Green Wave Analysis in a Tandem of Traffic-Light Intersections

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Abstract Green waves in urban settings are seen to be beneficial both for the drivers and the environment because they lower delays and reduce pollution. To obtain these benefits, it is important to determine the optimal network parameters, e.g., green splits and offsets, that yield the desired traffic behaviour. We consider two possibilities for network optimization. The first one is minimising the average lost time per vehicle; the second one is maximising the green wave efficiency. We compare the results with the MAXBAND algorithm, which is used at the design stage of arterial traffic light settings to facilitate green waves. Using our optimisation method, the average delay per vehicle can be reduced by up to 10% and more vehicles can encounter a green wave compared to the MAXBAND approach. The reduction of the average delay for the main stream reaches 20%.

Keywords: fixed traffic-light control, green wave, stochastic model

1. INTRODUCTION

The use of green waves in traffic control is considered to be a promising way of reducing delays and emissions. The coordinated intersections provide a higher capacity for the main directions and, as a result, give shorter delays. The traffic flows become smoother and the air pollutant emissions, such as CO_2, NO_x, are reduced in range of $10\% - 40\%$, see (*1*).

At the design stage of the urban traffic control, the traffic is often assumed to be deterministic and going through the network at free-flow speed, see, for example, MAXBAND (*2*) and TRANSYT (*3*). However, the delayed vehicles waiting at traffic intersections should also be taken into account. For example, the vehicles from side roads may arrive during the red time and form a queue, which interrupts the flow of vehicles moving with the green wave. These vehicles will also need to accelerate and, therefore, come to the next intersection much later than if they would go at free-flow speed. This may lead to further disruption of the green wave and, therefore, to longer delays. Moreover, it is also possible that the queue of vehicles waiting at an intersection is long and is not cleared during the green time. For these reasons, the real behaviour of the traffic should be taken into account at the design stage. One of the ways to do so is to use a traffic microsimulation. However, while it is a useful tool to predict network performance, it is time-consuming and, therefore, not very well suited for network optimisation purposes.

In this paper, we optimise a tandem of intersections operating under fixed-cycle traffic control. For this purpose, we use the stochastic model presented in (*4*). In this model, we take into account the randomness of the drivers behaviour and of the arrival processes to the network. It can be seen as an extension of the fixed-cycle traffic-light model (see (*5*)). In (*4*), the focus is on the difference between fixed and semi-actuated control. In the current paper, we analyse green waves. In particular, we introduce a new measure of green wave efficiency, namely the expected number of intersections for a vehicle that were passed without stopping. This means that a vehicle arrived during green time and there were no vehicles in the queue. For an ideal green wave, the expected number of such intersections is equal to the expected number of intersections in a route. In the worst case scenario, all of the vehicles need to stop and, therefore, our measure of green wave efficiency is equal to 0.

We also compare the performance of the settings with the highest green wave efficiency with the settings with the lowest average delay per vehicle. Both cases turn out to perform better in terms of green wave efficiency than the settings given by the MAXBAND algorithm, which seeks the broadest possible green wave, see (*2*). Using delay optimisation, the delays may be reduced by up to 10% compared to MAXBAND. For the main stream, the reduction reaches 20%. The results given by the model are validated using the microsimulation suite SUMO, see (*6*) for more details about SUMO.

The paper is structured as follows. In Section 2, we briefly present the model discussed in (*4*). Then we give a short description of both the MAXBAND and genetic algorithms, in Section 3. Finally, in Section 4, we present numerical results and, in Section 5, we conclude the paper.

2. MODEL

In this section, we briefly describe the model presented in (*4*). The model allows us to quickly compute the performance of the traffic-light settings. For one setting, we find the average delay and the green wave efficiency measure in about 5 seconds, while a reliable estimate using SUMO takes about 20 minutes (it includes 41 simulations).

2.1. Network

We consider the following network. There are three intersections at regular distances of 100 meters that form an arterial in west-east direction, see Figure 1. There are 24 lanes in total.

