# Voter discernment and candidate entry in pluralitarian election* 

ROGER D. CONGLETON ${ }^{1}$ \& BERNARD STEUNENBERG ${ }^{2}$<br>${ }^{1}$ Center for Study of Public Choice, George Mason University, Fairfax, VA 22030, U.S.A.;<br>${ }^{2}$ Faculty of Public Administration and Public Policy, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands

Accepted 4 January 1996


#### Abstract

The paper develops a sequential model of candidate entry into elections decided on the basis of plurality. We analyze the kinds of candidates who are most likely to enter elections and simulate several plausible myopic entry sequences under various assumptions about voter abilities to discern differences in candidate positions. In the cases examined, open elections for "important" positions attract the entry of more than two candidates. Moreover, myopic entry often generates electoral outcomes which depart from the median-mean outcomes of the conventional models. These results are consistent with the observed diversity of candidates in presidential and other significant primary elections which contrasts with many previous analyses of electoral entry.


## 1. Introduction

There is an extensive literature on elections between fixed numbers of candidates under wide variety of electoral rules and assumptions about the motivations of candidates and voters. However, until recently, little work has addressed the question of how candidates come to join an electoral contest. The problem of entry into elections is not mentioned, for example, in Mueller's (1989) extensive survey of the Public Choice Literature. Nor is the topic analyzed in the fundamental texts by Downs (1957), Black (1958) or Tullock (1967). ${ }^{1}$ During the past decade a number of political theorists have begun to explore various aspects of party and candidate entry into elections. Palfrey (1984, 1989), Brams and Straffin (1982), and Shepsle and Cohen (1990) use spatial models to analyze whether political parties can protect themselves from third party entry by nominating candidates that deviate from the median preference.

Many of these papers affirm the Duverger (1954) hypothesis that two party systems are stable equilibrium phenomena under pluralitarian electoral

[^0]institutions. Such results are consistent with the historical experience of the United States and Great Britain where the normal state of electoral affairs is characterized by two dominant political parties. However, the work sheds relatively little light on candidate entry in electoral contests where party affiliation is a less decisive factor for voters. For example, multiple-candidate elections are the norm rather than the exception in elections for important positions within political organization.

Fedderson, Sened, and Wright (1990) provide a possible explanation of multiple candidate single winner elections based on a Nash simultaneous candidate entry game, but their characterization of the equilibrium implies that all candidates take identical positions at median voter's ideal point. The latter conflicts with recent experience in primaries where candidates often taken a wide variety of positions - although it might be argued that the mean of the distribution of candidates tends to be at approximately the median of the overall relevant population of voters.

Previous models of candidate entry appear to capture essential features of equilibria in electoral contests with fully rational voters and candidates. So, evidently either the observed pattern of candidate entry is not behavior at full equilibrium or does not reflect fully rational and informed decisions by electoral participants.

This paper explores implications of less than fully informed decisions by candidates and voters. We analyze the kind of candidates who are likely to enter an election, simulate vote maximizing sequences of entry under different assumptions about voter abilities to discern differences between candidates, and attempt to determine whether or not the probability of electoral success falls to levels where entry ceases.

The model used as the basis of the simulation experiments is broadly similar to those of Palfrey (1984) and Fedderson, Sened and Wright (1990), but without their equilibrium focus. The simulation results demonstrate that sequential entry, together with imperfect discernment, is sufficient to generate electoral outcomes that depart from the usual median or mean results of modern voting models. In this respect, the results are similar to those developed by Palfrey (1984), but diverge from those of Fedderson, Sened, and Wright (1990), who identify the median of the distribution of voter ideal points as the equilibrium result in case of single-winner elections. ${ }^{2}$ The scenarios developed below suggest that new entrants are often very likely to win. This, of course, is the most likely explanation of entry, although it contrasts with Palfrey's (1984) results which may be more applicable to entry of long-lived political parties rather than entry by candidates. The simulations suggest that expected votes and the average probability of electoral success tend to fall as entry occurs. Moreover, in many cases the last candidate who
enters has a higher probability of success than previous entrants. As developed below, the decline in the probability of electoral success implies that there tends to be an upper bound on the number of candidates willing to enter a given electoral contest with finite numbers of voters and distribution of potential candidates.

The analysis is organized as follows. Section 2 develops a model of candidate decisions to enter electoral contests. Section 3 analyzes the effects of imperfect voter discernment on incentives for candidates who sequentially enter elections, and demonstrates that a third candidate can often win elections by adopting non-central policy positions. Section 4 uses simulation methods to demonstrate that the probability of electoral success tend to fall as entry occurs. Together with the analysis of section 2, this implies the existence of an upper bound on the number of candidates that will enter any particular election. Section 5 discusses the possibilities for strategic entry evident in the simulation results. Section 6 summarizes the results and suggests implications and extensions of the analysis.

## 2. Candidate type and the probabilistic prerequisites for entry

In order to examine some of the factors which influence candidate utility levels, consider the following model of a utility maximizing prospective candidate. Let $\mathrm{W}^{\mathrm{W}}$ be the candidate's wealth if he runs and achieves his electoral aim, normally winning the election. Let $\mathrm{W}^{1}$ be his wealth if he runs and is unsuccessful, and let $\mathrm{W}^{0}$ be his wealth if he does not run for office. C is the cost incurred if he participates in the campaign. The policy adopted if the prospective candidate runs for office and is successful is $\mathrm{G}^{\mathrm{w}}$, while $\mathrm{G}^{0}$ is adopted is he does not enter or is not successful. To simplify exposition, as conventional, we assume that candidate utility be defined over two goods: public service level G and personal income $\mathrm{Y}=\mathrm{W}-\mathrm{C}$.

