19

Equalizing vectors as a "tool" in H_∞-control

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Abstract

In this paper we present the equalizing vectors as an important "tool" in H_∞-control.

Keywords

Infinite-dimensional systems, Wiener algebra, H_∞-control, equalizing vectors, J-spectral factorization.

19.1 Introduction and main results

We consider the following class of transfer functions, known as the *Wiener algebra*

$$\hat{W} = \left\{ \hat{f} \mid \hat{f} = \hat{f}_1 + \hat{f}_2, \text{ with } \hat{f}_1, \hat{f}_2^{\sim} \in \hat{A} \right\},$$

where $\hat{A}$ (the proper-stable class of transfer functions) consists of all the Laplace transform of functions of the form

$$f(t) = \begin{cases} f_a(t) + f_0\delta(t), & t \geq 0, \\ 0, & t < 0, \end{cases}$$

with f_0 a complex number, $\int_0^\infty |f_a(t)|dt < \infty$, δ represents the delta distribution at zero, and $f^\sim(s) = \overline{f(-\bar{s})}$ for any complex number s. The elements of $\hat{W}$ are bounded and continuous on the imaginary axis, and their limit at infinity is well-defined. Note that proper rational functions which have no poles on the imaginary axis belong to the class $\hat{W}$. We denote by $\hat{A}^{n\times m}$, $\hat{W}^{n\times m}$, the classes of $n \times m$ matrices with entries in $\hat{A}$, $\hat{W}$, respectively, and by H_2 ($H_2^\perp$) the set of all vector-valued functions f analytic in the right (left) half-plane, such that $\sup_{\sigma>0}\int_{-\infty}^{\infty}\| f(\sigma+j\omega)\|^2\,d\omega<\infty$ $(\sup_{\sigma<0}\int_{-\infty}^{\infty}\| f(\sigma+j\omega)\|^2\,d\omega<\infty)$.

We say that a matrix-valued function $Z\in\hat{W}^{n\times n}$, defined on the imaginary axis, admits a J-spectral factorization if there exists a matrix-valued function V, invertible over $\hat{A}^{n\times n}$, such that

$$Z(s)=V^{\sim}(s)\begin{bmatrix} I_p & 0\\ 0 & -I_q\end{bmatrix}V(s)=V^{\sim}(s)JV(s)$$

on the imaginary axis. Here $p+q=n$ and $V^{\sim}(s)=\overline{V(-\overline{s})}^T$.

By definition, a vector u is an equalizing vector of $Z\in\hat{W}^{n\times m}$ if u is a nonzero element of H_2 such that Zu is in $H_2^{\perp}$. We show that $Z=Z^{\sim}\in\hat{W}^{n\times n}$ (with nonzero determinant on the imaginary axis) admits a J-spectral factorization if and only if Z has no equalizing vectors (see [1]). This extends the result from [5] to non-rational functions.

Using equalizing vectors, a simple frequency domain solution is obtained for the suboptimal Nehari extension problem for our class of infinite-dimensional systems. Moreover, the equalizing vector is fixing the solution of the Nehari extension problem in the direction of the eigenvector corresponding to the largest singular value of the Hankel operator. For the scalar case, an equalizing vector can be used to prove the uniqueness of the solution for the Nehari extension problem (see [3]). Similar technics can be used to solve the suboptimal Hankel-norm approximation problem (see [4]).

In order to provide necessary and sufficient conditions for the solvability of the standard H_∞-suboptimal control problem for systems with the transfer function in a subalgebra of the quotient field of the Wiener algebra, equalizing vectors play also a very important role (see [2]).

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