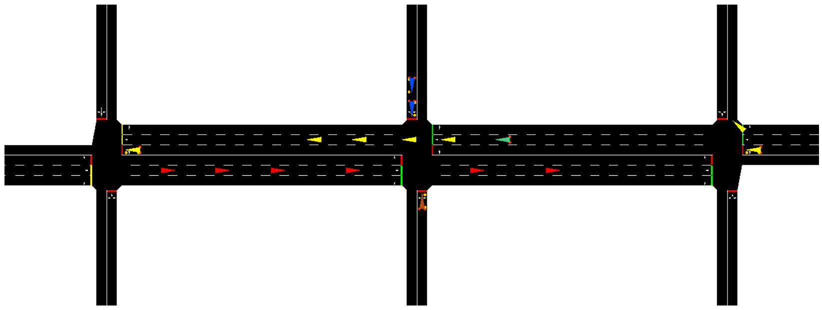

FIGURE 1 The tandem of three intersections in SUMO.

The arrival rate for the major roads (from west and east) is higher than the arrival rate for the minor roads (from north and south). We assume that most of the vehicles want to use the arterial road, i.e., the turning probability for the major road is low and for the minor road is high. We consider a fixed phase schedule, see Figure 2.

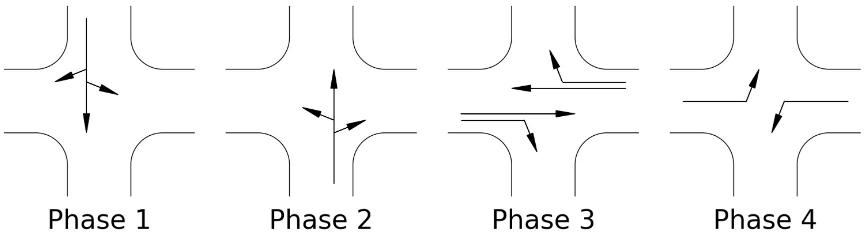

FIGURE 2 The phase schedule of the traffic-light settings.

The minor roads do not have lanes for the turns, and, therefore, has one phase for all movements. For the major roads, it is better to use a special phase for the left turns because of the low turning probability. In this way, the major directions have higher capacity.

2.2. Model Overview

In our model, we look at each lane of the network separately. The input for the model consists of the red and green times together with arrival rate or the departure processes from the upstream lanes. Using them, we find the average delay per vehicle in the queue and the probability of passing this intersection without stopping, i.e., the probability of encountering green light and no queue upon arrival.

In this paper, we consider fixed traffic control, i.e., the lengths of the red and green times are fixed. We assume that the delayed vehicles depart only at predetermined times after the beginning of the green time. However, due to the drivers distraction the departures of the delayed vehicles may be postponed by up to 2 seconds. We incorporate all possible uncertainties in the postponing of the first vehicle in the queue. The following vehicles are postponed by the same number of seconds as the first one. In Section 4, by comparing with SUMO, we show that this assumption gives a good approximation to reality. Note that using predetermined departure times allows us to model slow departures for the vehicles at the beginning of the queue. In (*4*), we determined the departure times and the probability of postponing the departures empirically by using SUMO. If the queue is empty during the green time, all vehicles that arrive during the rest of this green time can proceed without delay at a free-flow speed. These vehicles, in fact, encounter a green wave.

The arrivals come either from another intersection or from outside of the system. In the latter case, we assume that each second a vehicle may arrive with a certain probability, that the arrival rate is constant, and that the arrivals are independent of each other. We do not consider more than one arrival per second since the inter-arrivals times of vehicles to the lane of less than one second are unrealistic due to the safety reasons. The arrivals from the previous intersection we find by analysing the departure process from all upstream lanes. In this analysis, we incorporate the time needed to accelerate after the intersection. In both cases, we can model arrivals using a (non-stationary) Markov chain such that arrivals at second i depend only on the state of the Markov chain at this second and not on the previous arrivals, see (4) for more details. For model tractability, we assume that arrivals during different cycles are independent of each other. Therefore, the beginning times of the cycles are regeneration points of our Markov chain.

