finite planning period.

The specific literature on LTB decisions for spare parts can be classified according to the sourcing options that are used to satisfy demand after ceasing production. Some papers focus on service part demand forecast and propose methods to compute the all-time requirement for a part (Moore (1971); Ritchie and Wilcox (1977); Fortuin (1980); and Hong et al. (2008)). Teunter and Klein Haneveld (1998, 2002) study the conditions under which a multi-component LTB problem can be approximately decomposed into single component problems. Next, they determine the ordering policy and the optimal LTB quantity when there is an option to order again from external suppliers after the LTB decision, but at a much higher price. Cattani and Souza (2003) study the benefits of delaying an LTB order at the expense of extra costs for extending the production period. Bradley and Guerrero (2009) study sequential LTB decisions for various parts of a *single product* by making a decision for only one part at each decision point until the end of the life cycle of the product.

Other papers take into account additional sources of supply than LTB as mentioned in the introduction. Teunter and Fortuin (1999) consider retrieving of the parts from returned systems. They assume a push ordering policy due to negligible remanufacturing cost, and investigate a remove-down-to policy to dispose remanufactured parts in order to avoid excessive holding costs. They model demand as a discrete random variable, but an extension to continuous demand distributions is possible. Van Kooten and Tan (2009) consider the repair option (with possible condemnation during the repair process) next to an LTB order, and build a Markovian model. Krikke and Van der Laan (2011) study the repair of failed parts and retrieving parts from phased-out system next to an LTB order. Timing and quantity of the returns is deterministic and known. As service level, they use a maximum stock-out probability just before a return occurs. The last two papers both assume Poisson demand, and propose a Normal approximation for larger problem instances. Pourakbar et al. (2012) study the point in time at which repair is not worth considering anymore due to price erosion of the original product. Instead, the failed product is replaced by a new one. Pourakbar et al. (2014) consider phased-out systems as the supply source and extend the work of Krikke and Van der Laan (2011) with uncertainty in timing and quantity of the phased-out systems. The latter two papers are based on the assumption that demand is Poisson distributed. Inderfurth and Kleber (2013) consider extra production and retrieving parts from phased-out systems as alternative supply sources. Though they formulate the model for discrete demand, an extension to continuous demand is feasible.

There are only two papers that consider LTB decisions explicitly for continuous demand distribution with alternative sourcing options. Inderfurth and Mukherjee (2008) formulate a stochastic model with the option of extra dedicated production runs and retrieving parts from returned systems next to an LTB order. They assume that demand and returns are independent random variables, and use a newsvendor model to approximately determine the order-up-to levels. Shen and Willems (2014) study as supply alternatives: (i) part substitution (by a technically suitable alternative part), and (ii) line redesign (modifying the production line to accommodate a new part). To the best of our knowledge, there is no previous paper in which an LTB order decision with repair alternative has been studied for fast moving parts under a non-stationary continuous demand distribution over a finite planning horizon.

In recent years, quite some attention has been paid to the problem of jointly controlling manufacturing of new products and reusing returned products. An important issue is how to control the flow of the returned parts. Under a pull policy, a part is processed on request, whereas under a push policy processing occurs immediately. There are some situations with a short return lead time or high return costs in which it does not make sense to send all the parts immediately. Then, a pull policy is more appropriate. Nevertheless, there are other situations with low return costs, in which immediate return is better and then the products are quickly available for further processing (c.f. Van der Laan and Teunter (2006); Vercraene et al. (2014)).

In this paper, we propose accurate approximations for the total costs and parts availability under a given LTB quantity and a given set of repair-up-to levels in the remaining service period. Based on these approximations, we develop a computationally efficient optimization heuristic to determine the repair policy and the LTB quantity for fast moving parts under continuous demand. Furthermore, we consider two control policies, “push return - pull repair” and “pull return - push repair” and study the impact of the input parameters on the system performance and the conditions under which one of the two policies outperforms the other. Finally, we show that the continuous model can be used as a good approximation for the discrete model in Behfard et al. (2015) under specific conditions.

3. Model and notation

We first explain our model and assumptions, and then introduce the notation.

3.1. Model description and assumptions

We consider an LTB order and as an alternative the repair of failed parts where there is no fixed cost for repair jobs. The repaired parts are as good as new with the same value and failure behavior as new parts. Demand is non-stationary over time and follows a continuous distribution. To facilitate the analysis, we divide the service period T in equally sized discrete intervals. The demands in the different intervals are independent and can be satisfied by both new and repaired parts. Each unit of demand represents a failed part that can be returned and repaired with a certain probability. Arrival of ready-to-repair parts (from the return process) and ready-to-use parts (from the repair process) takes place only after the relevant lead time. Unmet demand is backordered during the finite service period, since contractual obligations enforce that the installed base should always be maintained until the end of the service period. During regular part supply, lost sales models are often applied, where lost sales refers to the use of emergency shipments from another location in the supply chain. As such options typically do not exist anymore after stopping regular supply and demand has to be satisfied one way or another, we have to rely upon backordering.

It is known that dynamic inventory models with no fixed ordering costs and perfect repairs (100% repair yield) follow a *base stock policy* (Zipkin (2000)). We assume that an accurate inspection on the reparability of failed parts in the field is feasible. That is, already in the field it is known which parts can be repaired and which not, and only the first category will be returned. The rest will be scrapped at negligible costs. Therefore, we can assume that the repair yield is 100% and apply a base stock policy. We have observed in practice that the initial diagnosis tends to be good (even though not perfect), and so the repair yield is typically very high. Note that inclusion of imperfect repair in our model would seriously complicate the computation of the base stock levels and the performance indicators.

The *objective* of our model is to find the LTB order quantity jointly with the dynamic repair policy that minimizes the total expected relevant costs over the remaining service period. The total relevant costs consist of procurement, holding, repair, return, and shortage costs, minus the salvage value of the parts remaining at the end of the service period. The *decision variables* of the model are the LTB quantity (Q) and the base stock levels ($s_t,\ t = 1, 2, \dots, T$) for the repair decisions over the remaining service period. That is, at the start of each interval, we may start a number of repair jobs that raises the inventory position to s_t , insofar sufficient ready-to-repair parts are available. As side results, we give as service levels per interval: (i) the probability of running out of stock, and (ii) the cumulative fill rate, i.e., the fraction of demand in the intervals 1,..,t that is satisfied from stock on shelf.

For ease of presentation, we assume that the initial stock level is zero, and so are the initial repair and return pipelines. However, the extension to positive initial levels is straightforward and will be explained at the end of Section 4.4.

We consider the following assumptions:

1) Both the return lead time and the repair lead time are deterministic and expressed as an integer number of intervals. It is a realistic assumption, as we have observed in the industry that typically an agreement is made with the third party repair shops to deliver the repaired parts within a predefined duration. This is in connection with our model, as we consider an accurate inspection on reparability of the failed parts prior returning from the field and only technically repairable parts are returned. Stochastic lead times significantly increases complexity of the analysis due to introducing another factor of uncertainty on calculating the inventory position after reordering and evaluation of the performance indicators while dealing with the capacity restrictions.
2) Ready-to-repair parts that are not repaired in a certain interval are kept on stock for possible future repair at negligible costs; this is supported by current practice where failed parts are typically valued at, e.g., €0,01 per piece.

We consider two different control policies for the return of failed parts and starting the repair jobs:

I. *A push policy for the return of failed parts from the field, and a pull policy for the repair of ready-to-repair parts.* That is, all failed parts that are repairable will be returned immediately after failure. These parts will be kept on stock at the repair facility, until the *installation* inventory position of ready-to-use parts drops below the base stock level for the corresponding period. Here, the installation inventory position is defined as the inventory level of ready-to-use parts (on-hand minus backorders), plus the number of failed parts being repaired, *excluding* the failed parts waiting for repair. Then, a number of repairs will be started to raise the inventory position of ready-to-use parts to the base stock level, insofar sufficient ready-to-repair parts are available. Here, the inventory replenishment lead time is the throughput time of the repair process only.

II *A pull policy for the return of failed parts from the field, and a push policy for the repair of ready-to-repair parts.* That is, the return of failed parts from the field is postponed until there is a need for ready-to-use parts. We can see this as a two-echelon serial inventory system, consisting of a first stock point with ready-to-repair failed parts, and a second stock point with ready-to-use parts. Because of the push policy for repair, the inventory at the second stock point is zero. In this setting, a number of parts are returned from the field that raises the *echelon* inventory position of ready-to-use parts at the repair shop to its base stock level, insofar sufficient ready-to-repair parts are available in the field. Here, the echelon inventory position is defined as the inventory level of ready-to-use parts, plus the number of failed parts being repaired, plus the number of failed parts being returned, *excluding* the failed parts in the field waiting for return. When a failed part arrives at the repair shop, repair is immediately started. The replenishment lead time of the second stock point with ready-to-use parts equals the sum of the return lead time and the repair lead time.

Because of the similarity between the two scenarios, we need exactly the same methodology for the analysis of both scenarios with as key differences: (i) the definition of inventory position, (ii) the definition of replenishment lead time, (iii) calculation of the expected number of returned failed parts that are dependent on the decision variables (base stock levels, LTB quantity) in Scenario II, whereas that is constant in Scenario I.

We first present the total cost function for the both scenarios, but for ease of presentation, we elaborate the model and the analysis only for Scenario I in Section 4. Note that the analysis of Scenario II is identical to that of Scenario I if we modify the definition of inventory position and replenishment lead time as explained above.

For both scenarios, we have a sequence of the events in each time interval (at the start, during, and at the end of an interval) as shown in Fig. 1:

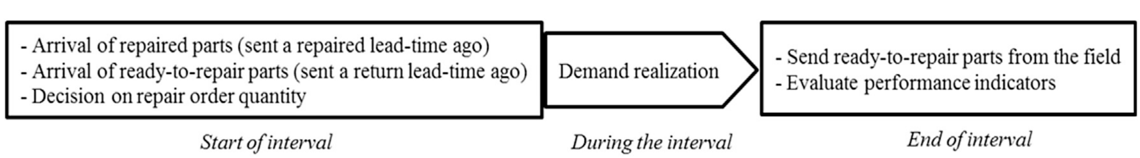

Fig. 1. Sequence of events at each time interval.