From a rational choice perspective, it is tautological that a candidate will run for office when the expected utility associated with the political contest exceeds that of not running. ${ }^{3}$ Here, the individual enters the election whenever a platform exists such that:

$$
\begin{equation*}
\mathrm{PU}\left(\mathrm{G}^{\mathrm{W}}, \mathrm{~W}^{\mathrm{W}}-\mathrm{C}\right)+(1-\mathrm{P}) \mathrm{U}\left(\mathrm{G}^{0}, \mathrm{~W}^{1}-\mathrm{C}\right)>\mathrm{U}\left(\mathrm{G}^{0}, \mathrm{~W}^{0}\right) \tag{1}
\end{equation*}
$$

Written this way, it is clear that the prospective candidate becomes more inclined to enter an election as the probability of winning increases, as the pecuniary advantage increases, as personal campaign costs decline, and as the personal value of the candidate's preferred policy relative to the alternative increases. ${ }^{4}$

The policy that maximizes the expected utility of running for office, $\mathrm{G}^{\mathrm{W}^{*}}$, can be found by differentiating the left-hand side of equation 1 with respect to candidate 2 's position, $\mathrm{G}^{\mathrm{W}}$, and setting the result equal to zero. $\mathrm{G}^{\mathrm{W}^{*}}$ satisfies:

$$
\begin{equation*}
\mathrm{P}_{\mathrm{G}} \mathrm{U}^{\mathrm{W}}+\mathrm{PU}_{\mathrm{G}}^{\mathrm{W}}-\mathrm{P}_{\mathrm{G}} \mathrm{U}^{1}=0 \tag{2}
\end{equation*}
$$

where $\mathrm{U}^{\mathrm{W}}$ is the utility realized by electoral success, and $\mathrm{U}^{1}$ is the utility associated with losing the election. (Subscripts denote partial derivatives with respect to the variable subscripted.) Equation 2 indicates that the prospective challenger's expected utility maximizing policy is partly a matter of the candidate's own policy preferences and partly a matter of the effect that alternative policies have on his probability of success.

Incentives for different kinds of candidates to run for office are determined partly by the range of values that $\mathrm{U}_{\mathrm{G}}^{\mathrm{W}}$ might take. In the case where the challenger is nearly indifferent between alternative policies, $\mathrm{U}_{\mathrm{G}}^{\mathrm{W}} \Rightarrow 0$, and the ideal position is that which maximizes the probability of electoral success. In the case where a candidate has strong policy preferences, $\mathrm{U}_{\mathrm{G}}^{\mathrm{W}}$ differs from zero over the full range of platforms, and the policy position adopted reflects the tradeoff between the policy interests of the candidate and his interest in being elected.

In the latter case, part of the cost of running for office is the cost of what Kuran (1987) terms preference falsification. The cost of preference falsification implies that, other things being equal, policy interested candidates whose own policy preferences have a high probability of success are more inclined to run for office than otherwise similar candidates with different policy preferences. The marginal cost of running for office is smaller for sincere candidates since $\mathrm{U}_{\mathrm{G}}^{\mathrm{W}}=0$ at their preferred policy. Such "sincere" candidates have the "right" preferences, and may simply adopt the platform which maximizes their own utility. Absent an equivalent "sincere" candidate, "pragmatic" individuals who bear the lowest personal cost for preference falsification have a relative cost advantage in positioning themselves for electoral success. ${ }^{5}$

Equations 1 and 2 also indicate a central role for the probability of electoral success function. As the probability of winning or affecting the outcome falls to zero, the expected utility of participating in the campaign approaches $\mathrm{U}\left(\mathrm{G}^{0}, \mathrm{~W}^{0}-\mathrm{C}\right)$ which is necessarily less than $\mathrm{U}\left(\mathrm{G}^{0}, \mathrm{~W}^{0}\right)$ as long as $\mathrm{C}>0$. In the case where the probability of influencing the electoral outcome is zero, the expected net advantage of candidacy is necessarily less than zero. Clearly , a potential candidate who anticipates no advantage from being a candidate will not run for office.

Equation 1 also implies the existence of a lowest probability of success sufficient to induce a candidate to enter an election. A candidate is indiffer-
ent between running and not running for office in the case where the left hand side of equation 1 equals the right-hand side. Solving this expression for P yields the lowest probability sufficient to induce a candidate to run for office, and yields a modest generalization of the Fedderson, Sened, and Wright (1990) characterization of sufficient conditions for entry:

$$
\begin{equation*}
\mathrm{P}^{\min }=\frac{\mathrm{U}\left(\mathrm{G}^{0}, \mathrm{~W}^{0}\right)-\mathrm{U}\left(\mathrm{G}^{0}, \mathrm{~W}^{1}-\mathrm{C}\right)}{\mathrm{U}\left(\mathrm{G}^{\mathrm{W}}, \mathrm{~W}^{\mathrm{W}}-\mathrm{C}\right)-\mathrm{U}\left(\mathrm{G}^{0}, \mathrm{~W}^{1}-\mathrm{C}\right)} \tag{3}
\end{equation*}
$$

A candidate runs for office whenever the probability of electoral success exceeds the ratio of his potential incremental utility losses to utility winnings. ${ }^{6}$
Denote the utility associated with winning as $\mathrm{U}^{\mathrm{W}}$, that of losing as $\mathrm{U}^{1}$, and that of not entering the election as $\mathrm{U}^{0}$, and partial derivatives with subscripts. The partial derivatives of $\mathrm{P}^{\mathrm{min}}$ are:

