Design and Realization of an Optimal Current Sensitive CCC

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***Abstract*— The sensitivity of an overlapped tube cryogenic current comparator (CCC), coupled to a SQUID by means of a flux transformer, is calculated and compared with measurements. Conditions for optimal coupling between the CCC and the SQUID are derived. Based on these, an optimal CCC for precision measurements of very small currents in the nanoampere range has been designed. The paper describes the construction and testing of some parts of the system. An essential element is a home-made dc SQUID with low current noise and low inductance input coil equal to that of the CCC overlapped shield, which we use as sensing coil.**

***Index Terms*—Cryogenic electronics, current comparators, current measurement, electrical metrology, SQUID's.**

I. Introduction

Since the discovery of single electron tunnelling (SET) effects [1], metrologists have considerably raised their interest in the measurement of very small currents in the picoampere range. Devices based on small Al-AlO_x-Al junctions can only be operated with sufficient accuracy at frequencies, f_c, up to 10 MHz, so that the device output current $I = ef_c$ is of the order of a few picoamperes.

Recently [2], macroscopic single electron transport phenomena at frequencies of GHz have been discovered in devices based on GaAs-AlGaAs heterostructures. Generation of high precision currents in the nanoampere range may be a reality in the near future.

The accurate measurement of such small currents is a challenging task. Cryogenic current comparators (CCC's) are ideal devices for scaling of (small) currents, but a first attempt [3] showed negative results in the sense that optimal coupling was not achieved in the flux transformer and that external noise limited the sensitivity of the system.

In a recent analysis of the ultimate current resolution of CCC's [4], it was proposed that their sensitivity could be improved by the use of low input coil inductance SQUID's.

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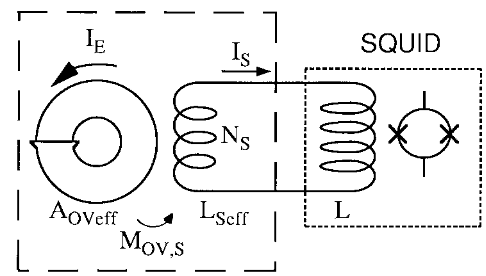

Fig. 1. Schematic representation of an overlapped tube Type I CCC. Indicated are the overlapped tube with effective self-inductance A_{OVeff}, the flux transformer sensing coil with N_S turns and effective self-inductance L_{Seff}, and the SQUID with input inductance L. The discontinuous lines indicate closed superconducting shields.

Based on that we started the design and construction of an optimal current sensitive CCC and its SQUID detector. The paper is divided into five sections. Section II deals with an analysis of an overlapped tube CCC coupled to a SQUID by means of a superconducting flux transformer. Numerical calculations of the CCC sensitivity are compared with experimental measurements. From this analysis the conditions for optimal coupling are derived. Section III describes the CCC dimensions, the effective inductance calculations of the CCC overlapped tube, and the design of a dedicated dc SQUID and its cryogenic insert. The results of testing a series of dc SQUID's designed for optimal coupling are presented in Section IV. Finally, the paper ends with the conclusions in Section V.

II. Calculation of the Sensitivity of a Type I CCC

The Type I CCC [5] is essentially composed of several primary windings inside a superconducting tube that is overlapped like a snake swallowing its own tail. When the CCC is operated, two opposite currents I_1 and I_2 are passed through the N_1 and N_2 turns of the primary windings and a Meissner current $I_E = I_1N_1 - I_2N_2$, equal to the magnetomotive force imbalance, appears in the inner surface of the superconducting overlapped tube and returns through the external surface. As shown in Fig. 1, in which the primary windings are not shown for simplicity, the current I_E induces a current I_S in the superconducting flux transformer. This transformer consists of a sensing coil having N_S turns with nominal self-inductance L_S (with effective self-inductance L_{Seff} due to the presence of the overlapped tube) and the SQUID input coil with self-inductance L. The SQUID acts as null detector for I_E. We shall define the normalized sensitivity of the CCC, $S_{\rm CCC}$, as the square root of the ratio of the energy in the SQUID input coil $(LI_S^2/2)$ to the available energy in the CCC $(A_{\rm OVeff}I_E^2/2)$ which, due to flux conservation in the flux transformer, is given by

$$S_{\rm CCC} = \frac{I_S}{I_E}\sqrt{L/A_{\rm OVeff}} = \frac{M_{{\rm OV},S}}{L_{\rm Seff}+L}\sqrt{L/A_{\rm OVeff}} \quad (1)$$