3.2. Notation

In the remainder of this paper, we use the following notation:

Input parameters:
t : interval index, $t \in \{1, 2, \dots, T\}$ with T the total number of intervals.
l_1: repair lead time.
l_2: return lead time.
h: holding cost per ready-to-use part (new or repaired) per time interval.
c_p: purchasing cost of a new part at the start of the planning period, $t = 1$.
$c_{r,t}$: repair cost for each repair job started in interval t. This cost may depend on time, for example because parts become older and more difficult to repair, or because lower repair resource utilization over time increases costs.
p_t: return cost for each part returned in interval t.
sv: salvage value per ready-to-use part remaining in stock at the end of the service period; this value may be negative, e.g., if environmental friendly disposal of the parts yields extra costs.
$c_{b,t}$: shortage cost per ready-to-use part at the end of interval t.
y_t: return yield, i.e., the fraction of failed parts that are returned from the field at the end of interval t and that are ready-to-repair.
State variables and derived states:
I_t: inventory position of the ready-to-use parts before the reorder decision at the beginning of interval t. Note that I_t is the only state variable and the rest can be derived based on it.
S_t: inventory position of ready-to-use parts after reordering at the start of interval t.
OH_t: on-hand inventory of ready-to-use parts at the end of interval t.
BO_t: shortage of ready-to-use parts at the end of interval t.
R_t: number of repair jobs started at the start of interval t.
K_t: number of ready-to-repair parts at the start of interval t.
Note that the definitions of R_t and S_t differ for Scenario (I) and (II).
Auxiliary variables:
D_t: demand in time interval t, a continuous random variable with density function $f_t(x)$ and cumulative distribution function $F_t(x)$.
$D_{t,\tau}$: cumulative demand in the intervals $t,..,\tau$, a continuous random variable with density function $f_{t,\tau}(x)$ and cumulative distribution function $F_{t,\tau}(x)$.
$r_t(D_t)$: random number of failed parts D_t that can be returned from the field at the end of interval t; we assume that it follows a binomial distribution with success probability y_t; demand is rounded to the nearest integer in case of real values. These parts are available for repair process at the beginning of interval $t + l_2$.
Decision variables:
s_t^*: optimal base stock level of ready-to-use parts at the beginning of interval t; $\boldsymbol{s}^* = (s_1^*, .., s_T^*)$; $\boldsymbol{s}^*$ is the vector containing the optimal base stock levels.
Q: number of parts procured at the start of the planning period as Last Time Buy quantity.
Performance measurements:
$TRC(Q, \boldsymbol{s}^*)$: total expected relevant costs as function of LTB quantity and base stock levels; it consists of the procurement, expected repair, holding, shortage, and return minus the salvage value.
β_t: cumulative fill rate in the interval 1,.., t.
α_t: probability of running out of stock at the end of interval t.

3.3. Model presentation

Our goal is to minimize the total expected relevant costs, given in (1), by determining the optimal base-stock levels, $\boldsymbol{s}^* = (s_1^*, .., s_T^*)$ and the LTB quantity Q.$TRC_I(Q, \boldsymbol{s}^*)$ consists of the purchasing cost of the LTB quantity at the beginning of the planning period, the expected holding, shortage and repair costs, and the return cost of the failed parts (used for repair), minus the salvage value of the remaining parts at the end of the service period. Repair jobs can be started only from interval $2 + l_2$ due to the return lead time and the first time availability of the ready-to-repair parts. Next, returning the ready-to-repair parts from the field does not make sense after interval T-l_1-l_2-1, since any repair after interval T-l_1 cannot be used before the end of the planning period. For Scenario I, we find

$$TRC_I(Q, \boldsymbol{s}^*) = Q \cdot c_p + \sum_{t=1}^{T} h \cdot E[OH_t] + \sum_{t=1}^{T} c_{b,t} \cdot E[BO_t] + \sum_{t=2+l_2}^{T-l_1} c_{r,t} \cdot E[R_t] + \sum_{t=1}^{T-l_1-l_2-1} p_t \cdot E[D_t] \cdot y_t - sv \cdot E[OH_T] \cdot \tag{1}$$

The total expected relevant costs function for Scenario II is very similar to (1). The only difference is in calculating the expected return cost that should be in connection to the repair decisions:

$$TRC_{II}(Q, \boldsymbol{s}^*) = Q \cdot c_p + \sum_{t=1}^{T} h \cdot E[OH_t] + \sum_{t=1}^{T} c_{b,t} \cdot E[BO_t] + \sum_{t=1}^{T-l_1-l_2-1} (p_t + c_{r,t+l_2+1}) \cdot E[R_t] - sv \cdot E[OH_T] \cdot \tag{2}$$

A core element of our approach is that we can determine the optimal base stock levels $\boldsymbol{s}^*$ independently of the LTB quantity Q. The key reason is that if unused failed parts are never discarded, the base stock levels for infinite supply and finite supply are equal (Behfard et al. (2015)). Once we have found the base stock levels, we can perform a numerical search over Q to find the minimum total expected relevant costs. In Section 4.3, we will show that the evaluation of the cost functions is not trivial and we need some approximations for the various cost elements. Due to the capacity restrictions (limited number of ready-to-repair parts), there is uncertainty in the inventory position after reordering that is the key in the evaluation of the total expected relevant costs $TRC(Q, \boldsymbol{s}^*)$. This uncertainty introduces strong correlations among the different random variables through the planning period.

To find the optimal base stock levels $\boldsymbol{s}^*$, we use the stochastic dynamic programming (SDP) recursions as in (3) and (4). The ***decision variables*** are the base stock levels s_t at the beginning of each interval $t = 1,..,T$. We define the *state* as the inventory position before reordering I_t at the beginning of the *stage t*. The *action* is increasing the inventory position up-to s_t, if the inventory position before ordering I_t is lower than s_t. Applying a backward recursion, we define the *value function* $V_t(I_t)$ as the minimum expected costs from the start of interval t until the end of the planning period T. The value function in period t consists of the ordering cost, the expected holding and shortage costs a lead time later in interval $t + l_1$, plus the unconditional value function in the interval t+1 given I_{t+1}. We use the shorthand notation $X^+ = \max\{X, 0\}$. The value function $V_t(I_t)$ is connected to $V_{t+1}(I_{t+1})$ as follows:

$$V_t(I_t) = c_{r,T} \cdot (s_t - I_t) + E\left[h.(s_t - D_{t,t+l_1})^+ + c_{b,t} \cdot (D_{t,t+l_1} - s_t)^+\right] + E_{D_t}\left[V_{t+1}(I_{t+1})\right] \tag{3}$$

$$I_{t+1} = s_t - D_t \tag{4}$$

At the end of the planning horizon, we have a salvage value of the items remaining in inventory, so the value function equals $V_{T+1}(I_{T+1}) = -sv.I_{T+1}$. Note that these recursions are valid for both Scenarios (1) and (2), provided that we use the appropriate definitions of inventory position and replenishment lead time.

Solving this SDP for a *discrete* state space with limited size is straightforward. However, here we deal with *continuous* demand, and consequently the state space is continuous. In Section 4.2, we describe more details on how to adapt the SDP formulation to make it applicable for our case.

4. Approach and performance analysis

In this section, we first describe our approach to find the optimal base stock policy and the LTB quantity. Then, we elaborate on the main differences compared to the approach of Behfard et al. (2015) for discrete demand. We explain how to determine the optimal base stock levels in Section 4.2. Next in Section 4.3, we derive the expressions to evaluate the performance indicators for given values of the decision variables. We propose the algorithm to find the near-optimal LTB quantity in Section 4.4.

4.1. Approach

As stated before, we determine the base stock levels independent of the LTB quantity. This leads to the following procedure to find the optimal LTB quantity (Fig. 2):

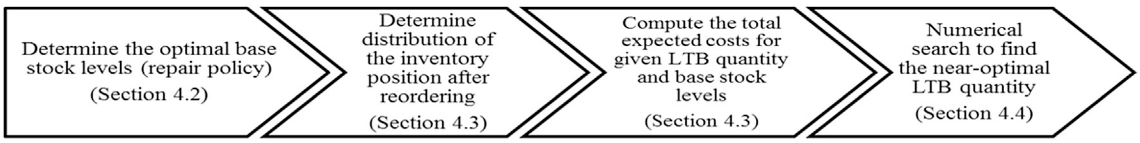

Fig. 2. Procedure to find the near-optimal LTB quantity and the repair policy.

1) We determine near-optimal base stock levels using Stochastic Dynamic Programming (SDP), assuming that there are ample ready-to-repair parts. We first describe the algorithm for zero lead times, and next explain how it can be extended to non-zero lead times. This approach is commonly used in the literature for ease of explanation (Zipkin, 2000).
2) We derive expressions for evaluating the performance indicators (i.e., total expected costs, fill rates and cycle service levels) for given values of the decision variables (LTB quantity and the base stock levels). In this analysis, the distribution of the inventory position after reordering S_t has an important role in the computation of performance indicators. S_t is a random variable and not necessarily equal to the base stock level s_t^* for two reasons. First, the number of ready-to-repair parts is finite, and so there might be insufficient number of parts to reach the base stock level. Second, the inventory position due to the LTB order is typically *higher* than the base stock levels in the first periods of the planning period. Then, repair is not initiated.
3) We apply a numerical search to find the near-optimal LTB quantity that minimizes the total expected costs.

The key differences with study of Behfard et al. (2015) due to the use of continuous demand distributions are as follows:

a) The state space is continuous. Therefore, we cannot directly apply SDP to determine the base stock levels. Therefore, we propose an approximation for computing the value functions when determining the base stock levels.
b) We use continuous approximations for the distributions of S_t ($t = 1,..,T$), the inventory position after the reorder decision at the start of interval t. Computation of the total expected costs consists of multiple double and triple integrals that we will reduce to single integrals for the sake of a fast numerical evaluation.

4.2. Base stock levels for repair decisions

There are three approaches to find the base stock levels for dynamic inventory models with infinite supply, depending on the demand behavior over time (Zipkin, 2000):

1) *The demand is constant or inclining over time*:

The optimal base stock level in interval t minimizes the current period's costs and is not influenced by future demands and costs (a so-called myopic policy). The optimal level base stock level s_t^* is the value satisfying $F_t(s_t^*) = \frac{c_{b,t}-(c_{r,t}-c_{r,t+1})}{c_{b,t}+h}$ (Zipkin, 2000).

2) *The demand is declining over time without significant drops*:

The myopic policy, as defined above, provides a near-optimal base stock level. It can be shown that this is an upper bound for the optimal base stock level.