2.3. Queue Length Results

With the previous assumptions, it is possible to find the probability generation function (pgf) of the queue length at the beginning of the cycle by considering the changes to the pgf of the queue length during one cycle. In this paper, we give just a brief explanation of the results, for more details and proofs see (4). The pgf of the queue length at the beginning of a cycle has the following rational form:

$$X_0(z) = \frac{\sum_{j=0}^{n-1} x_j f_j(z)}{z^n - A(z)},$$

where n is the maximum number of delayed departures per cycle, x_j, $j = 0, \dots, n-1$, are the probabilities of having j vehicles in the queue at the beginning of a cycle. The function $A(z)$ is the pgf of all arrivals during one cycle multiplied by the pgf of lost capacity due to the randomness of the first delayed departure, i.e., by $\sum_{s=0,1,2} \kappa_s z^{n-n_s}$, where n_s is the maximum number of delayed departures in the case when the departures are postponed by s seconds, and κ_s is the probability of this event. Informally speaking, each function $f_j(z)$, $j = 0, \dots, n-1$, is the difference between what happens if the queue length at the beginning of the cycle is j vehicles and what would happen if the queue would be longer than n vehicles. The difference is caused by the fact that the queue may become empty during the green time if it was not long enough at the beginning of the cycle. For the sake of clarity, we omit the complicated formula for $f_j(z)$ and refer to (4) for more details.

All the components of the pgf except for x_j can be found by considering the arrival process and the lengths of green and cycle times. The probabilities x_j follow from the analyticity of the pgf, see (4). After finding x_j, we determine the average queue length at the beginning of the cycle. Using it, we find the evolution of the average queue length during the cycle. As a result, we get the average queue length at an arbitrary second of the cycle. Finally, using Little's law, we find the average delay per vehicle.

2.4. Extra Delay Due to Acceleration

The delay acquired using our model is shorter than the time lost by vehicles. The reason for it is that the departing vehicles need to accelerate after the intersection. Using SUMO for a single lane, we can find the arrival moments to the downstream intersection depending on the departure number of the vehicle, i.e., depending whether it is the first, second, etc. departure from the lane. The arrival moments are calculated starting from the beginning of the corresponding green time. Then, using the departure and arrival moments, we find the time lost due to acceleration, which also depends on the departure number. Note that a delayed departure prevents the following arrivals from proceeding with the free-flow speed. Finally, to find the average extra delay, we need the probabilities to have exactly k delayed departures, which we find using the arrival process and the probabilities x_j obtained earlier.

3. DESIGN OF GREEN WAVES

In this section, we consider two ways to set the parameters of a tandem of traffic lights in order to make a green wave. The first one is the MAXBAND algorithm, which assumes traffic flows to be deterministic. The second one is the genetic algorithm, which uses results given by the model described in Section 2 to find sub-optimal settings. For simplicity, we fix the cycle length, phase schedule and the speed limits for both algorithms. In this way, we can focus on the differences between deterministic (MAXBAND) and stochastic (our model) approaches. Moreover, fixing the cycle length allows us to compare the algorithms for different loads.

3.1. MAXBAND Algorithm

In its simplest form, the MAXBAND algorithm, (2), finds optimal offsets between consecutive intersections for given settings of the traffic lights. We choose the same settings for each intersection. For our network and the chosen phase schedule (see Section 2), the problem of maximising the bandwidth can be formulated as follows:

$$\max b + k\bar{b}$$

$$w_i + b \leqslant g, \quad (1)$$

$$\bar{w}_i + \bar{b} \leqslant g, \quad (2)$$

$$(w_i + \bar{w}_i) - (w_{i+1} + \bar{w}_{i+1}) + (\bar{\tau}_i + \tau_{i+1}) = 2t + m_i c, \quad (3)$$

$$m_i \in \mathbb{Z},$$

$$b, \bar{b}, w_i, \bar{w}_i \geqslant 0. \quad (4)$$

Here b ($\bar{b}$) is the west-east (east-west) bandwidth, k is the so called target ratio of the bandwidths, w_i ($\bar{w}_i$) is the time-gap between the west-east (east-west) green band and the following (previous) red time at intersection i, t is the travel-time from one intersection to another, g is the green time, c is the cycle time, τ_i ($\bar{\tau}_i$) is the queue clearance time, and m_i is the so called loop integer. In Figure 3, we show the interrelation between these variables. All times are given in seconds. The target ratio between the bandwidths b and $\bar{b}$ is usually chosen to be equal to the ratio of the traffic flows.

For the optimal values of $b, \bar{b}, w_i, \bar{w}_i$, the offset of intersection $i+1$ from intersection i is equal to $w_{i+1} - \tau_{i+1} + t - w_i$.