where $M_{{\rm OV},S}$ is the mutual inductance between the superconducting overlapped tube and the sensing coil. $A_{\rm OVeff}$ is the effective inductance of the overlapped tube when surrounded by the superconducting closed shield (shown in Fig. 1). This shield protects the CCC and sensing coil against external noise. It has been demonstrated [6] that, if the sensing coil is wound very close to the inner or to the outer surface of the overlapped tube, $M_{{\rm OV},S}$ and $L_{\rm Seff}$ can be approximated by

$$M_{{\rm OV},S} \approx N_S A_{\rm OVeff}, \quad L_{\rm Seff} \approx L_S(1-k') + N_S^2 A_{\rm OVeff} \quad (2)$$

where k' is the coupling constant between the real sensing coil and its image with respect to the contiguous superconducting wall of the overlapped tube. Substituting (2) in (1) we have

$$S_{\rm CCC} \approx \frac{N_S\sqrt{A_{\rm OVeff}L}}{L_S(1-k') + N_S^2 A_{\rm OVeff} + L}. \quad (3)$$

From (3) it becomes clear that the first step to optimize the sensitivity of a CCC is to build a sensing coil arrangement for which $k' = 1$. In the following we will refer to this particular situation as the "ideal coupling" case for which (3) can be written as

$$S_{\rm CCC} = \frac{x}{x^2+1} \quad (4)$$

where $x = N_S\sqrt{A_{\rm OVeff}/L}$ is a dimensionless parameter. Then, for ideal coupling the normalized sensitivity is a universal function of x and has an optimum value, $(S_{\rm CCC})_{\rm OPT} = 0.5$, for $x = 1$ (i.e.: $(N_S)_{\rm OPT} = \sqrt{L/A_{\rm OVeff}}$). This is illustrated by the continuous line in Fig. 2.

When thin wire is used for the sensing coil, the nominal inductance of a single turn is typically ≈60 nH/cm of diameter. This is about one order of magnitude greater than the inductance of an overlapped tube, typically ≈8 nH/cm of mean diameter. Therefore, for a thin wire sensing coil of N_S turns, L_S can be one order of magnitude greater than $N_S^2 A_{\rm OVeff}$. Typical k' values vary from 0.7 to 0.9 when N_S varies from 1 to 10 [6], [7]. Then the first term in (2) can no longer be neglected and an undesirable sensitivity reduction can exist (see (3)). This is experimentally demonstrated by the measurements shown in Fig. 2 where filled squares correspond to measured $S_{\rm CCC}$ values of an overlapped tube CCC (a rectangular torus with 27 mm of mean diameter, 16 mm height, 7 mm width, and $A_{\rm OVeff} = (11 \pm 0.8)$ nH when surrounded by a superconducting cylindrical shield of 45 mm diameter [4]) in which different sensing coil turn numbers were used ($N_S = 1, 2, 4, 6, 8, 10, 12, 15$, and 20). The wire diameter was 140 μm, the spacing between turns was about 0.2 mm and the distance of the wire to the overlapped tube about 30 μm. In these experiments a commercial rf SQUID from Oxford

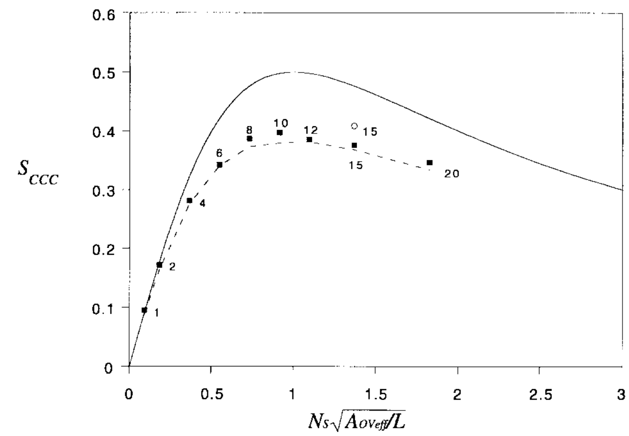

Fig. 2. Normalized sensitivity of the CCC as a function of the dimensionless parameter $N_S\sqrt{A_{\rm OVeff}/L}$. The solid line is the theoretical curve for ideal coupling ($k' = 1$) between overlapped tube and sensing coil [(4)]. The squares are measured sensitivities in a test setup where N_S is varied. The dashed curve corresponds to (3) for this particular case. The circle with $N_S = 15$ is obtained when foil instead of wire is used for making the sensing coil. For $N_S = 1$ the experiment was done using the overlapped tube as single turn sensing element, and with the coil wound on the inner and on the outer surface of the overlapped tube. The three values coincide within the experimental error.