$$
\begin{align*}
\mathrm{P}_{\mathrm{C}}^{\min } & =\left[\mathrm{U}_{\mathrm{Y}}^{1}\right] /\left[\mathrm{U}^{\mathrm{W}}-\mathrm{U}^{1}\right]-\left[\mathrm{U}^{0}-\mathrm{U}^{1}\right]\left[\mathrm{U}_{\mathrm{Y}}^{\mathrm{W}}-\mathrm{U}_{\mathrm{Y}}^{1}\right] /\left[\mathrm{U}^{\mathrm{W}}-\mathrm{U}^{1}\right]^{2} \\
& =\left\{\left[\mathrm{U}^{0}-\mathrm{U}^{1}\right] \mathrm{U}_{\mathrm{Y}}^{\mathrm{W}}+\left[\mathrm{U}^{\mathrm{W}}-\mathrm{U}^{0}\right] \mathrm{U}_{\mathrm{Y}}^{1}\right\} /\left[\mathrm{U}^{\mathrm{W}}-\mathrm{U}^{1}\right]^{2}>0  \tag{4.1}\\
\mathrm{P}_{\mathrm{W}}^{\min } & =-\left[\mathrm{U}^{0}-\mathrm{U}^{1}\right]\left[\mathrm{U}_{\mathrm{Y}}^{\mathrm{W}}\right] /\left[\mathrm{U}^{\mathrm{W}}-\mathrm{U}^{1}\right]^{2}<0  \tag{4.2}\\
\mathrm{P}_{\mathrm{W} 0}^{\min } & =\left[\mathrm{U}_{\mathrm{Y}}^{0}\right] /\left[\mathrm{U}^{\mathrm{W}}-\mathrm{U}^{1}\right]>0  \tag{4.3}\\
\mathrm{P}_{\mathrm{W} 1}^{\min } & =\left[-\mathrm{U}_{\mathrm{Y}}^{1}\right] /\left[\mathrm{U}^{\mathrm{W}}-\mathrm{U}^{1}\right]+\left[\mathrm{U}^{0}-\mathrm{U}^{1}\right]\left[\mathrm{U}_{\mathrm{Y}}^{1}\right] /\left[\mathrm{U}^{\mathrm{W}}-\mathrm{U}^{1}\right]^{2} \\
& =\left[\mathrm{U}_{\mathrm{Y}}^{1}\left(\mathrm{U}^{0}-\mathrm{U}^{\mathrm{W}}\right)\right] /\left[\mathrm{U}^{\mathrm{W}}-\mathrm{U}^{1}\right]^{2}<0 \tag{4.4}
\end{align*}
$$

The partial derivatives of $\mathrm{P}^{\text {min }}$ indicate that the probability threshold for entry increases as total cost C increases, and declines as the pecuniary advantage of office increases. Other things being equal, candidates will accept a lower probability of success in the pursuit of an office with a relatively large direct or indirect salary associated with it. Similarly, candidates with a relatively low cost of participation - because they are sincere, have relatively low preference falsification costs, or are for other cost advantages - will enter with lower probabilities of success. The $\mathrm{P}^{\mathrm{min}}$ of sincere candidates is below that of pragmatic candidates which in turn is below that of strategic candidates, other things being equal.

Equation 4.3 indicates that wealthier prospective candidates enter only if they have higher probabilities of success than less wealthy ones. Such candidates run only if they have strong policy interests. Equation 4.4 indicates
that the required probability of success falls as wealth associated with losing an election increases. Elections where wealth losses are minor, or where personal wealth may increases as a consequence of campaign fame, attract more candidates than ones where losses may be substantial, other things being equal.
$P^{\text {min }}$ varies systematically across candidates. Each potential candidate has a minimum probability of victory sufficient to induce his candidacy. The maximum number of candidates who may enter the contest can be determined for a given distribution of candidates by comparing each candidate's $\mathrm{P}^{\mathrm{min}}$ with their actual probability of electoral success. Entry occurs as long as the probability of electoral success is above $\mathrm{P}^{\mathrm{min}}$ for at least one of the remaining prospective candidates. Consequently, analysis of the probability that successive entrants may win an election provides a general indication of incentives for candidates to run for elected office.

## 3. Voter discernment, entry and success in single-winner elections

We now turn our attention to the manner in which entry affects the probability of electoral success under different characterizations of voter discernment. Voters are assumed to have single peaked preferences, and vote for the candidate that adopts the position which appears to be closest to their ideal point. The issue space of interest may be thought of as alternative proportional income tax rates used to increasing levels of public services. Voter ideal points are assumed to be uniformly distributed along the [0,1]-interval of a single dimension. We do not focus on specific policies over which candidates may taken positions but rather the number of candidates who may enter an election and the consequent dispersion of candidate positions which may represent different public service levels financed by a proportional tax on income.

We depart from standard treatments of voter behavior by assuming that voters are imperfectly informed about candidate positions and consequently are not always able to distinguish between the positions of candidates whose positions yield approximately the same expected utility. This cognitive assumption provides a behavioral rationalization for the perturbation parameter $\varepsilon$ used in the limit equilibria proofs of Palfrey and Shepsle et al., and provides a bridge between fully rational and non-rational voting models.

Each voter's ability to distinguish between candidate positions is imperfect. We denote by $\delta$ the lower bound on a voter's ability to discriminate among distances to candidate positions. If the distances from a voter's ideal point to the nearest candidate positions differ by less than $\delta$, voters are effectively indifferent between the candidates. We assume that in such cases,
voters cast their votes randomly with equal probabilities among these best, albeit indistinguishable, candidates positions. ${ }^{7}$ As long as candidate positions differ by $\delta$ or more, the candidate closest to a voter's ideal point receives that voter's vote with probability one. If there are n candidates within the zone of effective indifference the probability that any of these candidates receives the voter's ballot is $1 / \mathrm{n}$. ${ }^{8}$

Candidates run for office if the expected utility of doing so exceeds that of not running. As in a conventional spatial model, the probability that a candidate wins the election or influences the outcome in the manner desired is a determined by the positions of other candidates and the distribution of voter ideal points. A candidate who believes himself to be at last to enter the election achieves the highest probability of winning by taking the position that maximizes expected votes. The zone of voter non-discernment affects candidate incentives to enter elections by altering the expected votes associated with alternative policy positions and the probability of electoral success. The latter influences a candidate's decision to run for office or not. ${ }^{9}$

Successive candidates are assumed to adopt the position which maximizes their expected vote share given the positions of candidates already in the election. This assumption generates a simple Nash-like dynamic for elections, and also allows us to characterize the effect that successive entry has on the probability of electoral success. However, entry behavior of this sort is clearly myopic in that candidates do not take future entry into account when they decide whether to enter an election or not. In effect, we have assumed that candidates know their own circumstances and the positions taken by previous entrants but do not know the distribution of critical P's for other potential candidates. Consequently, successive entrants take the strategies of all other players inside and outside the game as given, in much the same manner as players choosing strategies in non-cooperative games are assumed to. More fully rational behavior on the part of candidates is very difficult to characterize. Predicting future entry is complex and, may not even be feasible given the various impossibility results of this literature (see, for example, Greenberg and Shepsle, 1987).