3) *The demand is declining over time with significant drops*:

A myopic policy does not provide a good solution, and we need to use SDP.

As we observed in practice (Koopman, 2011), supply disruptions happen at the end of the life cycle while there are significant drops in demand due to phasing out of the systems, or in the middle of the life cycle while number of systems are steady (not growing). Surprisingly, we also observed that disruption might even occur early in the life cycle while the systems are still being produced and sold to the customers, so the installed base may be inclining. This appears to be not an exception. As a result, a simple myopic policy is not a good heuristic, and we have to apply SDP.

In the literature, there are other heuristics for the optimization of dynamic inventory problems, such as look-ahead policies, and linear decision rules (Truong (2014), Levi et al. (2007), Chen et al. (2008)). These heuristics are applicable to cases where the demand is non-stationary and correlated over the different periods. Heuristics are also used when information on certain variables, such as the demand distribution, is incomplete. In our case, the demand is independent over time intervals, and their distributions are known. Therefore, we only need a good approximation with continuous state space.

We have two options: (i) discretize and truncate the state space, or (ii) approximate the value function by a simple continuous function based on the evaluation of a limited number of support points. As the first option is time-consuming due to the large number of the feasible points, we focus on the second option. We use a *backward* recursion, and approximate the value functions by piecewise linear functions. Let us introduce the function $m_t(s_t)$:

$m_t(s_t)$: The minimum expected costs from the start of interval t until the end of the planning horizon when the base stock level s_t is used and the inventory position before ordering is $I_t = 0$.

We first explain the procedure to determine the base stock level in the last interval T, and then for any interval $t < T$. Recall that at the end of the planning horizon, we have a salvage value of the parts remaining in inventory, so the value function equals $V_{T+1}(I_{T+1}) = -sv.I_{T+1}$. Note that the definition of $V_t(I_t)$ is given in (3). Zipkin (2000) shows that we can find the optimal s_t^* at any interval t by minimizing $m_t(s_t)$. The relation between $m_t(s_t)$ and $V_t(I_t)$ is given in (6):

$$m_t(s_t) = c_{r,t}.s_t + E\left[h.(s_t - D_t)^+ + c_{b,t}.(D_t - s_t)^+\right] + E_{D_t}\left[V_{t+1}(s_t - D_t)\right]. \quad (5)$$

$$V_t(I_t) = \begin{cases} -c_{r,t}.I_t + m_t(s_t^*) & \text{for } I_t \le s_t^*, \\ -c_{r,t}.I_t + m_t(I_t) & \text{otherwise}. \end{cases} \quad (6)$$

For further properties on these functions, we refer to Appendix A. For interval T, the derivative of $m_T(s_T)$ to s_T, denoted by $m'_T(s_T)$, is given by the following expression:

$$m'_T(s_T) = c_{r,T} - sv + h\cdot F_T(s_T) + c_{b,T}\cdot[F_T(s_T) - 1]\cdot \quad (7)$$

Similar to the newsvendor problem, we see that $m_T(s_T)$ is convex, since $m''_T(s_T) = (h + c_{b,T})\cdot f_T(s_T) \ge 0$. Therefore, we find the optimal base stock level by solving the equation $m_T^{'}(s_T) = 0$. We find that s_T^* is equal to the minimum value of s_T satisfying: $F_T(s_T) = \frac{c_{b,T}+sv-c_{r,T}}{c_{b,T}+h}$. Next, we compute the value function $V_T(I_T)$.

It is straightforward to compute $V_T(I_T)$ for all values $I_T \le s_T^*$, since $m_T(s_T^*)$ is a constant. From (7), we see that $m_T^{'}(0) = c_{r,T} - sv - c_{b,T}$ and that $\lim_{s_T \to \infty} m_T^{'}(s_T) = c_{r,T} - sv + h$. That is, the function becomes linear as $s_T \to \infty$. Therefore, it seems reasonable to approximate $V_T(I_T)$ by a piecewise linear function $\tilde{V}_T(I_T)$ for all values $I_T > s_T^*$ (Appendix A). We need this approximate function for the next step of the backward recursion to find our approximate function $\tilde{m}_{T-1}(s_{T-1})$. As the state in the next interval (stage) is given by $I_T = s_{T-1} - D_{T-1}$, we find:

$$\begin{aligned}\tilde{m}_{T-1}(s_{T-1}) = c_{r,T-1} \cdot s_{T-1} + E\big[h \cdot (s_{T-1} - D_{T-1})^+ \\ + c_{b,T-1} \cdot (D_{T-1} - s_{T-1})^+\big] + E_{D_{T-1}}\big[\tilde{V}_T(I_T)\big]\end{aligned} \tag{8}$$

For any interval $t < T$, we follow the steps below to determine the base stock levels:

i. Compute the (approximate) expression for $m_t(s_t)$, defined as $\tilde{m}_t(s_t)$, as in (8).
ii. Determine the near-optimal base stock level s_t^* by first computing the derivative $\tilde{m}_t^{'}(s_t)$, and then searching for the value of s_t that changes the sign of the derivative. We do this, because $\tilde{m}_t(s_t)$ is not differentiable at the support points, and so the piecewise linear function typically has a minimum value in one support point (or possibly in two adjacent support points having exactly the same function value).
iii. Derive the approximate value function $\tilde{V}_t(I_t)$ using a piecewise linear function.

We checked the accuracy of the piecewise linear function approximation by comparing the results to the discretization method. We found almost the same results, but with much lower computation times (a factor 10). We refer to Appendix A for further details on how we use piecewise linear functions, the convexity of $\tilde{m}_t(.)$, and the related derivatives.

For non-zero lead times, a decision in interval t influences the holding and the shortage costs at the end of interval $t + l_1$ (a repair lead time later). Therefore, we can still apply the abovementioned algorithm, if we evaluate the single period costs (expected holding and shortage costs) based on the lead time demand. Then, the last decision is made at time T-l_1.

4.3. Performance evaluation given the LTB quantity and the base stock levels

To evaluate the key performance indicators (total expected relevant costs, cumulative fill rates, and cycle service levels), we need the probability distribution of the inventory position after reordering at the start of each interval S_t. The availability of ready-to-repair parts $min((s_t^* - S_{t-1} + D_{t-1})^+, K_{t-1} - R_{t-1} + r_{t-l_2-1}(D_{t-l_2-1}))$ is the key factor in determining the distribution of S_t ($S_t = S_{t-1}$- D_{t-1}+ R_t). The recursive stochastic equations are not straightforward to evaluate due to a strong correlation between the inventory position after reordering, the number of the ready-to-repair parts, and the size of the repair orders in subsequent time intervals. Therefore, we derive a simple approximate probability distribution for S_t (First approximation). Since this first approximation does not always provide accurate results, we will introduce a correction variable (Second approximation).

4.3.1. First approximation of distribution S_t

We use the cumulative supply of ready-to-repair parts in the intervals {1,..,t-1-l_2} and the cumulative demand in the intervals {1,..,t-1} to find the approximate probability distribution. We denote the associated random variable by $\widehat{S}_t$. We define three possible cases for $\widehat{S}_t$ at each interval t:$\widehat{S}_{t1} < s_t^*$ (Case 1), $\widehat{S}_{t2} > s_t^*$ (Case 2), and $\widehat{S}_{t3} = s_t^*$ (Case 3). These cases depend on the demand, supply, and the base stock levels as follows:

$$\begin{aligned}&[1]: \widehat{S}_{t1} = Q - D_{1,t-1} + \sum_{i=1}^{t-l_2-1} r_i(D_i), && Q - D_{1,t-1} + \sum_{i=1}^{t-l_2-1} r_i(D_i) < s_t^* \\ &[2]: \widehat{S}_{t2} = Q - D_{1,t-1}, && s_t^* < Q - D_{1,t-1} \le Q, \\ &[3]: \widehat{S}_{t3} = s_t^*, && Q - D_{1,t-1} \le s_t^* \le Q - D_{1,t-1} + \sum_{i=1}^{t-l_2-1} r_i(D_i).\end{aligned} \tag{9}$$

In Case 1, the inventory position is equal to the maximum inventory position if *all* the ready-to-repair parts have entered repair. In Case 2, the inventory position without any repair exceeds the target level s_t^*. In Case 3, there are sufficient ready-to-repair parts to raise the inventory position to the base stock level s_t^*. Since there is a probability mass at $\widehat{S}_{t3} = s_t^*$, we fit two *separate* continuous probability distribution functions for Case 1 and Case 2. We define $(g_t^{(1)}(.), G_t^{(1)}(.))$ and $(g_t^{(2)}(.), G_t^{(2)}(.))$ as the probability density function (pdf) and the cumulative density function (cdf) for Case 1 and Case 2, respectively. We denote the probability mass in Case 3 by $pr(\widehat{S}_{t3} = s_t^*)$. Let N_t be the maximum value that the inventory position after reordering can take at the beginning of interval t, i.e., $N_t = \max\{s_t^*, Q\}$. We compute the probability mass in s_t^* such that the total probability is equal to one:

$$pr(\widehat{S}_{t3} = s_t^*) = 1 - G_t^{(1)}(s_t^*) - (G_t^{(2)}(N_t) - G_t^{(2)}(s_t^*)) \tag{10}$$

We need the probability distribution of $\widehat{S}_{t1}$ and $\widehat{S}_{t2}$ for the evaluation of the performance indicators. Therefore, we use (9) to compute the first two moments of $\widehat{S}_{t1}$ and $\widehat{S}_{t2}$, and fit an approximate distribution to these moments.

$$\begin{aligned}&E[\widehat{S}_{t1}] = Q - E\Big[E\Big[D_{1,t-l_2-1} - \sum_{i=1}^{t-l_2-1} r_i(D_i) \big| D_{1,t-l_2-1}\Big]\Big] - E[D_{t-l_2,t-1}], \\ [1]:\quad &Var[\widehat{S}_{t1}] = Var\Big(E\Big[D_{1,t-l_2-1} - \sum_{i=1}^{t-l_2-1} r_i(D_i) \big| D_{1,t-l_2-1}\Big]\Big) + \\ &\qquad E\Big[Var\Big(D_{1,t-l_2-1} - \sum_{i=1}^{t-l_2-1} r_i(D_i) \big| D_{1,t-l_2-1}\Big)\Big] + Var[D_{t-l_2,t-1}].\end{aligned} \tag{11}$$

$$[2]:\quad \begin{aligned}&E[\widehat{S}_{t2}] = Q - E[D_{1,t-1}], \\ &Var[\widehat{S}_{t2}] = Var[D_{1,t-1}].\end{aligned} \tag{12}$$

By defining three possible Cases for $\widehat{S}_t$ as in (9), it may happen for *declining* base stock levels that an inventory position *before* reordering exceeds s_t^* even though $Q - D_{1,t-1} \le s_t^*$. This happens if $Q - D_{1,t-1} + \sum_{n=2+l_2}^{t-1} R_n > s_t^*$, with $\sum_{n=2+l_2}^{t-1} R_n > 0$ (i.e., at least some repairs have been started in the previous intervals). Case 2 in (9) does not cover these cases, since we assume that no repair job has been started yet. This causes underestimation of the density in Case 2 $(\widehat{S}_{t2} > s_t^*)$, and overestimation of the density in Case 3 $(\widehat{S}_{t3} = s_t^*)$. Therefore, a second approximation is required to correct this issue by removing the overestimated density in Case 3 and allocating it as a correction to the underestimated density in Case 2. As a result, in Case 1 the distribution of $\widehat{S}_{t1}$ is well approximated and no correction is required. In the next subsection, we elaborate on this correction.