3.2. Genetic Algorithm for the Optimal Settings

Since the space of possible settings grows exponentially with the number of intersections, we chose a genetic algorithm to find a sub-optimal solution. This approach is widely used in optimizations of traffic settings (see, e.g., (3, 7, 8)). Compared to local search, this approach considers a wide selection of solutions and, therefore, avoids the problem of local optimum.

We consider two objectives: minimising the average delay per vehicle and maximising the green wave efficiency, i.e., maximising the expected number of intersections passed without stopping in a route of an arbitrary vehicle. We use the following genetic algorithm. For each set of arrival rates, we consider a population of 140 settings. For each setting, we compute the objective functions. Then, we generate a new population of the same size by combining settings of the previous generation in the following way. For each of objective, we produce 70 new settings. The existing settings with better values of the objective are more likely to be used. The combination of two settings is produced by operations "crossover" and "'mutation".

For the crossover operation, we produce the "chromosome" of settings and offsets. The settings for each intersection are translated to 3 numbers which represent the division of the extra green time (the total green time without the minimal green time) between phases. Each chromosome contains

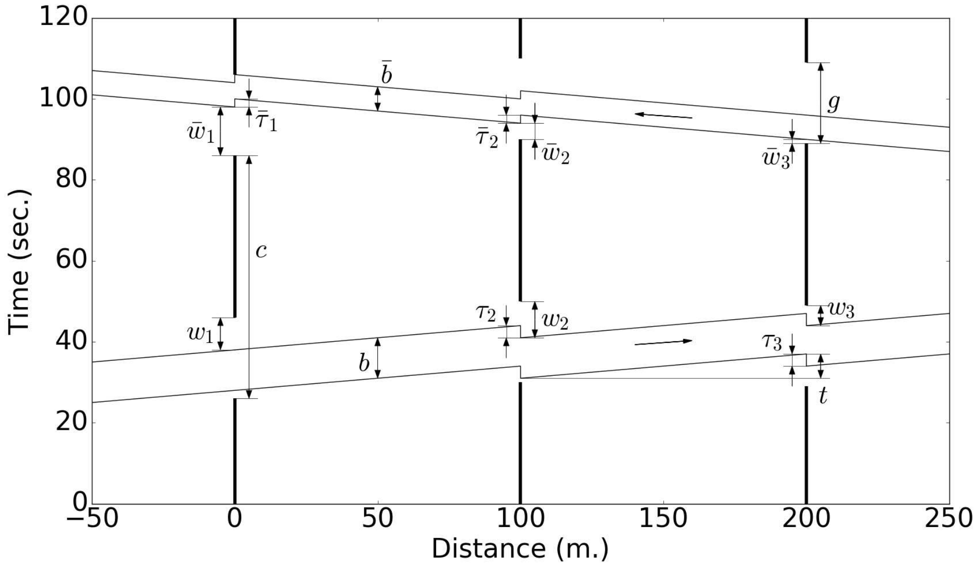

FIGURE 3 Space-time diagram for the MAXBAND algorithm. The bold lines show the red times for the major directions.

information of these numbers and offsets. The crossover operation combines a part of the chromosome of one "parent" with the other part of the chromosome of the second "parent". The rest of these settings is combined to be the second "child". The moment of switching from one parent to another is chosen randomly. Then with a certain probability (0.4) a mutation occurs. It changes one of the settings parameters. If the offset parameter is chosen, then it is changed to a random offset. If the phase is chosen, then we exchange one second between this phase and the following or previous phase, if it is possible (the resulting settings provide enough time for each lane). This procedure continues for 30 generations. For each objective, the settings with the best performance are further improved by choosing the best neighbour until the settings cannot be improved anymore.

4. NUMERICAL RESULTS

In this section, we compare the settings obtained by the algorithms described in Section 3. We also validate results of our model using the microsimulation suite SUMO.

4.1. Network

In our network, we fix the cycle length of traffic lights (60 seconds) and speed limits (60 km/hour). The arrival rates to the minor roads (from north and south) and from east are 20% and 50% of the arrival rate from west, respectively. The probabilities of turning north or south on the main road are both equal to 0.08. The turning probabilities of the minor roads are chosen so that to compensate for the departures from the major roads, i.e., the probability of turning east is 0.4 and of turning west is 0.2. We set the switch (yellow) time to be 6 seconds. We consider 5 loads from 0.5 to 0.9. For a lower load, it is better to shorten the cycle length since most of the delay is caused by waiting for the end of the red time.