The entry scenario explored is an open seat election in which as many candidates may enter as desire, but where there will be a single-electoral winner. This is the typical case in primary elections for a party out of power. It is also the case for general elections after the retirement or death of incumbents. Moreover, it is the usual focus of analyses of candidate competition in spatial models. There are no entry barriers for candidates, and the number of candidates is not restricted to two or three. Every entrant chooses a policy position which cannot be changed. In this, active candidates in our model face the same type of problem as political parties that choose a party manifesto. Once


Figure 1. Probability of a second candidate being elected when the first candidate is positioned at the median voter's ideal position and voters only imperfectly estimate candidate positions.
a manifesto is chosen, the position of a party with respect to most political issues cannot be changed easily and has to be regarded as fixed.

How will entry develop? A first candidate enters the election as long as $\mathrm{U}\left(\mathrm{G}^{\mathrm{W}}, \mathrm{W}^{\mathrm{W}}-\mathrm{C}\right)>\mathrm{U}\left(\mathrm{G}^{0}, \mathrm{~W}^{0}\right)$ for some candidate. The first candidate is assured of victory as long as a second candidate does not enter. We arbitrarily assume that the first candidate has the lowest $\mathrm{P}^{\mathrm{min}}$ and takes the policy position $1 / 2$, that is to say the first entrant is the most willing to run and takes the median voter's position. Other entrants and positions are also possible, but this combination is consistent with the conventional Nash analysis of player strategy choices away from equilibrium and with the logic of entry used by economists to analyze Ricardian economies. It thus seems a reasonable place to start our analysis. ${ }^{10}$

The discernment parameter determines the range of platforms that can potentially win the election. The probability of being elected function faced by a second entrant is depicted in Figure 1 for the case where voter ideal points are uniformly distributed within a discrete interval and each voter has the same relatively small discernment parameter, $\delta$.

Regardless of the size of discernment parameter $\delta$, the probability that the second candidate wins is at most fifty percent given that the first candidate takes the median-voter position. The range of viable policy positions are those between $\mathrm{G} 1-\delta$ and $\mathrm{G} 1+\delta$. Other policy positions necessarily lose, and consequently will not be adopted by candidates for whom campaigning is a costly process. The second entrant can not win the election outright, but
enters only if his benefits and costs are such that the ratio of possible net gains to net losses is greater than 2 . If the office is of little value and/or the cost of running a viable campaign is relatively large, the first candidate may be the only candidate that enters the election. ${ }^{11}$

If discernment parameter $\delta$ is small, the principal reason that a second candidate enters an election can not be policy concerns. To have a chance of winning the election, the second candidate necessarily adopts a policy that is very close to the first candidate's position. Having effectively identical platforms, there is little policy advantage gained by winning office. Votes for two "close" candidates are apportioned randomly, and each has the same probability of electoral success and expected votes. In such circumstances, prospective challengers are unlikely to be among the most successful people from business, science, or entertainment insofar as such persons have relatively little to gain in wealth or prestige from even a fairly prominent political post. Rather, potential second candidates will be those who stand to gain the most personal wealth from winning or running for office.

The effect of discrimination parameter $\delta$ on the viability of candidate platforms is most easily demonstrated with two extreme cases. Consider the case where $\delta>1$. Here, voters are unable to distinguish between any candidate positions, and vote randomly for all candidates. In this case, it is clear that a candidate's position does not affect his probability of electoral success at all. Now consider the other extreme, where $\delta=0$. Here, the usual deterministic model of voter behavior obtains, and candidate positions determine the probability of victory. For example, if Candidate 1 adopts the median voter's position, the second candidate must also adopt this position if he is to have a non-zero probability of victory.

The effect that discernment parameter $\delta$ has on candidate entry is also most clear for the extreme cases. If $\delta>1$, voters are insensitive to candidate positions over the whole range of interest, and the probability of success is simply $1 / \mathrm{n}$, where n is the number of candidates running for office. On the other hand, if $\delta=0$, the probability of success is not a monotone decreasing function of the number of candidates already in the election. For example, a third candidate who faced two candidates at the median-voter position could achieve a $1 / 3$ probability of success by also adopting the median-voter position. However, the third candidate can better by adopting a position a bit to the right or left of the median voter. In this case, the first two candidates split about half of the voters, while the third candidate obtains the rest. New entrants can generally do at least as well as candidates already in the election unless prior entrants have, in some manner, coordinates their positions to block further entry. ${ }^{12}$


Figure 2. Probability of a third candidate being elected when the first and second candidates take different positions but are indistinguishable from one another.

Intermediate levels of $\delta$ imply that candidate positions have intermediate effects on the probability of candidate success as some voters vote deterministically and others stochastically. Figure 2 characterizes the probability of being elected function faced by a third candidate for the case where the first two candidates have taken somewhat different, although similar policy positions. As in Figure 1, voter ideal points are assumed to be uniformly distributed within the interval of interests, and each voter is assumed to have the same discernment parameter. Candidate 2 is located at a position within $\delta$ of the first candidate's position, G1.