4.3.2. Second approximation of distribution S_t

We define the random variable $CF_t \ge 0$ (correction variable) as the gap between the inventory position *before reordering* $(\widehat{S}_{t-1} + CF_{t-1} - D_{t-1})$ and the *base stock level* s_t^* at time t. Let us define $\tilde{S}_t$ as the inventory position after reordering by adding a correction variable to $\widehat{S}_t$:

$$\tilde{S}_t = \widehat{S}_t + CF_t, \tag{13}$$

$$CF_t = \left(\widehat{S}_{t-1} + CF_{t-1} - D_{t-1} - s_t^*\right)^+ \tag{14}$$

So the correction variable CF_t depends on the demand in the previous period, D_{t-1}, and upon the demand in preceding periods through CF_{t-1} (which depends on D_{t-2} and CF_{t-2}, etc.). The correction variable CF_t does not depend upon demand in the current period D_t. The correction variable may appear for the first time in interval τ, as soon as $s_\tau^* < s_{\tau-1}^*$ (i.e., only for declining base stock levels) and once repair jobs have been started before. We need the probability density and distribution function of the correction variable in interval t, denoted by $h_t(.)$ and $H_t(.)$, respectively. Here $h_t(0)$ denotes the probability mass in the point 0. After finding these densities and distributions, we find the second approximation of the inventory position after reordering $\tilde{S}_t$ as function of the distribution $\widehat{S}_t$ and CF_t. We define $(\tilde{g}_t^{(1)}(.), \tilde{G}_t^{(1)}(.))$ and $(\tilde{g}_t^{(2)}(.), \tilde{G}_t^{(2)}(.))$ as the pdf and cdf for Case 1 and 2, respectively, and $pr(\tilde{S}_t = s_t^*)$ for Case 3:

$$\begin{array}{lll} [1] & \tilde{g}_t^{(1)}(y) = g_t^{(1)}(y) & \text{for } y < s_t^*, \\ [2] & \tilde{g}_t^{(2)}(y) \approx g_t^{(2)}(y) + h_t(y - s_t^*), & \text{for } y > s_t^*, \\ [3] & pr(\tilde{S}_t = s_t^*) = pr(\widehat{S}_t = s_t^*) - (1 - h_t(0)), & \text{for } y = s_t^*. \end{array} \tag{15}$$

As mentioned earlier, no correction variable is needed for Case 1 ($y < s_t^*$), so $\tilde{S}_t = \widehat{S}_t$ in Case 1. Therefore, we only need to remove the overestimated density for $y = s_t^*$ (Case 3) and allocate that probability mass to the values $y > s_t^*$ (Case 2).

To determine the distribution of CF_t, we differentiate between three situations:

$$\begin{array}{ll} CF_t = \left(\widehat{S}_{t-1} - D_{t-1} - s_t^*\right)^+, & \text{for } \quad CF_{t-1} = 0 \text{ and } \widehat{S}_{t-1} < s_{t-1}^*, \\ CF_t = \left(s_{t-1}^* - D_{t-1} - s_t^*\right)^+, & \text{for } \quad CF_{t-1} = 0 \text{ and } \widehat{S}_{t-1} = s_{t-1}^*, \\ CF_t = \left(s_{t-1}^* + CF_{t-1} - D_{t-1} - s_t^*\right)^+, & \text{for } \quad CF_{t-1} > 0 \text{ and } \widehat{S}_{t-1} = s_{t-1}^*. \end{array} \tag{16}$$

We use (15) and (16) to determine the pdf of CF_t. By recursively using the pdf of the correction variable $h_t(.)$ and the pdf of the inventory position $\tilde{g}_t^{(1)}(.)$, we derive the following expression for the pdf of CF_t:

$$h_t(z) = \int_{s_t^*}^{s_{t-1}^*} \tilde{g}_{t-1}^{(1)}(y) . f_{t-1}\left(y - z - s_t^*\right) dy +$$
$$pr\left(\tilde{S}_{t-1} = s_{t-1}^*\right) \cdot f_{t-1}\left(s_{t-1}^* - z - s_t^*\right) + \int_0^{s_t^* - s_{t-1}^*} h_{t-1}(u) . f_{t-1}\left(y + u - z - s_t^*\right) du. \tag{17}$$

As numerical integration is inefficient due to the recursive nature of $h_t(.)$ and $h_{t-1}(.)$, we use an approximation by computing the first two moments of CF_t, and fitting a suitable distribution.

4.3.3. Two-moment approximation for the distribution of CF_t

We first compute the probability mass $h_t(0)$, and then derive expressions for the first two moments of CF_t as in (16). Since the first time that the correction variable appears is interval τ (i.e., repair jobs started and the base stock level is declining), the correction in the preceding equals $CF_{\tau-1} = 0$ and $CF_\tau = \left(\widehat{S}_{\tau-1} - D_{\tau-1} - s_\tau^*\right)^+$ according to (16). The probability mass at interval τ when $\widehat{S}_{\tau-1} - D_{\tau-1} - s_\tau^* \le 0$ can be computed as:

$$\begin{aligned} h_\tau(0) = & \int_{-\infty}^{s_{\tau-1}^*} g_{\tau-1}^{(1)}(y) \cdot \left(1 - F_{\tau-1}\left(y - s_\tau^*\right)\right) dy \\ & + pr\left(\widehat{S}_{\tau-1} = s_{\tau-1}^*\right) \cdot \int_{s_{\tau-1}^* - s_\tau^*}^{\infty} f_{\tau-1}(x) dx. \end{aligned} \tag{18}$$

Next, we compute the first two moments of CF_τ using standard numerical integration. For strictly positive values of CF_τ, we fit a distribution to the first two moments of $(CF_\tau | CF_\tau > 0)$:

$$E[CF_\tau | CF_\tau > 0] = \frac{E[CF_\tau]}{1 - h_\tau(0)}, \quad E\left[CF_\tau^2 \middle| CF_\tau > 0\right] = \frac{E\left[CF_\tau^2\right]}{1 - h_\tau(0)}. \tag{19}$$

We choose to fit a Gamma distribution with shape and scale parameters (k, θ) based on the moments in (19) (Tijms, 1986). We stress that this approximation does not depend on the shape of the demand distribution. We denote $h_\tau(.)$ in (20) as a mixture of the probability mass and the pdf of $(CF_\tau | CF_\tau > 0)$. We scale the Gamma function for the strictly positive correction variables, since we have a probability mass at $CF_\tau = 0$:

$$h_\tau(f) = \text{Gamma}(k, \theta, f).(1 - h_\tau(0)) \quad \text{for } CF_\tau > 0 \tag{20}$$

The complete distribution of CF_τ is given by (18) and (20). For any interval $t > \tau$, we find the distribution of CF_t recursively using the same procedure: First compute the point mass, and next fit a two-moment approximation. Note that in Equations (18) and (19), we should also take into account $CF_{t-1}>0$ for any $t > \tau$, according to the various situations as defined in (16).

Using the approximate probability distribution of S_t, we can evaluate the performance indicators and find the LTB quantity using the procedure that we will describe in Section 4.4.

4.3.4. Expected total relevant costs and service levels

$TRC_I(Q, \boldsymbol{s}^*)$ as in (1) and (2) consists of the several components, namely the purchasing cost, the expected holding, shortage, and repair costs minus the salvage value, plus the return cost. Below, we derive expressions for each cost component.

4.3.5. Expected on-hand inventory

In the intervals $t \in \{1, \ldots, l_1 + l_2 + 1\}$, demands should be covered by the LTB quantity solely, since repairs cannot be completed due to the return and the repair lead time. Given that the first ready-to-repair parts are available at the end of interval 1, the first time that repaired parts can be available for use is at the start of interval $2 + l_2 + l_1$. We compute $E[OH_t]$ from the repair lead time demand and the actual inventory position after reordering a repair lead time ago:

$$E[OH_t] = \begin{cases} E[(Q - D_{1,t})^+], & \text{for } 1 \le t \le l_1 + l_2 + 1, \\ E[(S_{t-l_1} - D_{t-l_1,t})^+], & \text{for } l_1 + l_2 + 2 \le t \le T. \end{cases} \tag{21}$$

We evaluate (21) for $1 \le t \le l_1 + l_2 + 1$ using standard numerical integration. However, for $l_1 + l_2 + 2 \le t \le T$ that the evaluation becomes more difficult due to two separate probability density functions for $\tilde{S}_t > s_t^*$ and $\tilde{S}_t < s_t^*$ as in (15), we find (22) (online Appendix 1):

$$\begin{aligned} E[OH_t] = & \int_0^{s_{t-l_1}^*} \left(\tilde{G}_{t-l_1}^{(1)}\left(s_{t-l_1}^*\right) - \tilde{G}_{t-l_1}^{(1)}(y)\right) . F_{t-l_1,t}(y)\, dy \\ & + \int_0^{N_t} F_{t-l_1,t}(x) . \left(\tilde{G}_{t-l_1}^{(2)}(N_t) - \tilde{G}_{t-l_1}^{(2)}(y)\right) dy \\ & - \int_0^{s_{t-l_1}^*} F_{t-l_1,t}(x) . \left(\tilde{G}_{t-l_1}^{(2)}\left(s_{t-l_1}^*\right) - \tilde{G}_{t-l_1}^{(2)}(y)\right) dy + \left(\tilde{G}_{t-l_1}^{(2)}\left(s_{t-l_1}^*\right)\right. \\ & \left. - \tilde{G}_{t-l_1}^{(1)}\left(s_{t-l_1}^*\right)\right) . \left(\int_0^{s_{t-l_1}^*} F_{t-l_1,t}(x) dx\right). \end{aligned} \tag{22}$$

For a special case of the distributions $F_{t-l_1,t}$ and ($\tilde{G}_{t-l_1}^{(i)}$ $i = 1, 2$) (i.e., Coxian and Normal distribution respectively), we find a closed-form expression for $E[OH_t]$ (Online Appendix 2).