4.2. MAXBAND Approach

In the considered example, the cycle length, c, is 60 seconds, the travel time between intersections, t, is 6 seconds and the target ratio, k, is 0.5. For the considered loads (between 0.5 and 0.9), the number of arrivals from side roads is, on average, between 0.64 and 1.16 vehicles on the west-east direction and between 0.32 and 0.58 vehicle on the east-west direction. Hence, the values of τ_i and $\bar{\tau}_{i+1}$ are bounded by 5 seconds, which is enough for one vehicle to depart from the lane. Note that in fact the queue may be longer, but this fact can not be embedded in MAXBAND. In the case of the lowest load (0.5), the green time of phase 3 (for the major road) is not more than 30 seconds. Hence, the sum $w_i + \bar{w}_i$ is bounded by cycle length. As a results, if any of m_i is not equal to 0, then the optimal value of $b + k\bar{b}$ for such m_i is too low to be the optimal for all possible m_i. Therefore, the only solution in terms of m_i is $m_i = 0$. In this case, we can easily find optimal w_i, $\bar{w}_i$, b and $\bar{b}$ analytically.

Using (3), we find $(w_1+\bar{w}_1) = (w_2+\bar{w}_2)+2t-\tau_2-\bar{\tau}_1$ and $(w_1+\bar{w}_1) = (w_3+\bar{w}_3)+2(2t-\tau_2-\bar{\tau}_1)$ since $\tau_2 = \tau_3$, $\bar{\tau}_1 = \bar{\tau}_2$. Hence, using (1) and (2), we get

$$b + \bar{b} \leqslant 2g - 2(2t - \tau_2 - \bar{\tau}_1) - (w_3 + \bar{w}_3).$$

The right hand-side of the latter equation has the highest value if $w_3 = \bar{w}_3 = 0$. Thus, from (1), (2) and (4), it follows that the optimal values for b and $\bar{b}$ are $b = g$, $\bar{b} = g - 2(2t - \tau_2 - \bar{\tau}_1)$ and the offsets of the second and third intersection from the first one are equal to $t - \tau_2$ and $2(t - \tau_2)$, respectively. Note that $\bar{\tau}_i$ for each i does not influence the offsets. Using our model, we found that the best value for τ_i is 0, which can not be interpreted as queue clearance time anymore. Compared to results for $\tau_i = 5$, the average delay is reduced by 10%. In this example, we fix the settings of the traffic lights such that the system is still stable for high load. Such choice can be seen as an alternative of Webster formula, see (9), for the discrete settings. Each phase is provided with enough time to maintain stability, and this time is independent of the load.

TABLE 1 The Optimal Settings (in sec.) for Load 0.8

Green wave design	Intersection 1 phases				Intersection 2 offset	phases				Intersection 3 offset	phases			
	1	2	3	4		1	2	3	4		1	2	3	4
MAXBAND	7	7	20	2	6	7	7	20	2	12	7	7	20	2
opt. delay	7	7	20	2	10	7	7	20	2	18	7	7	20	2
opt. green wave	7	8	18	3	10	7	7	20	2	17	7	7	20	2

4.3. Average Delay per Vehicle

In this subsection, we consider the average delay per vehicle, see Figure 4. The results for optimal delay give up to 10% improvement compared to MAXBAND. The results for the optimal green wave are better than MAXBAND for load 0.9. For a medium load (0.5, 0.6), this optimization favours the major directions and the delay on the minor roads is very high. For a higher load (0.7, 0.8), the behaviour is less clear. In Table 1, we give, as an example, the optimal settings for load 0.8. Note that almost in every case the total green time is divided between phases in the same way. In our network, it seems to be the best way to do it. In fact, the settings for the optimal delay always have such phases independently of the load. If we fix such phases and optimise only the offsets, the results for optimal green wave become indistinguishable from the results for optimal delay.