Consider the incentives of a third candidate, C. By choosing a position between the previous candidates, Candidate 3 obtains a probability of success equal to $1 / 3$. When $\delta$ is relatively small, $0<\delta<1 / 6$, a prospective entrant can do better than the previous candidates as in the deterministic case discussed above. By moving a bit more than distance $\delta$ away from the candidate nearest to the median voter's ideal point, the first candidate, Candidate 3 receives somewhat more than $1 / 3$ of the votes, while the other two candidates share the remaining voters who cast their votes stochastically.

If Candidates 1 and 2 exactly split the remaining votes, Candidate 3 wins the election because the other two candidates each receive less than $1 / 3$ of the total votes. However, because many voters cast their votes stochastically
among Candidates 1 and 2 , there is a small chance that one of them will get essentially all of the votes not received by candidate 3 and win the election. (The votes of Candidate's 1 and 2 are distributed according to the binomial distribution.) Consequently, the probability that Candidate 3 wins is one minus the probability that either of the two central candidates receives more votes than Candidate 3's combined deterministic and stochastic vote. Elementary statistics informs us that if two candidates each have a .5 chance of receiving any vote cast, the probability that either candidate gets $2 / 3$ of this pool of votes rapidly approachs zero as the number of voters becomes large. ${ }^{13}$

Voters with ideal points within $\delta / 2$ of the midpoint of the relevant candidate positions are also indifferent between candidates. ${ }^{14}$ Consequently, Candidate 3 necessarily receives some stochastic votes even if he locates further than $\delta$ from the other candidates. The smaller $\delta$ is, the smaller is the number of voters in the middle whose votes are apportioned randomly between all three candidates. The smaller is the stochastic vote, the less likely it is that one of the other candidates obtains by chance a sufficient share of these votes to win the election.

It bears noting that the possible entry of a third candidate eliminates the strong central tendency of electoral outcomes in majoritarian elections. ${ }^{15}$ The third candidate can take positions in ranges to the left or right of the other candidates, and still have a better chance of winning than the first two candidates. Many alternative policy positions yield greater probabilities of success than those of the first two candidates. Moreover, the third candidate does not have to maximize expected votes to have a relatively high probability of electoral success.

The viability of non-central third candidate policies is not an artifact of parameter $\delta$. For example, consider the case where $\delta=0$, and where Candidates 1 and 2 have taken identical positions. Here, any third candidate position between $1 / 6$ and $5 / 6$ other than $1 / 2$ will be very likely to receive majority support. ${ }^{16}$ In cases where $\delta$ is large, say for example $\delta=1 / 3$, there are no longer non-central strategies that have a relatively high probability of success, but there are many extreme strategies that have the same expected electoral payoff as more central strategies. With $\delta$ large, more extreme strategies become indistinguishable from central strategies.

## 4. Natural limits on entry

In ordinary markets, entry stops when anticipated rates of return fall below opportunity cost rates of return for prospective entrants. In a political market, entry stops when the probability of electoral success falls below that
required to secure an additional entrant. Generally, the probability of success falls as the number of candidates increases because the number of voters casting votes stochastically tends to increase with the number of candidates, and expected vote shares diminish. If candidates have fairly similar utility functions and campaign costs then $\mathrm{P}^{\text {min }}$ will be similar for all candidates and entry will cease at the same time for all potential candidates. On the other hand, if policy preferences, campaign costs, wealth, differ among potential candidates, then the number of potential candidates will diminish gradually as entry takes place.

The effect of entry on the probability of electoral success is most clear in the case where discernment parameter $\delta$ is large, $\delta>1$. In such cases, all voters vote stochastically, and as the number of candidates, $n$, becomes large, the probability of success necessarily becomes small. $1 / \mathrm{n}$ approaches zero as n increases. With $\delta$ large, candidate positions are relatively unimportant, and entry ceases when the probability of success falls below that acceptable to other prospective candidates. Consequently, as the desirability of the elective office increases, the number of candidates who will run for office increases since $P^{\text {min }}$ falls.
For deterministic and intermediate level of $\delta$, the relationship between the number of candidates and probability of success for an entrant is less transparent. No monotonic relationship between number of candidates and the probability of electoral success exists in these cases. This has already been suggested in the previous analysis (see Figures 1 and 2), where the probability of success for the first and third entrants were much higher than that of the second entrant. ${ }^{17}$ Because the probability of electoral success is not a well behaved concave function, analysis of candidate strategies must rely upon numerical methods even in the relatively tractable cases examined here where candidates are assumed to adopt positions as if no further entry will occur. The non-concavity of the probability of winning and expected vote share functions is not an artifact of the sharpness of the discernment parameter but rather a property of spatial voting in sequential entry models. (See Palfrey, 1986)

A computer program was written in Basic to tabulate expected votes associated with different dispersions of candidate positions under different assumptions about the voter discernment parameter. To gain an initial impression of incentives for entry, the program places successive candidate at a position which maximizes expected votes. The computer program arbitrarily assigns candidates to leftmost vote maximizing position in cases where two or more policy positions have equal probabilities of electoral success.

Figure 3 depicts the series of expected vote functions faced by successive entrants. As suggested in the previous illustrations, the functions are


```
+Candidate 3 - Candidate 4 }->\mathrm{ Candidate 5
Candidate 6 Candidate 7 - Candidate 8
```

Figure 3. Expected vote functions faced by successive vote-maximizing candidates at time of entry when voters imperfectly discern candidate positions.

$\rightarrow$ Candidate $1 \square$ Candidate $2 *$ Candidate 3

+ Candidate $4 *$ Candidate $5+$ Candidate 6

Figure 4. Expected votes received by all candidates as successive vote maximizing candidates enter the electoral contests.