4.3.6. Expected backorders

Using the fact that the inventory level (on hand stock minus backorders) equals the inventory position a lead time ago minus the lead time demand, cf. Zipkin (2000), we have:

$$E[OH_t] - E[BO_t] = E\left[S_{\max(t-l_1,1)}\right] - E\left[D_{\max(t-l_1,1),t}\right]. \tag{23}$$

4.3.7. Expected number of repairs

We approximate the expected number of repairs started at time t as:

$$E[R_t] \cong (E[S_t] - E[S_{t-1}] + E[D_{t-1}])^+. \tag{24}$$

4.3.8. Service levels

Now it is straightforward to compute the cumulative fill rates and the cycle service levels:

$$\beta_t = 1 - \frac{\sum_{i=1}^{t} E[BO_i]}{\sum_{i=1}^{t} E[D_i]}, \quad \alpha_t = \Pr\{S_{t-l_1} - D_{t-l_1,t} > 0\}. \tag{25}$$

Using the expressions for all the performance indicators, we tackle the cost optimization problem using a straightforward numerical search.

4.4. Algorithm to find the optimal LTB quantity and bases stock levels

Let us summarize our algorithm to determine the repair policy and the optimal LTB quantity. We apply a numerical search (bisection) to find the value Q^* that minimizes $TRC(Q, \boldsymbol{s}^*)$. For the initial lower bound (LB) and upper bound (UB) of Q in the bisection search, we use $LB = \sum_{t=1}^{l_1+l_2+1} E[D_t]$ and the minimum value that satisfies $F_{1,T}(UB) \geq \frac{\max(c_{b,t})}{\max(c_{b,t})+h}$. LB refers to the expected demand until the moment that first ready-to-use parts are available, and UB refers to the number of parts required for a single period problem without repair. Define the midpoint as $MP = \lceil (LB + UB)/2 \rceil$. This yields the following algorithm:

Step 1 Compute the near-optimal base stock levels $\boldsymbol{s}^* = \{s_1^*, s_2^*, \ldots, s_{T-l_1}^*\}$ (Section 4.2).

Step 2 Find the distribution of the actual inventory position after reordering for the base stock levels $\boldsymbol{s}^*$ (found in *Step 1*), and for each value $Q \in \{LB, MP, UB\}$ (Section 4.3).

Step 3 Compute the total expected relevant costs $TRC(Q, \boldsymbol{s}^*)$ for each value $Q \in \{LB, MP, UB\}$ using (1) or (2) (depending on the considered scenario) and the base stock levels $\boldsymbol{s}^*$ (Section 4.3).

Step 4 If convergence to the optimal total expected costs is satisfactory (i.e., the relative difference between the calculated total expected costs for $Q = LB$ and $Q = UB$ is less than a predefined threshold ε), choose *the near-optimal* $Q^* = MP$ and go to *Step 6*. Otherwise, go to *Step 5*.

Step 5 Choose a support point (SP) equal to $\lceil (MP + UB)/2 \rceil$ to determine the search section in the next step. Find the distribution of the actual inventory position after reordering for the base stock levels $\boldsymbol{s}^*$ (*Step 1*) and the value $Q = SP$, then compute $TRC(SP, \boldsymbol{s}^*)$. If $TRC(SP, \boldsymbol{s}^*) > TRC(MP, \boldsymbol{s}^*)$, then replace the values of the search triplet points as $Q \in \{LB, \lceil (LB + MP)/2 \rceil, MP\}$. Otherwise, replace the values as $Q \in \{SP, \lceil (SP + MP)/2 \rceil, MP\}$. In either case, go to *Step 2*.

Step 6 Compute the service levels for Q^* using (25).

We conjecture that the cost function has a single minimum. Due to complexity of the analysis, we were not able to prove this. Though, extensive numerical experiments did not reveal any counter example.

So far in our analysis we have assumed zero stock level and zero pipelines of in-return and in-repair parts at the beginning of the planning period. In case that there are initial ready-to-use parts while making the LTB decision, we need to deduct the procurement cost of those parts while computing $TRC(Q, \boldsymbol{s}^*)$ and finding the optimal LTB quantity. If there are parts in the repair pipeline or the return pipeline, we further need to modify two parts of our model. First, we take into account arrival of those parts in (9) and (11) while determining the approximate distribution of the inventory position after ordering $\widehat{S}_t$. Second, in computing the performance indicators, we need to modify the expected repair, expected on-hand inventory, and expected returns. Repair jobs can be started from $t = 1$ and not from $t = l_2+2$ in (1), since there might be available ready-to-repair parts right from the beginning. For the on-hand inventory, we need to consider the arrival sequence of repair jobs in the pipeline for evaluating (21). This yields a modification in the intervals $1 \leq t \leq l_1+1$ and $l_1+2 \leq t \leq T$. For the expected returns, we should add up the parts in return pipeline for the return costs in (1).

Table 2
Input parameters in the numerical experiments.

Varying parameters	Value 1	Value 2
Repair cost per part	50% of the new part price	200% of the new part price
Shortage cost per part/interval	300	600
Total expected demand	500	1500
Return yield	0.6	0.8
Return lead time	1 (1 month)	3 (3 months)
Repair lead time	1 (1 month)	3 (3 months)

5. Model validation and insights

We first check the accuracy of our approximations by comparison to discrete event simulation (Section 5.1), followed by managerial insights that we obtained from numerical experiments in 5.2. In 5.3, we examine under which conditions Scenario (I) outperforms Scenario (II). Finally, in 5.4 we study when our proposed heuristic for fast moving parts can be used as a good approximation for slow moving parts to benefit from faster computation times.

5.1. Accuracy of the model

We built a simulation model in Plant Simulation,[1] and we compare the cost and the service levels from our approximate method to the simulated values with the same base stock levels, LTB quantity, and demand pattern. We consider 128 problem instances for Scenario (I). In all instances, the planning period between LTB and the remaining service period is equal to 10 years (a value that we encountered in practice), divided in 120 intervals of 1 month. The price of a new part is €100, and any part left over at the end of the service period has no value. The holding cost per part per year equals 25% of the new part price, and the return cost per part is negligible. For simplicity, we assume that the repair cost and the shortage cost per part per interval are constant over time. We vary the other key input parameters as stated in Table 2. Table 3 shows two different yearly mean demand patterns with accumulated values of 500 and 1500 and coefficient of variation (cv) per interval. The mean demand per interval (1 month) is equal to the mean yearly demand divided by 12. The variance of demand per interval is derived by multiplying demand per interval by the given variance to mean ratio (20 and 30). In practice, the demand uncertainty increases with the number of periods ahead due to the inaccuracy involved in long-term forecasts. We assume that demand is Gamma distributed, as this distribution is also suitable when demand is highly uncertain. All combinations yield 128 problem instances for which we find the near-optimal LTB quantity and the base stock levels.

In Table 4, we compare the estimated performance indicators to the results from simulation with 50,000 replications in terms of mean absolute percentage errors. We see that the total expected costs and the individual cost components (the repair costs, the obsolescence costs, and the holding costs) are well approximated. For the shortage costs, we see larger errors, especially when shortage cost is high, i.e., when the overall fill rate is high. However, the low values of the expected shortages make the errors relatively large, but typically small in absolute sense. Next, we

[1] www.plm.automation.siemens.com/.

Table 3
Pattern of the expected yearly demand.

Year	1	2	3	4	5	6	7	8	9	10
Mean demand 1 (500)	80	77	71	65	58	49	40	30	20	10
cv per interval (1)	1.73	1.76	1.83	1.92	2.03	2.21	2.44	2.82	3.46	4.89
cv per interval (2)	2.12	2.16	2.25	2.35	2.49	2.71	3	3.46	4.24	6
Mean demand 2 (1500)	240	230	215	195	175	150	120	90	60	25
cv per interval (1)	1	1.02	1.05	1.1	1.17	1.26	1.41	1.63	2	3.09
cv per interval (2)	1.22	1.25	1.29	1.35	1.43	1.54	1.73	2	2.44	3.79

Table 4
Relative error of the results from the model compared to the simulation (%).

	High overall fill rate		Low overall fill rate	
	Mean absolute percentage error	90% percentile	Mean absolute percentage error	90% percentile
Total costs	0.47%	0.83%	0.38%	0.80%
Shortage costs	5.1%	9.5%	3.2%	6%
Obsolescence costs	1.4%	3.3%	1.8%	4.1%
Repair costs	0.6%	1.25%	0.57%	1.17%
Holding costs	0.12%	0.17%	0.10%	0.14%

observed that the approximation accuracy improves for higher variations of demand. For details, see online Appendix 3 for the results of 128 problem instances with lower coefficients of variation.

To check how much the near-optimal LTB quantity from the model differs from the true optimum, we find the optimal LTB quantity by performing a numerical search with the simulation model using the same base stock policy. We find a maximum relative difference of 1% in the LTB quantity and a maximum relative difference 0.9% in the total expected costs compared to the results of our model. We conclude that the LTB quantity and the performance indicators are very close to the optimum value indeed.

The average runtime for computing the near-optimal base stock policy and evaluating all the performance indicators for a single LTB quantity varies between 6 s (mean demand 500) and 40 s (mean demand 1500). The average runtime for the joint optimization of the LTB quantity and the repair policy varies between 60 and 400 s. The computation times are measured using a computer with a 2.83 GHz Core 2 quad processor.