The comparison with SUMO shows that the results of our model are very accurate except for a very high load (0.9). The reason for it is that we assume independence between arrivals in different

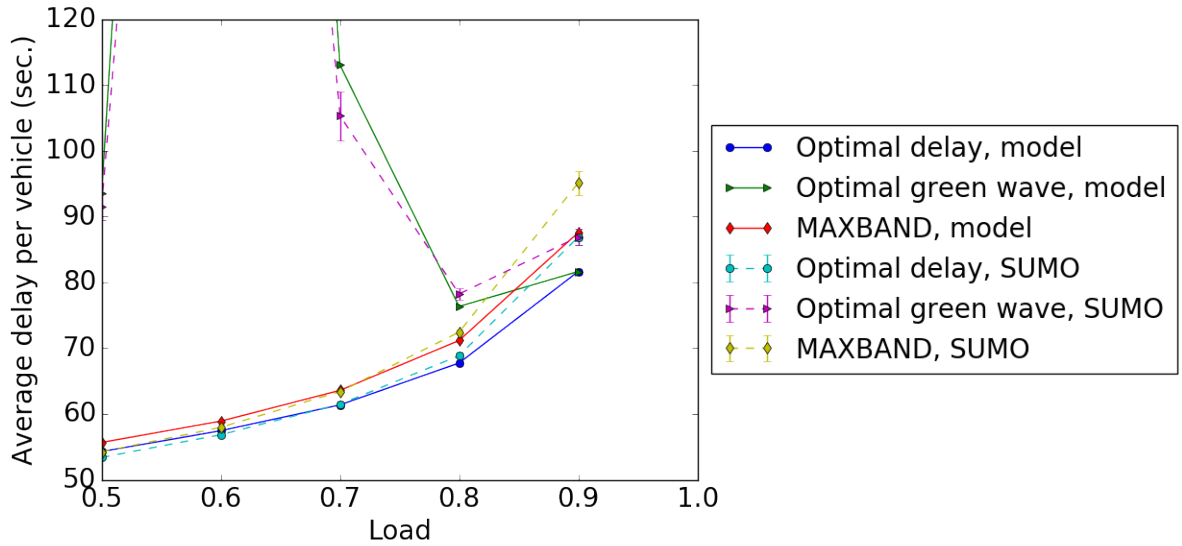

FIGURE 4 The average delay per vehicle as a function of load depending on the green wave design (optimisation of average delay, green wave measure or MAXBAND). The results of the model are compared with SUMO.

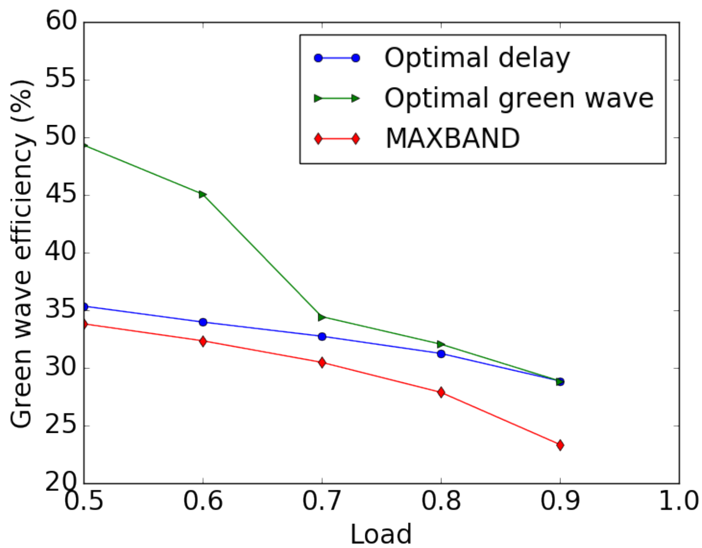

FIGURE 5 The green wave efficiency as a function of load depending on the green wave design (optimisation of average delay, green wave measure or MAXBAND).

cycles. However, in case of high load, the departures from a lane are strongly correlated, and therefore, the arrivals from upstream lanes are also correlated. Note that operating under such load is undesirable, and, in this case, it is beneficial to lengthen the cycle length.

4.4. Green Wave Efficiency

In this subsection, we consider the measure of green time efficiency, i.e., the average number of intersections passed without stopping in an arbitrary route. We compare three ways of green wave design in terms of this measure. As can be seen from Figure 5, the results of genetic algorithm with both objectives give much better green waves than MAXBAND. In case of high load, the difference is 1.23 times.