Figure 5. Expected vote shares of vote maximizing candidates at time of entry.
non-concave and overlap over several ranges. Generally the maximum number of votes available to new entrants falls as successive entry occurs. Note, for example, that the expected number of votes for the eighth candidate lies everywhere below that of the first five candidates. This is a consequence of sharing votes with more alternative candidates.

Figure 4 plots the expected votes received by the first six candidates as entry occurs. Note that each candidate's expected vote diminishes with entry and, moreover, that electoral results become more equal as entry continues. Thus, in the limit, vote maximizing entrants appear to replicate the simple case developed above with a large discernment parameter. Note also that the expected vote of an entrant is always above average, but is not always greater than that of the previous entrant. For example, candidates 4 and 6 receive fewer votes than candidates 3 and 5 and thus face a rather small probability of winning the elections. If candidates will enter only if they expect to win the election - that is, have the highest expected vote - only three candidates will enter the election. ${ }^{18}$ This result depends on the first entrant's position and the value of $\delta$. If the first entrant takes a non-central location, the number of candidates that may enter becomes larger. ${ }^{19}$ If the discernment parameter becomes large, the probability that an entrant is successful falls more rapidly which tends to reduce entry.

Figure 5 depicts the expected vote shares of successive entrants. Expected vote shares approach $1 / n$ in the limit, where $n$ is the number of candidates running for office. Equal vote shares imply equal probability of electoral success, which in this case approaches $1 / \mathrm{n}$ in the limit. An implication of this pattern is that there is always some finite number of candidates which will reduce the probability of electoral success below that required for further candidate entry, $\mathrm{P}^{\mathrm{min}}$, for any election. ${ }^{20}$

## 5. Conclusion and summary

The range of possible entry scenarios naturally varies with the particular assumptions made about the motivation (tastes, ideology) and strategic calculations of prospective candidates. We have analyzed incentives for candidate entry into open elections in a setting where candidates and voters have similar goals and both are imperfectly informed. Insofar as candidates are assumed to take positions which maximize the probability of electoral success, but disregard the possibility of further entry, the model of candidate entry analyzed is in the middle of the range of rational and non-rational models of candidate decision making that might be analyzed.

Our results suggests: (1) that the natural limit to entry will often exceed two candidates, which is the most analyzed form of election; (2) that three candidate elections do not have strong central tendencies, either to the mean or median; (3) voter abilities to distinguish between candidate positions has important effects on the extent to which elections exhibit a strong central tendency. The less voters are able to discern the relative merits of candidate positions the more discretion candidates have to take non-central positions. These results differ substantially from those of previous analyses of entry insofar as they imply a richer range of possible electoral outcomes.

The broad range of electoral possibilities found is chiefly of interest because such patterns are observed in real elections. "Winner take all" primary elections for important positions often attract a half dozen or more candidates taking a wide range of policy positions. The model developed here does well at rationalizing such a broad range of candidate positions, although it does less well at characterizing specific electoral outcomes. Right and left of central electoral outcomes do occur, witness the recent success of "right to life" and "green" candidates in Republican and Democratic primaries, but they are by no means universal.

In general, our results suggests the relevance of the extensive literature on two-candidate elections is more dependent on unstated institutional and technological constraints than is generally acknowledged. Two candidate final elections may reflect unstated advantages of being affiliated with dominant
parties, or various economies of scale in campaigning for office, rather than the existence of stable two candidate equilibria under free entry. Electoral contests between two serious candidates do not appear to reflect general incentives for individual candidates to enter elections.