Instruments, with a measured effective input inductance in closed loop operation of $L = (1.3 \pm 0.1)$ μH, was used.

The maximum experimental value is about 20% smaller than the ideal one, this difference being explained by the nonoptimum coupling $(k' < 1)$. In fact the dashed line in Fig. 2 is the calculated curve using (3) when, for each N_S value, $k'(N_S)$ and therefore $L_{\rm Seff}(N_S)$ have been calculated [6]. From these results it can be concluded that ideal coupling can not be obtained using thin wire sensing coils.

To improve the coupling, different sensing coil configurations were tried. For $N_S = 15$ (circle in Fig. 2) the experiment was repeated using a lead foil (0.125 mm thick and 12 mm width) coil. Since this coil had a lower L_S value than a thin wire coil, the contribution of the first term in (2) was smaller and an improvement of 8% was obtained. For the same reason other authors [8] have predicted CCC sensitivity improvements when considering thick wires.

For $N_S = 1$, the experiment was done using the overlapped tube as secondary single turn loop, in which case $k' = 1$ by definition. However, the result could not be distinguished from that obtained with a single turn coil wound on the inner or on the outer surface of the overlapped tube. The expected improvement in the coupling was not detected because, in this case, $L_{\rm Seff}$ $(N_S = 1) = A_{\rm OVeff} \ll L$ (see (3)). Nevertheless, the experiments clearly establish that the overlapped tube can be used as secondary single turn coil, which eliminates problems arising from relative mechanical vibrations of the sensing coil with respect to the overlapped tube. A requirement for this simple configuration to be used in an optimum CCC is that a SQUID with input coil inductance equal to $A_{\rm OVeff}$ has to be available. Since commercial SQUID's have L in the μH range and, for practical size reasons $A_{\rm OVeff}$ should no be greater than 100 nH, a custom-made SQUID is required.

Taking into account the above results we decided to design and build SQUID's with low input coil inductance, while

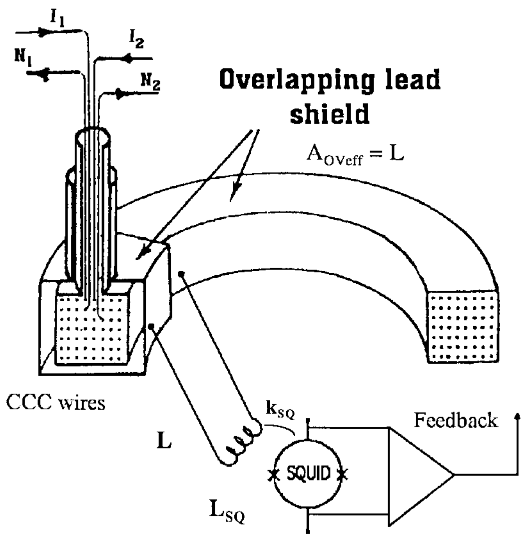

Fig. 3. Schematic representation of a CCC in which the overlapped lead shield of the windings is used as sensing coil.

keeping or increasing their energy resolution, to be used in optimal current sensitive CCC's.

III. Design of the CCC

An analysis of the published sensitivity data of present CCC's [4] shows that they are far from optimal in current sensitivity. The main reasons are the nonideal coupling between the sensing coil and the overlapped tube and poor matching of the effective inductances of the flux transformer, L_{Seff} and L, both contributing to $S_{\mathrm{CCC}} < 0.5$.

We propose to solve these problems and to improve the SNR by using the overlapped tube as sensing coil. This is schematically depicted in Fig. 3. In this case $I_S = I_1 N_1/2$ when $L = A_{\mathrm{OVeff}}$. Then, assuming that the SQUID is the only noise source, the mean square value of the equivalent input current noise in one of the primary windings, $\langle I_1^2 \rangle$, will be given by

$$\langle I_1^2 \rangle = 8\langle \varepsilon_{\mathrm{SQ}} \rangle / \left(N_1^2 k_{\mathrm{SQ}}^2 A_{\mathrm{OVeff}} \right) \tag{5}$$

where $\varepsilon_{\mathrm{SQ}}$ is the intrinsic energy resolution of the SQUID and k_{SQ} the coupling factor between L and the self-inductance of the SQUID loop, L_{SQ}.