We also study whether it is worthwhile to use larger (aggregated) intervals to benefit from faster computations. For this purpose, we use the same set of numerical experiments, in which the only difference is the length of an interval, i.e. 3 months instead of 1 month. We only consider those problem instances that resemble the same length for the lead times in both of the experiments, i.e. the instances with a lead time of 3 intervals (each equal to 1 month) in the first experiment compared to the instances with a lead time of 1 interval (equal to 3 months) in the second experiment. The demand in each interval equals to the cumulative demand in 3 subsequent intervals. As in the first set of experiments, we assume that demand is independent over intervals. We compared the computation times, LTB quantities and the total costs in those instances. The computation times are on average 5 times smaller, but we find a larger LTB quantity (on average 9%), since we have less opportunities for intermediate replenishment decisions. As a consequence, we observe on average 5% higher total costs. We conclude that it is better to use intervals that are not too large, because this facilitates quick reactions to changes in the inventory position by conducting repairs.

In Appendix B, we implement our proposed heuristics based on the data from two industry cases and show the decision variables (near-optimal LTB quantity and base stock levels) and the side results.

5.2. Insights from the model

We study the impact of the key parameters such as repair cost, return yield, return and repair lead time, and demand variability on the performance indicators. We find similar key insights as in Behfard et al. (2015), such as: i) the postponement advantage of the repair option, i.e., repair option is worth considering though it can be more expensive than buying a new part, ii) high sensitivity of the repair policy and LTB quantity to the lead times, though aggregating return and repair lead time into just one lead time has small impact on the LTB quantity, iii) high sensitivity of the repair policy and LTB quantity to demand variability.

In addition, we study the impact of the return yield on: 1) the total expected costs and 2) discard rate of failed parts for different values of the repair cost. Consider the problem instance with a mean demand of 1500 with CV in the range of [1–3.1], and a shortage cost of 600 as in Section 5.1. Both the return lead time and the repair lead time are equal to 3 months. Fig. 3 shows the impact of the return yield on the reduction of the total expected costs as function of the repair cost. We see that a high return yield leads to a significant reduction in the total expected costs, even if the repair cost is higher than buying a new part. However, the reduction in the total costs diminishes for very high return yields. When the repair cost is higher than buying a new part, it is not worthwhile to spend much effort to improve the return process (i.e. increase the return yield), since the reduction in the total expected costs is not significant.

In Fig. 4, we study the percentage of failed parts that should be discarded during the planning period for various return yields as function of the repair cost. When there is no repair option (yield = 0), all failed parts are discarded. When the repair option is available (yield>0), the discard rate of failed parts is significantly reduced, even for high repair costs. Therefore, repair also has a strong impact on the sustainability of these types of supply chains. We observe 50%–90% reduction in obsolescence at the end of the planning period compared to the case when there is no repair.

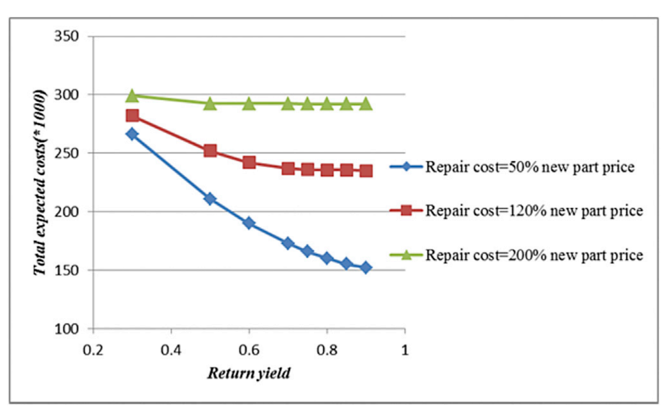

Fig. 3. Impact of the return yield on reduction of the total costs as function of the repair cost.

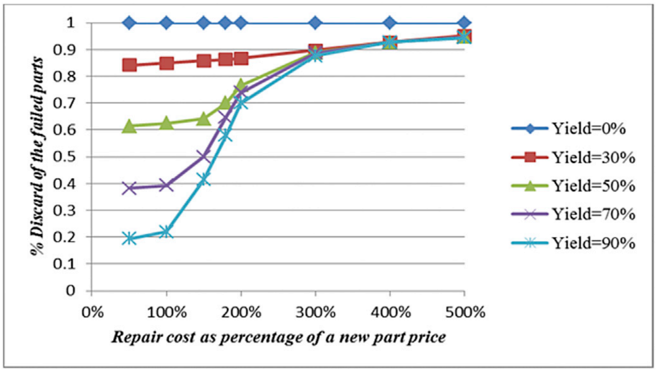

Fig. 4. Impact of the return yield on reduction of discard rate failed parts as function of the repair cost.

5.3. Comparing Scenario (I) to Scenario (II)

As we explained in Section 1, we can use two control policies in our model, i.e., Scenario (I) with push return - pull repair and Scenario (II) with pull return - push repair. It is worthwhile to study under which conditions which scenario outperforms the other. As one of the insights from the model, aggregating the return lead time and the repair lead time into just one repair lead time hardly changes the LTB quantity (maximum five units differences in the numerical experiment). We perform an extensive numerical experiment over the key parameters (return lead time, demand variability, and return yield). In each instance, the mean demand is 1500, and the shortage cost is equal to €600 per part per interval. Each interval is equal to 1 month.

Fig. 5 shows the impact of the return cost on the relative difference in the total expected costs of Scenario (I) versus Scenario (II). When the return lead time is zero, Scenario (I) with a push return always yields *higher* total costs than Scenario (II), since all the ready-to-repair parts are returned from the field. Due to zero return lead time, it is not necessary to consider a push return policy. When the return lead time is strictly positive, there is a break-even point in the return cost. That is, for the return cost lower than the break-even point, Scenario (I) outperforms Scenario (II), and for higher return cost Scenario (II) outperforms Scenario (I). Fig. 5 shows this break-even point as return costs being 70% of the product value, when the return lead time is 1 month. In Figs. 6–8, we study impact of the return lead time, the return yield, and the demand variability on the break-even point.

We see that the break-even point increases when each of the parameters increases (and all other parameters remain the same). That is, Scenario (II) outperforms Scenario (I) only for relatively large values of the return cost (recall that the new part price equals €100). Fig. 6 shows that the return lead time has a larger impact on the break-even point than the demand variability and the return yield. It is because in Scenario (II), the base stock level at interval t should cover larger demand over the interval $[t, t + l_1 + l_2]$. Therefore, the costs of inventory and backorders are larger in Scenario (II) due to the longer lead times (return + repair lead time instead of repair lead time only). It is remarkable that for all of the studied parameters (return lead time, return yield, and demand variability), increments in the break-even points decrease when the parameter increases.

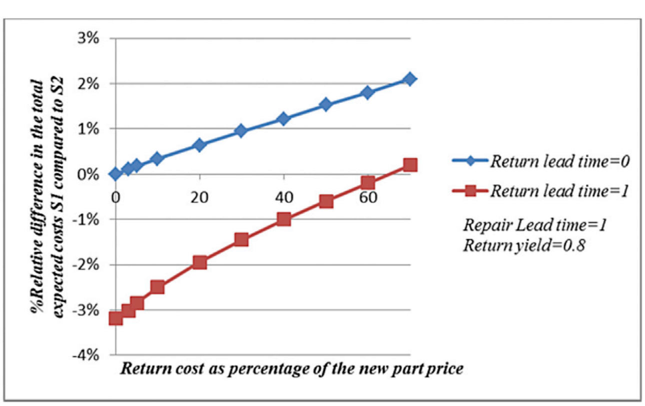

Fig. 5. Impact of the return cost on the relative difference in the total costs.

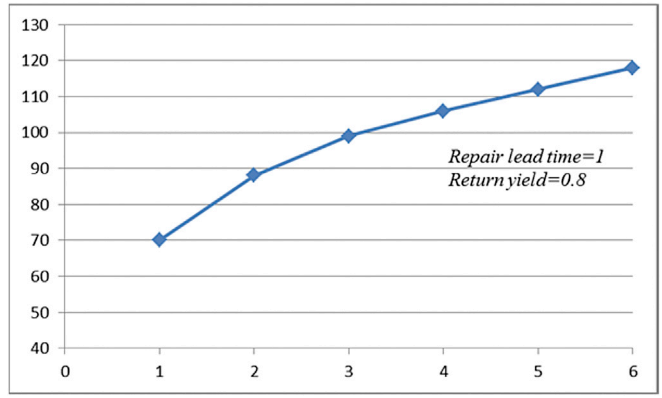

Fig. 6. Impact of the return lead time on the break-even point.

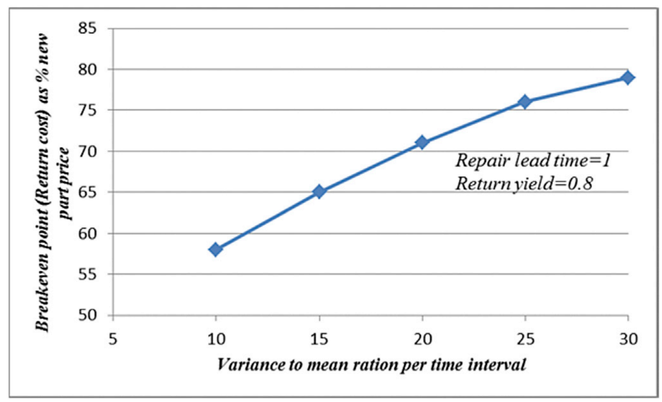

Fig. 7. Impact of the demand variability on the break-even point.

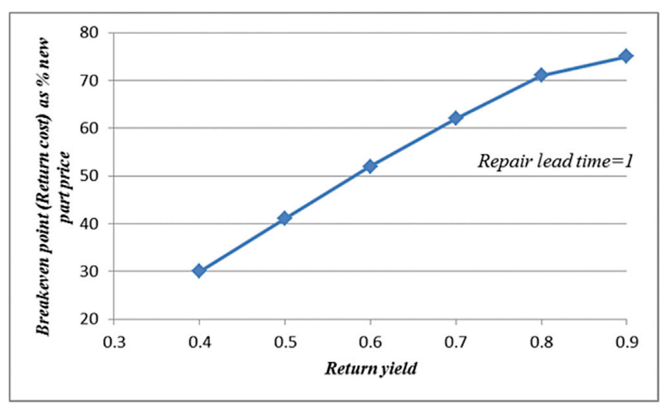

Fig. 8. Impact of the return yield on the break-even point.