## Notes

1. In his pioneering paper on spatial competition Hotelling (1929) presented a simple model of a firm's decision to enter a market. Using a one-dimensional representation, the model led to the conclusion that competitors will cluster at the median of the market. Eaton and Lipsey (1975), Prescott and Visscher (1977), and Shepsle (1991) analyze more general spatial market settings. A nice survey of the political literature on entry is found in Shepsle and Cohen (1990) and Shepsle (1991).
2. Greenberg and Shepsle (1987) also identify the median ideal point as "one-equilibrium", that is the outcome of a single winner election. This result generalizes Black's (1958) median-voter theorem. If the number of winning positions is two or more, Greenberg and Shepsle (1987: 529) establish an impossibility result for so-called k-equilibria in the sense that there exist societies for which there is no equilibrium. Shepsle and Cohen (1990: 33-35) push this analysis further for 2-equilibria. They identify distributions of voters for which such an equilibrium exists. In this paper our attention is restricted to the case of single-winner elections.
3. Entry in plurality political contests is somewhat different from entry in competitive economic markets. In a competitive economic market firms will enter as long as rates of return are greater thant they are in other markets, and the market of interest will be shared among competing firms. Although left implicit, the opportunity cost of an economic entrepreneur is generally assumed to be entry into another comparable market. In such cases, an entrepreneur enters a market when the risk adjusted expected rate of return exceeds that of his opportunity rate of return. In pluralitarian political markets the alternative sacrifices may be another political office or employment in the private sector. Moreover, rather than considering a range of possible profit outcomes, electoral outcomes tend to be dichotomous. There are clear winners and losers. However, to the extent that prospective candidates are similarly motivated by anticipated advantages, the decision to enter an electoral choice is fundamentally similar to that faced by an economic entrepreneur.
4. Of course, policy interests might be based indirectly on wealth consideration. A more complete analysis of entry incentives would allow the possibility that a candidate's wealth is a function of the policies adopted, although the results would not be substantially different from those developed below. For example, name recognition may help the candidates own business enterprises as well as improve prospects in future elections.
5. Preference falsification is Kuran's (1987) term for adopting public positions which differ from one's true private preferences. Candidates who falsify their private policy preferences may also have a more difficult time being elected, insofar as voters value consistency in their political agents.
Moreover, strong policy preferences also allows the possibility that "idealistic" candidates may enter an election although their personal wealth will be diminished by electoral success $\left(\mathrm{W}^{\mathrm{W}}<\mathrm{W}^{0}\right)$. Such policy driven candidates may be "sincere" in that they adopt platforms that they hope will win, or they may be "strategic" in that they run on platforms that they hope will indirectly change the electoral outcome in a favorable manner. A policy-oriented candidate may also influence electoral outcomes without winning the election. In the simulations developed below, it is often possible for a "strategic" challenger to falsify his preferences and significantly change the electoral outcome from one side of the median preference to the other by appropriately sharing the votes of prior entrants.
6. Note that given a model of voter behavior, it is possible to calculate probabilities of electoral success, and thus political entrepreneurship might be considered in principle more analogous to an insurance market than to Knightian entrepreneurship. However, a candidate can not generally diversify away the risk because he or she may run for only one or two offices at a time.
A continuum of possible outcomes might also be said to occur in elections insofar as there are benefits associated with simply being a candidate. For example, candidates generally become better known during the course of an election which can enhance future electoral prospects. To simplify analysis, we ignore non-wealth benefits associated with merely running for office. If campaigning itself generates utility for prospective candidates, there may be entrants who are unconcerned with prospects for electoral success in the current election. Our analysis focuses exclusively on what might be called serious candidates who are motivated by their prospects for electoral success.
7. This characterization of voter choice is similar to the classical statistical approach to hypothesis testing. Given some target level of confidence (or risk) estimates within the implied confidence interval are said to be statistically indistinguishable from one another. Another approach is found in Downs (1957), who introduces an expected party differential. This is the difference in expected utility between parties by a voter. If this differential is equal to zero, a voter abstains in Downs' model. In our model voters will still cast their votes, but they pick a candidate randomly.
8. This approach to modelling voter behavior seems to be more consistent with neoclassical consumer theory than the usual stochastic models insofar as voters only fail to perfectly optimize over candidates who lack clear advantages over one another.
9. We neglect the effect that campaign resources have on a candidate's probability of success. As demonstrated by Austen-Smith (1987) and Congleton (1986), this effect can be modeled as a matter of candidate positions. The principal effect of adding consideration of campaign resource effects is to make the probability of electoral success a somewhat more complex function of candidate position.
10. Although our focus in this paper is on open seat elections, we also ran simulations with non-central starting points. The reputation of an incumbent candidate may prevent her from changing positions. In cases where the incumbent has adopted a non-central platform in a previous election, the "first" candidate may not be at the median voter's preferred policy. We found only minor differences in the entry scenarios developed below for elections where the first candidate took a non-central position.
11. Uncontested entry may also occur in cases where there are asymmetries in the consideration of candidates. For example, the existence of an incumbent advantage reduces the probability of a successful challenge and thereby reduce the likelihood of entry.
12. This depends in part on the strategy used by prospective entrants. If the third candidate locates at $1 / 4$ and the fourth candidate locates at $3 / 4$, as part of a blocking strategy, then a fifth candidate cannot do as well as either the third or fourth candidates.
13. The cumulative binomial probability that a single candidate gets $2 / 3$ of the votes when he has a .5 chance of getting any particular vote is .254 for 9 stochastic voters, .194 for 12 stochastic voters, and .119 for 18 stochastic voters. In such cases, the probability that a position just to the left of G1-d would win is a bit greater than $.49, .60$, and .75 respectively. In the continuous case depicted in Figure 1, there are effectively infinite stochastic voters.
14. To see this consider a voter whose ideal point is between two candidate positions G1 and G3. Let the distance from the voter's ideal point, V, to the left most candidate (G3) be d'. This voter will be indifferent between the two candidates if:

$$
\begin{aligned}
& \|\mathrm{V}-\mathrm{G} 3|-| \mathrm{G} 1-\mathrm{V}\|<\delta \quad \text { or substituting } \mathrm{V}=\mathrm{G} 3+\mathrm{d}^{\prime}, \\
& \left|2 \mathrm{~d}^{\prime}+\mathrm{G} 3-\mathrm{G} 1\right|<\delta \quad \text { solving for a range of d' yields } \\
& (\mathrm{G} 1-\mathrm{G} 3) / 2-\delta / 2<\mathrm{d}^{\prime}<(\mathrm{G} 1-\mathrm{G} 3) / 2+\delta / 2
\end{aligned}
$$

