Call the variety of algebras V congruence distributive if the lattice of congruences of every member of V is distributive. The following theorem gives an answer to problem (2) for certain congruence distributive varieties.

THEOREM. *Let V be a congruence distributive variety generated by its finite member, and assume that every member of V has a one-element subalgebra. Then if* Amal(V) *is an elementary class, then it is closed under reduced products. Thus, in that case* Amal(V) *is determined by Horn sentences.*

EXAMPLE AND COUNTEREXAMPLE. It is well known that a lattice L is modular iff L does not have a pentagon N as its sublattice (N is a five-element lattice generated by x, y, and z such that $x < y$, and z is noncomparable with x and y). It is shown by Bergman that if V is a lattice variety generated by a finite modular lattice, then Amal(V) is not elementary, which gives a negative answer on problem 1.

On the other hand, if V is generated by N, then Amal(V) is an elementary class determined by Horn sentences.

VLADIMIR V. RYBAKOV, *Schemes of theorems for first-order theories as propositional modal logic.*
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Our aim is to look from a general point of view, which adjoins closely to the approach of [1] and [2], at schemes of theorems for first-order theories. The language of logic of schemes (LS) has a countable set z_i of variables (for formulas), usual logical connectives, a countable set of additional unary logical connectives $\forall x_i$ (which simulate quantifier) and a countable set of 0-place logical connectives (or logical constants) $(x_i = x_j)$ (which simulate equality). The formulas of LS are built up on pointed variables by these connectives as usual.

Let T be a first-order theory. Then $\mathrm{LS}(T) := \{A(z_i) \mid A(B_i) \in T$ for arbitrary formulas B_i in the language of $T\}$. For a class of models K, $\mathrm{LS}(K) := \mathrm{LS}(\mathrm{Th}(K))$. It is clear that for any T $\mathrm{LS}(T)$ forms polymodal propositional logic which extends the polymodal analog $S5(\infty)$ of Lewis system $S5$ ($\Box_i = \forall x_i$). The notions "class of models K_1 has in class of models K_2 expressible weak track" is introduced which is designed by $K_1 \preceq K_2$.

THEOREM 1. *If $K_1 \preceq K_2$ and K_1 has hereditary undecidable theory, then* $\mathrm{LS}(K_2)$ *is undecidable.*

If we consider the LS language without $x_i = x_j$, then it is easy to extract from [2] that LS, even for pure predicate calculus, will be undecidable too.

THEOREM 2. *There exist decidable finitely axiomatizable first-order theories with not recursively enumerable logic of schemes.*

THEOREM 3. *If K is finite class of finite models, then* $\mathrm{LS}(K)$ *is decidable and admissibility of inference rules for* $\mathrm{Th}(K)$ *is decidable too.*

A complete description of recursive completeness of LS's is given by

THEOREM 4. *A first-order theory T has a decidable logic of schemes iff $T = \bigcap_{1 \le i \le n} \mathrm{Th}(M_i)$, where the M_i are finite (so T is almost trivial).*

Thus polymodal propositional logics $\mathrm{LS}(T)$ extending logic $(S5(\infty) + (\Box_i \Box j p \equiv \Box j \Box_i p)$ are, as a rule, undecidable.

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ANTONINO SALIBRA and GIUSEPPE SCOLLO, *A reduction scheme by preinstitution transformations.*
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Preinstitutions are introduced in [5] as a weakening of institutions in the sense of [2]. Briefly, a preinstitution consists of a category of signatures on which, for each signature Σ, a set-valued functor gives the Σ-sentences, a set-valued functor gives the Σ-models, and a binary satisfaction relation defines validity of Σ-sentences in Σ-models.

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A preinstitution transformation $\mathscr{T}\colon \mathscr{I} \to \mathscr{I}'$, with $\mathscr{I}, \mathscr{I}'$ preinstitutions, sends $\mathscr{I}$-signatures to $\mathscr{I}'$-signatures (by a functor), $\mathscr{I}$-presentations (sets of sentences) to $\mathscr{I}'$-presentations (by a natural transformation), and $\mathscr{I}$-models to $\mathscr{I}'$-model classes (again, by a natural transformation), such that satisfaction is invariant (in both directions) under the transformation.

The main motivation for introducing these notions was the experience gained in [4], relating to the translation of a number of logics of frequent use in computer science into equational type logic [3]. We detected a striking commonality over the different translations, concerning representation of models, translation of sentences, and structure of completeness proofs. The search for a more general framework, where that commonality could be factored out, was just as natural.

Somewhat beyond our original target, it turns out that preinstitution categories, with preinstitution transformations as morphisms, give rise to a 'soft' model-theoretic framework for the lifting of classical properties, such as compactness, from 'abstract logics' in the sense of abstract model theory, to a model-independent notion of logical system.

The model-theoretic concepts of compactness and Löwenheim-Skolem properties are then investigated in [6] for that 'soft' framework. Two compactness results are so obtained: a more informative refinement of the compactness theorem for preinstitution transformations, and a theorem on natural equivalences with an abstract form of first-order preinstitutions. These results rely on notions of compact transformation, which are introduced in [6] as an arrow-oriented generalization of the classical notions of compactness. Moreover, a notion of cardinal preinstitution is introduced in [6], and a Löwenheim-Skolem preservation theorem for cardinal preinstitutions is presented.

The aforementioned results indicate a fruitful adaptability of the reduction scheme, as outlined for abstract logics in [1], to the softer framework of preinstitution categories. In this framework, the adapted form of the reduction scheme tells that downward inheritance descends along the morphisms, that is, inheritance by preinstitution $\mathscr{I}$ of model-theoretic properties enjoyed by preinstitution $\mathscr{I}'$ is guaranteed by the existence of a suitable transformation $\mathscr{T}\colon \mathscr{I} \to \mathscr{I}'$.

Further evidence of the applicability of the reduction scheme by preinstitution transformations is expected from a generalization of our compactness results to (κ, λ)-compactness—which is of interest in the investigation of infinitary logics. Such a generalization is under study.

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For two varieties V, V_1, by $V \equiv V_1$ we mean that V is term definitionally equivalent with V_1. Further, $V = \mathbf{Rd}(V_1)$ means that V consists of the reducts *to the language* of V of members of V_1. Similarly, for $V = \mathbf{SRd}(V_1)$, etc., RQPA_ω and RPA_ω are the classes of representable quasi-polyadic algebras and representable polyadic algebras (of dimension ω) respectively. RQPEA_ω is RQPA_ω with equality, and similarly for RPEA_ω; cf. [HMT II].

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