5.4. Approximating the discrete model with the continuous model

In the model of Behfard et al. (2015) with discrete demand, the computation time explodes when total demand over the planning horizon increases (say ≥1000). In this subsection, we study when we can use

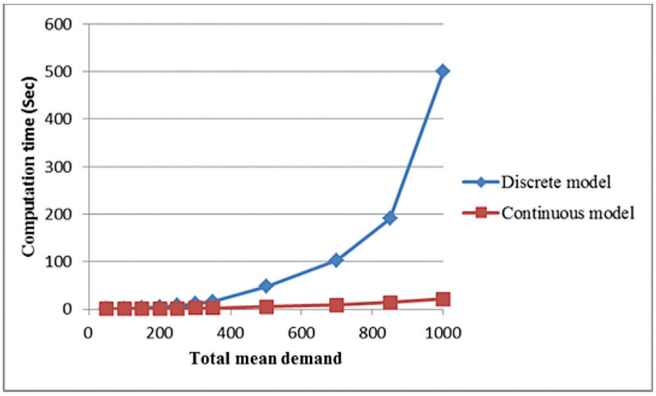

Fig. 9. Computation time from the discrete and the continuous model.

our model with continuous demand to approximate the LTB quantity for slow movers, and benefit from a faster computation. We compare the discrete model with Negative binomial demand to the continuous model with Gamma demand, where the mean and the variance are the same. In the discrete model, the return yield and the repair yield are separate parameters. Therefore, as an approximation we merge them into one return yield parameter in the continuous model by multiplying the repair yield and the return yield. We compare: i) the computation time, and ii) the optimal LTB quantity for different demand levels and coefficient of variation as in Table 5 (for higher variations, computation of the discrete model quickly explodes).

We consider problem instances with a planning horizon of 10 years. Both the return and the repair lead time are equal to 1 interval (two months). The shortage cost per part per interval is equal to €600, the repair cost is equal to €70 and the value of the part is €100. Both the return yield and the repair yield are assumed to be 0.9. Fig. 9 shows the computation time to evaluate a single scenario in terms of repair policy and performance indicators, for different demand levels:

We see that computation time quickly increases for the discrete model when the total demand over the planning horizon becomes large. However, the increase is far slow for the continuous model.

Fig. 10 shows the relative difference in the optimal LTB quantity computed for different demand levels using the discrete and the continuous model:

We find that for low mean demand, the continuous model does not yield a good approximation for the optimal LTB quantity. The reason is that a Gamma distribution differs significantly from a Negative binomial distribution. Then, the continuous approximation is not valid anymore. Nevertheless, for large mean demand (total mean demand >300), the continuous model provides a good approximation.

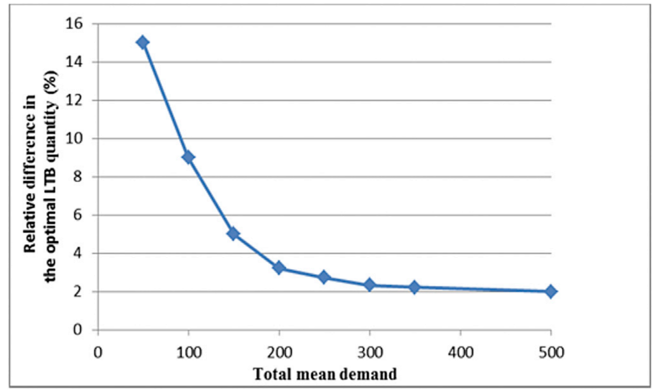

Fig. 10. Relative difference in optimal LTB quantity from the discrete and the continuous model.

We also examine impact of the demand variability and the return yield on the optimal LTB quantity computed by the discrete and the continuous model. We observe that higher demand variability leads to a relatively large difference (about 8%) in the computed LTB quantities. A high variability of demand and a high skewness of the continuous distribution lead to a lower base stock levels and a lower LTB quantity. We find relatively small difference in the optimal LTB quantities when we decrease the return yield (max 3% difference), since LTB order is main source of supply in such case.

6. Conclusion and direction for further research

In this paper, we consider the repair of failed parts as an alternative source of supply to an LTB order. We propose heuristics to find the near-optimal LTB quantity and the repair policy for fast moving parts with continuous demand distribution. Our model is applicable for two different return and repair policies: (I) a push return and a pull repair policy and (II) a pull return and a push repair policy. We find the following insights form the model:

- Improving the return yield has significant impact on reduction of total expected costs and discard rate of failed parts. This is also true for a repair cost higher than the cost of buying a new part. This highlights the importance of reusing the returned parts for sustainability purposes. Note that reductions are less significant when yield reaches a decent level (about 80%).
- For short return lead times, Scenario (I) with push return always outperforms Scenario (II) with pull return. However, for long return lead times, there is a break-even point in the return cost after which

Table 5
Pattern of the expected yearly demand.

Year	1	2	3	4	5	6	7	8	9	10
Mean demand (300)	48	46.2	42.6	39	34.8	29.4	24	18	12	6
CV per interval	0.86	0.88	0.91	0.96	1.01	1.1	1,22	1.41	1.73	2.44
Mean demand (500)	80	77	71	65	58	49	40	30	20	10
CV per interval	0.67	0.68	0.71	0.74	0.78	0.85	0.94	1.09	1.34	1.89
Mean demand (700)	112	107.8	99.4	91	81.2	68.6	56	42	28	14
CV per interval	0.56	0.57	0.6	0.62	0.66	0.72	0.8	0.92	1.13	1.6
Mean demand (900)	144	138.6	127.8	117	104.4	88.2	72	54	36	18
CV per interval	0.5	0.51	0.53	0.55	0.58	0.63	0.7	0.81	1	1.41
Mean demand (1000)	160	154	142	130	116	98	80	60	40	20
CV per interval	0.47	0.48	0.5	0.52	0.55	0.6	0.67	0.77	0.94	1.34

Scenario (II) outperforms Scenario (I), i.e., for return cost higher than the break-even point it is beneficial to apply pull return policy.
- The return lead time, the return yield, and the demand variability all have an impact on the break-even point for the choice between Scenario I (push-pull) and Scenario II (pull-push). We observe that the return lead time has the largest impact on the break-even point.
- Finally, we find that our proposed continuous model can be used as an accurate and a fast approximation for the discrete model of Behfard et al. (2015) to find the LTB quantity and the repair policy for high levels of demand (>300) and high yield rate (>0.8). Moreover, it is notable that our model yields more accurate results for higher variations in demand.

This work can be further extended to a pull policy for both the returns and the repairs when lead times are not negligible. Then we get a two-echelon serial system with base stock levels and/or dispose-down-to levels for the return process as well as the repair process. It makes the analysis more complex, particularly in the computation of the inventory positions after reordering. Considering more alternative sources of supply in addition to the repair of the failed parts is another interesting option for further investigation.

Acknowledgments:

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Supplementary data

Supplementary data related to this article can be found at https://doi.org/10.1016/j.ijpe.2017.12.012.

Appendix A. Approximate the value function to determine the optimal base stock levels

For determining the base stock levels (Section 4.2), we should compute the value function $V_t(I_t)$ for any value of I_t:

$$V_t(I_t) = \begin{cases} -c_{r,t}.I_t + m_t(s_t^*), & \text{for } I_t \le s_t^*, \\ -c_{r,t}.I_t + m_t(I_t), & \text{otherwise}. \end{cases} \quad (A.1)$$

For all values $I_t \le s_t^*$, it is straightforward to compute $V_t(I_t)$. However, we need an approximation for all values $I_t > s_t^*$ when demand is declining. We use a piecewise linear function as an approximation. The piecewise linear function $\tilde{V}_t(I_t)$ is based on a set of support points $(x_k, V_t(x_k))$ for $k = 0,1, \ldots,n$. The spaces between the support points are not necessarily equidistant:

$$\tilde{V}_t(I_t) = \begin{cases} -c_{r,t}.I_t + m_t(s_t^*), & \text{for } I_t \le s_t^*, \\ a_1.I_t + b_1, & \text{for } x_0 = s_t^* < I_t \le x_1, \\ a_2.I_t + b_2, & \text{for } x_0 = s_t^* < I_t \le x_1, \\ \ldots \\ a_{n+1}.I_t + b_{n+1}, & \text{for } x_{n-1} < I_t \le x_n = UB_t. \end{cases} \quad (A.2)$$

For the piecewise linear interpolation with two support points $(x_k, V_t(x_k))$ and $((x_{k+1}),\ V_t(x_{k+1}))$, the approximate function $\tilde{V}_t(I_t)$ is:

$$\tilde{V}_t(I_t) = V_t(x_k) + \frac{V_t(x_{k+1}) - V_t(x_k)}{(x_{k+1} - x_k)}.(x - x_k), \text{ for } \quad s_t^* < x_k < I_t \le x_{k+1}. \quad (A.3)$$

For the approximation, we are mainly interested in the values $s_t^* < I_t \le s_{t-1}^*$, since I_t cannot be higher than the base stock level in the previous interval when demand is declining ($I_t = S_{t-1} - D_{t-1}$). Because we still do not know the optimal base stock level s_{t-1}^* to determine the exact upper bound for I_t at interval t, we define UB_t as an approximate upper bound for s_{t-1}^* (and so for I_t). Since a myopic policy gives an upper bound for the optimal base stock level, we can compute UB_t from myopic policy and the minimum value of x that satisfies: $UB_t = F_{t-1}(x) \ge \frac{c_{b,t-1} - (c_{r,t-1} - c_{r,t})}{c_{b,t-1} + h}$ (Zipkin, 2000).

We use a bisection search to determine the support points for the piecewise linear approximation. At each iteration, we compare the computed value function from the approximated function at the middle point of the piece with the actual value function at the same point, and we check whether the relative error is lower than an acceptable error (ε). If the error was not acceptable, we continue by making more pieces of smaller distances. We can adjust ε to have higher or lower accuracy in our approximation. Our numerical experiments showed that $\varepsilon = 1$ provides fairly good results compared to high accuracy with $\varepsilon = 0.1$.