Voters with ideal points within $\delta / 2$ of the mid point of the two-candidate positions are effectively indifferent between the candidate positions.
15. It bears noting that while many positions substantially to the left or right of the median voter's ideal point are politically viable for third entrants, very extreme positions are not. In order to expect to win, the third candidate has only to anticipate getting more than $1 / 3$ of the votes cast. To see the bounds on viable extreme policies, let d' be the distance between the platforms of the first two candidates on the policy dimension. Candidates 1 and 2 are located approximately at $1 / 2$ and expect to split the support of voters who do not vote for Candidate 3. An extreme candidate will get all of the votes of voters whose ideal points are more extreme than that taken by the candidate.
In order to win from the left, Candidate 3 has to expect to get the votes up to and including some voters with an ideal point greater than $1 / 3$. For this to occur he must be no further away from the voters with position $1 / 3$ than any other candidate is. When Candidate 1 adopts a position of $1 / 2$, this implies that Candidate 3 would not adopt a position to the left of $1 / 6$. Similar logic applies for positions to the extreme right except that Candidate 3 cannot locate further right than $5 / 6$ - d' since Candidate 2 takes a position at $1 / 2+d^{\prime}$. The furthest position to the left that can ever generate $1 / 3$ of the expected votes is $G=1 / 6$. The furthest position to the right that can ever be successful is $G=5 / 6$ which occurs when d' $=0$.
16. In a richer analysis, it would be possible for Candidate 3 to misjudge the size of d, and locate too closely to one of the candidate positions. In this case, the middle candidate is most likely to win. For example, in the case depicted, where Candidate 2's position is similar but not identical with Candidate 1's, there is a range of policy positions where Candidate 3 may locate within distance $\delta$ of Candidate 2 but not Candidate 1, which increases the chance that Candidate 2 wins the election. In this case, Candidate 2 would split votes with Candidate 1 on the left and split them with Candidate 3 on the right. Third entrants might tend to locate substantially further from the middle of the distribution than strictly necessary to avoid such mistakes.
17. The properties of this function naturally depend upon the particular sequence of entries taken. For example, the probability of a third candidate's success would be smaller than that of the second, if the first and second candidates locate at $1 / 3$ and $2 / 3$ (which are blocking positions if $\delta=0$ ). However, with $\delta>0$, the probability of success does not fall to zero, and a third candidate may enter in any case. Once a third candidate enters, a fourth can enter and do as well as any of the announced candidates.
18. Note, however, that the entry of a fifth candidate makes candidate 4's position the most electable since it now gets more votes than positions 3 or 5 . Assessment of the likelihood of additional entry consequently plays a role, not only in decisions to enter or not but also with respect to which positions will ultimately prove most electable. In an extreme case of perfect foresight, Osborne (1992) demonstrates that in a population with only three prospective candidates, only one will enter the election. The second knows that if he enters he will surely lose to the third. So neither the second or third candidate ever enters the electoral contest.
19. Simulations for cases where the first candidate takes a non-central position were also ran, although not reported in detail in the paper. For $\delta=0.15$, the number of successful entrants is 3 if the first candidate is located at $0.5,4$ if he is located at $0.40,4$ if located at 0.20 , and 8 if located at 0 . Simulation results also affirm the effect that varying $\delta$ has on entry. The number of successful entrants falls to 2 if $\delta \geq 0.25$.
20. It bears noting in passing that strategic candidates or coordinated pairs of candidates may enter in settings where a normal candidate would not. A "strategic candidate" does not expect to win but rather to change the result in some desired way. For example, candidate 4 may pave the way for entry by a strategic candidate 5 who prefer's candidate 4 's position to that of candidate 3 . The entry of candidate 5 causes candidate 3 to loose a substantial share of its voter support, and causes candidate 4 to become the new expected winner. Candidate 4 receives an expected vote larger than candidate 3 but below that of
candidate 4. Such possibilities suggest that incentives to enter campaigns, and the effects of entry on electoral results may be complex even in relatively straightforward models if candidates coordinate entry decisions.

## References

Austin-Smith, D. (1987). Interest groups, campaign contributions and probabilistic voting. Public Choice 54: 123-140.
Black, D. (1958). The theory of committees and elections. Cambridge: Cambridge University Press.
Brams, S.J. and Straffin, P.D. (1982). The entry problem in a political race. In P.C. Ordeshook and K.A. Shepsle (Eds.), Political equilibrium. Boston: Martinus Nijhoff.
Congleton, R.D. (1986). Rent-seeking aspects of political advertising. Public Choice 49: 249263.

Davis, O.A., Hinich, M.J. and Ordeshook, P.C. (1970). An expository development of a mathematical model of the electoral process. American Political Science Review 64: 426-448.
Downs, A. (1957). An economic theory of democracy. New York: Harper and Row.
Duverger, M. (1954). Political parties. New York: Wiley.
Eaton, B.C. and Lipsey, R.G. (1975). The principle of minimum differentiation reconsidered: Some new developments in the theory of spatial competition. Review of Economic Studies 42: 27-49.
Enelow, J.M. and Hinich, M.J. (1984). The spatial theory of voting. Cambridge: Cambridge University Press.
Fedderson, T.J., Sened, I. and Wright, S.G. (1990). Rational voting and candidate entry under plurality rule. American Journal of Political Science 34: 1005-1006.
Greenberg, J. and Shepsle, K.A. (1987). The effect of electoral rewards in multiparty competition with entry. American Political Science Review 83: 525-537.
Hay, D.A. (1976). Sequential entry and entry-deterring strategies in spatial competition. Oxford Economic Papers 28: 240-257.
Hotelling, H. (1929). Stability in competition. Economic Journal 29: 41-57.
Knight, F.H. (1971 [1917]). Risk, uncertainty, and profit. Chicago: University of Chicago Press.
Kuran, T. (1987). Preference falsification, policy continuity and collective conservatism. Economic Journal 97: 642-665.
Mueller, D.C. (1989). Public choice II. Cambridge: Cambridge University Press.
Osborne, M.J. (1992). Candidate positioning and entry in a political competition. Mimeo. McMaster University.
Palfrey, T.R. (1984). Spatial equilibrium with entry. Review of Economic Studies 51: 139-156.
Palfrey, T.R. (1989). A mathematical proof of Duverger's law. In P.C. Ordeshook (Ed.), Models of strategic choice in politics. Ann Arbor: University of Michigan Press.
Prescott, E.C. and Visscher, M. (1977). Sequential location among firms with foresight. Bell Journal of Economics 8: 378-393.
Rothschild, R. (1979). The effect of sequential entry on the choice of location. European Economic Review 12: 227-241.
Shepsle, K.A. (1991). Models of multiparty electoral competition. New York: Harwood Academic.
Shepsle, K.A. and Cohen, R.N. 91990). Multiparty competition, entry and entry deterrence in spatial models of elections. In J.M. Enelow and M.J. Hinich (Eds.), Advances in the spatial theory of voting, 12-45. Cambridge: Cambridge University Press.
Tullock, G. (1967). Towards a mathematics of politics. Ann Arbor: University of Michigan Press.
Wagner, R.E. (1966). Pressure groups and political entrepreneurs. Public Choice 1: 161-170.


[^0]:    ${ }^{*}$ Previous versions of the paper were presented at the 1992 Public Choice Society meeting, the European Public Choice Society Meeting, and the Economics Seminar of the University of Amsterdam. The authors would like to thank the participants of these sessions and an anonymous referee for numerous helpful comments.