A. CCC Dimensions

For practical reasons, we have limited the number of primary windings N_1 to 2×10^4. A_{OVeff} can be made large by having a small cross-section of the windings (using 65 μm diameter thin Nb-Ti superconducting wire) and by choosing a large diameter for the CCC. A_{OVeff} increases roughly linearly with diameter, until the CCC is approaching the lead shield put around it for shielding external magnetic fields. When the CCC is close to this shield, image effects reduce A_{OVeff} [6]–[8]. Since our cryostat has an inner diameter of 160 mm, and space is required for liquid helium filling and magnetic shielding, the diameter of the lead shield around the CCC was chosen to be 126 mm. Numerical simulations of our configuration showed that the optimal diameter occurred at approximately 70% of the shield diameter, providing that the height of the shield is sufficient. The value of A_{OVeff} at this optimal point is approximately 55 nH.

B. SQUID Sensor

The SQUID sensor plays a crucial role in the system. Given the value of A_{OVeff}, L should also be 55 nH. $\langle \varepsilon_{\mathrm{SQ}} \rangle$ is proportional to L_{SQ}, the self-inductance of the SQUID loop, which thus should be as small as possible. However, a lower practical limit is set by the requirement $L = n^2 L_{\mathrm{SQ}}$, with n the number of turns of the SQUID input coil.

If we take into account that $\langle \varepsilon_{\mathrm{SQ}} \rangle = 9k_B T L_{\mathrm{SQ}}/R_S$ [9], where k_B is the Boltzmann constant and R_S the shunt resistor in the SQUID junctions, and considering the optimization condition, $A_{\mathrm{OVeff}} = L = n^2 L_{\mathrm{SQ}}$, then we have $\langle I_1^2 \rangle = 72k_B T/k_{\mathrm{SQ}}^2 N_1^2 n^2 R_S$. Substituting a value of $R_S \approx 2\ \Omega$, $k_B = 1.38 \times 10^{-23}$ J/K and $T = 4.2$ K will give a current resolution of $45/k_{\mathrm{SQ}} N_1 n$ pA/$\sqrt{}$Hz. For a SQUID input coil with $n = 50$ turns and $k_{\mathrm{SQ}} \approx 1$, a current resolution of ≈1 pA/$\sqrt{}$Hz per turn in the primary windings could be obtained.

C. Insert

Apart from the normal cryogenic functionality, the insert supporting the CCC has been specially designed for minimization of external influences. To reduce mechanical vibrations special attention was paid to the rigidity of the insert. In addition, springs are present that mechanically fix the bottom part of the insert in the tail of the cryostat. Shielding of external magnetic fields is realized by two μ-metal shields (one at low temperature) and two lead shields. All wires, especially those of the CCC, are mechanically fixed to the rods supporting the bottom part of the insert.

IV. Realization

Since the SQUID forms the key element of our system, we started with the fabrication of a series of dc SQUID's with input coil inductance $L = 20, 40, 80$, and 160 nH and SQUID loop inductance, L_{SQ}, ranging from 100 to 250 pH. The SQUID's were made with Nb/Al technology using photolithography techniques [10]. They were electrically characterized by measuring current-to-voltage ratio, voltage-to-flux ratio, and flux noise versus frequency. All experimental values of the SQUID parameters fitted reasonably well with the design parameters [11]. As an example, the theoretical value for the equivalent current noise at the input coil of the 200 pH SQUID ($n = 20$, $L = 80$ nH) is ≈1 pA/$\sqrt{}$Hz. The measured value was almost twice as high, due to the room-temperature read-out electronics. When a double relaxation oscillation SQUID (DROS) [12] was used for readout the measured value approached the theoretical one. Transferring this to the current resolution of the CCC, we expect to find a value of ≈0.1 fA/$\sqrt{}$Hz for $N_1 = 2 \times 10^4$. Therefore, a current resolution of up to 10^{-7} nA/$\sqrt{}$Hz in the white noise region could be reached. Finally, the "ideal coupling" predicted when the overlapped tube is used as sensing coil has recently been confirmed experimentally [13]. In that experiment the connection between the overlapped tube and the SQUID was made with lead foil so that its inductance was negligible (<3 nH).

V. Conclusion

A CCC with an optimal current resolution for the measurement of small currents in the nanoampere level, aimed at amplification of the current generated by single electron transport devices, has been designed. A complete model for the calculation of the sensitivity of a CCC coupled to a SQUID by means of a flux transformer has been used. Ideal coupling between the CCC windings and the SQUID is realized by using the CCC overlapped tube as sensing coil. DC SQUID's with input coil inductance covering the range of the overlapped tube self-inductance have been successfully made and tested. Test measurements of the complete system are in progress.

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