To know whether the main directions benefit from the acquired settings, we used SUMO to find the average delay per vehicle arriving from the west or east, see Figure 6. We see that the delay in west-east direction is much shorter than for the east-west direction. What happens is that the green wave is possible, basically, only in one direction, and since the west-east direction has a higher weight, the settings favour this direction. The settings for optimal delay reduce delay for both directions compared to MAXBAND except for load 0.5, for which the east-west direction gets a significant reduction of delay with a small rise in the delay for the west-east direction. For load 0.9, the delays for the west-east direction are reduced by 20%. The benefits of the optimal green wave efficiency settings for some directions are not as significant as the deterioration of the delay for other directions. This type of optimisation can be used only for a restricted search space.

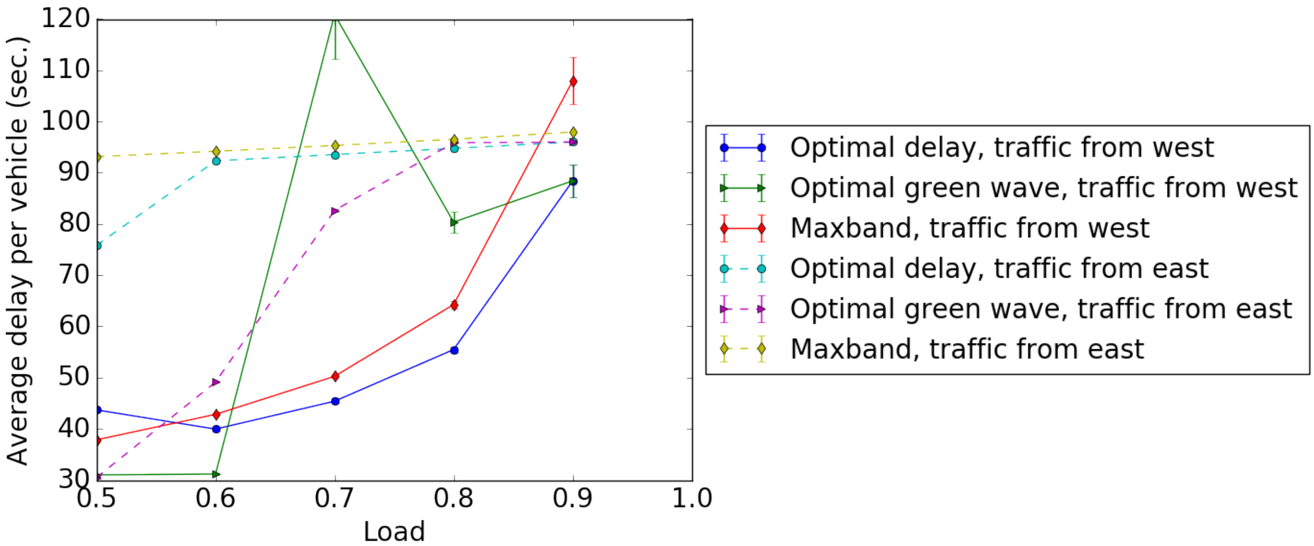

FIGURE 6 The average delay per vehicle as a function of load depending on origin of the vehicle and the green wave design (optimisation of average delay, green wave measure or MAXBAND). The results are given by SUMO.

5. CONCLUSIONS

In this paper, we have considered the problem of green wave design. We have used an analytical model, which takes the randomness of traffic into account, to find the network performance depending on the traffic-light settings. The time-efficiency of this model allowed us to optimise these settings. We have introduced a green wave efficiency measure and optimised it and the average delay per vehicle using a genetic algorithm. The results of the optimisation show a significant improvement compared to the MAXBAND algorithm. The optimisation with each of the objectives provides such settings that the vehicles more often experience a green wave. However, the setting with better green wave efficiency may be very disadvantageous for some lanes. The results of the model were also validated by comparison with the microsimulation suite SUMO. We conclude that the combination of our model with the delay optimisation technique is very beneficial for the traffic and can be used at the design stage of the traffic control.

Future work will be on considering different traffic control policies. For example, adaptive control using the detectors' information for a more efficient use of the cycle time. It is also important to investigate the properties of the objectives. In this way, it is possible to find a smarter optimisation algorithm.

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