In general, for any $t < T$, we find near-optimal base stock levels from the approximate equation $\tilde{m}_t(s_t)$, and by computing its derivative:

$$\tilde{m}_t(s_t) = c_{r,t} \cdot s_t + E\left[h \cdot (s_t - D_t)^+ + c_{b,t} \cdot (D_t - s_t)^+\right] + E_{D_t}\left[\tilde{V}_{t+1}(s_t - D_t)\right]. \quad (A.4)$$

$$\tilde{m}_t'(s_t) = c_{r,t} + (h + c_{b,t}).F_t(s_t) - c_{b,t} + \phi_{t+1}. \quad (A.5)$$

The parameter α_{t+1} is derived from the piecewise linear approximation. Because the derivative does not exist in support points, we look for a value s_t where the sign of the derivative changes:

$$\phi_{t+1} = -c_{r,t+1}.\left(1 + F_t\left(s_t - s_{t+1}^*\right)\right) - f_t\left(s_t - s_{t+1}^*\right).\left(c_{r,t+1}.s_{t+1}^* - m_{t+1}\left(s_{t+1}^*\right)\right) + \sum_{i=0}^{n-1}(a_{i+1}.[F_t(s_t - x_i) - F_t(s_t - x_{i+1})] + (a_{i+1}.x_i + b_{i+1}).f_t(s_t - x_i) - (a_{i+1}.x_{i+1} + b_{i+1}).f_t(s_t - x_{i+1})). \tag{A.6}$$

Next, we derive the approximate value function as:

$$\tilde{V}_t(I_t) = \begin{cases} -c_{r,t}.I_t + \tilde{m}_t\left(s_t^*\right), & \text{for } I_t \leq s_t^*, \\ -c_{r,t}.I_t + \tilde{m}_t(I_t), & \text{otherwise}. \end{cases} \tag{A.7}$$

Note that $\tilde{m}_t(.)$ is convex if $\tilde{V}_{t+1}(.)$ is convex, which is true if $\tilde{m}_{t+1}(.)$ is convex. Since $V_{T+1}(.)$ is convex, all the value functions are convex as long as we use a convex approximation of the value function.

To check the accuracy of this approximation, we performed numerical experiments under various scenarios: i) demand pattern (constant, inclining, declining, and a mixture), ii) zero lead time, and positive lead times, and iii) high and low level of demand. Next, we computed the base stock levels for the same problem instances using discretization. Comparison shows that both methods provide almost the same base stock levels. However, the computation times of our proposed method are much lower than of the discretization method (a factor 10). It is remarkable that the number of the required pieces to approximate the value function varies between two and five pieces within our experiments. This means that the piecewise approximation is accurate enough.

Appendix B. Model results for industrial cases

In this section, we present two cases and implement our proposed heuristics to calculate the near optimal LTB quantity and the base stock levels for the repair jobs. These are two cases from a company that is active in printing machines. Due to confidentiality of key information, we have modified the data slightly and kept the part identifications anonymous. Still, the data patterns and cost parameters are realistic. We divide the planning horizon in intervals of one month. We assume that the demand per month within each year is independent and identically distributed. So the monthly demand has as mean value: the forecast demand in that year divided by 12, and as coefficient of variation: the coefficient of variation of demand in that year multiplied by $\sqrt{12}$. Note that the demand over the repair lead time of 2 months is always 30 or larger, which satisfies the rule of thumb given by Silver et al. (2017) for applying a continuous demand distribution (mean lead time demand should be at least 10).

For both cases, the following information holds:

- The return cost of failed parts from the field is negligible.
- The salvage value of parts remaining in inventory at the end of the service period is zero.
- Return yield = 0.7, return lead time = 1 month
- Repair yield = 0.9, repair lead time = 2 months
- Annual holding cost = 27% of the original part price
- Repair cost = 70% of the original part price

For clarity of presentation, we assume that no ready-to-repair parts exist at the time of the LTB decision.

Case 1:

The period from the LTB decision until the end of the service period is 10 years (120 months). The original part price at the time of LTB is €45. For the shortage cost per part backordered at the end of each month, there is no clear monetary indication in industry. Therefore, we show the decision variables and key performance indicators based on € 250 per backordered part, and provide a sensitivity analysis on this parameter next. The total forecast demand over the 10 year planning horizon is 2502 parts. In Table B1, we give the declining demand pattern over these 10 years, both in terms of mean and coefficient of variation of the demand per year.

Table B1
Pattern of the expected yearly demand.

Year	1	2	3	4	5	6	7	8	9	10
Mean demand	324	310	290	280	259	240	222	205	192	180
CV	0.2	0.3	0.4	0.6	0.8	0.9	1	1.2	1.4	1.6

Our model leads to the following values for the decision variables and key performance indicators:

Near-Optimal LTB quantity = 1570. The base stock levels for the repair decisions are shown in Figure B1.

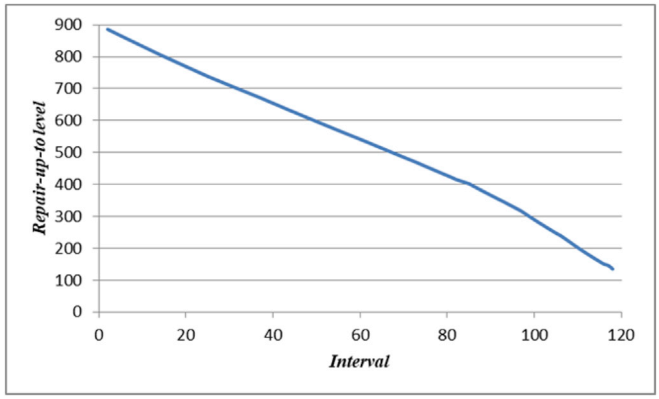

Fig. B1. Repair-up-to levels Case 1.

The overall cumulative fill rate equals 96% and the number of parts to be disposed at the end of the service period is expected to be 6% of the total part supply (number of parts procured at the LTB decision plus the expected number of repaired parts). Further, we find that 43% of the demand is satisfied by repaired parts. In this case, part repair is relatively expensive, and under normal circumstances the company would not repair these parts. Our model reveals that it is attractive to deploy the repair option. If the repair option is not used, the expected total costs are twice as high, and 35% of the parts procured at LTB would be disposed at the end of the service period (so about six times more).

Figure B2 shows the overall cumulative fill rate, the fill rate in the last year, and the percentage demand satisfied by repairs based on the various shortage costs per part. This figure can be used as an indication for the decision makers to set their inventory parameters based on the desired fill rate. Larger shortage cost will yield to higher fill rate particularly in the last interval in expense of higher total costs due to larger LTB at the beginning. We also observe that the usage of the repair option decreases with an increasing fill rate.

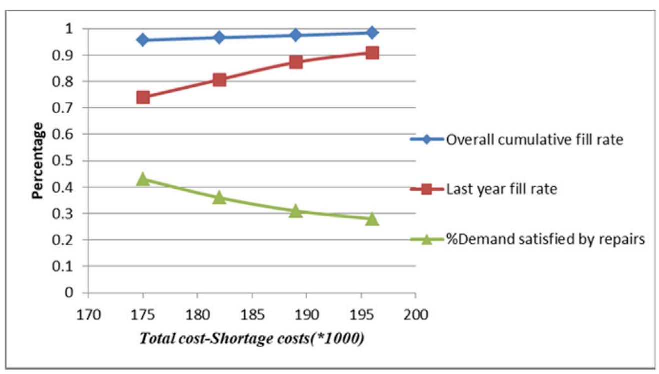

Fig. B2. KPI's based on the various shortage costs.

Case 2:

The period from the LTB decision until the end of the service period is 5 years (60 months). The original part price at the time of LTB is €10. We show the decision variables and key performance indicators based on €50 per backordered part, and next provide a sensitivity analysis on the shortage costs. The total forecast demand over the 5 years planning horizon is 4327 parts. In Table B2, we give the declining demand pattern over these 5 years, both in terms of mean and coefficient of variation of the demand per year.

Table B2
Pattern of the expected yearly demand Case 2.

Year	1	2	3	4	5
Mean demand	1016	940	835	806	730
CV	0.2	0.4	0.6	0.9	1

Our model leads to the following values for the decision variables and key performance indicators:

Near-Optimal LTB quantity = 2600. The base stock levels for the repair decisions are shown in Figure B3.

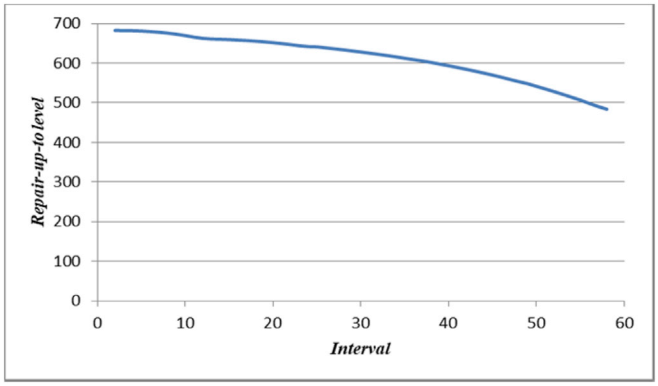

Fig. B3. Repair-up-to levels Case 2.

The overall cumulative fill rate equals 96% and the number of parts to be disposed at the end of the service period is expected to be 3% of the total part supply (number of parts procured at the LTB decision plus the expected number of repaired parts). Further, we find that 38% of the demand is satisfied by repaired parts. As in the previous case, part repair is relatively expensive, and under normal circumstances the company would not repair these parts. Our model reveals that it is attractive to deploy the repair option. If the repair option is not used, the expected total costs are about 1.5 times more, and the expected dispose rate at the end of the service period is 25% (about 8 times more). Compared to the first case, we observe that the expected number of parts disposed at the end of the service period is somewhat lower, and a somewhat lower fraction of demand is served using repair of failed parts. This is due to a shorter remaining service period.

Figure B4 shows the overall cumulative fill rate, the fill rate in the last year, and the percentage demand satisfied by repairs based on the various shortage costs per part. as the previous case, the larger shortage cost will yield to higher fill rate particularly in the last interval in expense of higher total costs due to larger LTB at the beginning and using less from the repair option.

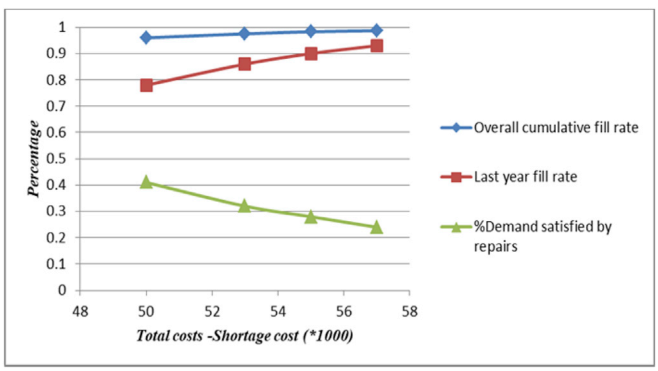

Fig. B4. KPI's based on the various shortage costs.

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