A Basic Course in Network Analysis: Part II—Types of Problems and Sequencing

H. Vos and F. F. G. de Bruin

Abstract—**In a previous paper an introductory course to Electric Network Analysis was described. Some educational measures showed passing rates and students' insight to be unsatisfactory. In this paper we analyze problems that students have to solve. Examples of the difficulties that students have are presented. Several kinds of problems can be distinguished. Difficulties in concepts and some measures to support concept formation are discussed. Finally, a possible change in the overall sequence of problems is proposed.**

I. Introduction

IN a previous paper [1] an introductory course to Electric Network Analysis was described covering both time domain and frequency domain analysis. The passing rates for this course are quite low.

Shortcomings of the students' performance are of various origins. In the previous paper the lack of overview, as it showed up during examinations, was discussed. This is the most serious problem because attainment of an overview is the main objective of the course.

This overview should enable the students to make a correct choice among the three methods of the course (differential equations, convolution integral, transfer function) in overview problems as discussed in [1]. Of course, students also should be able to find a correct procedure to solve the problem and to execute this procedure.

Difficulties on a lower level probably reduce the chance that students get an overview of the subject matter. These lower level inadequacies showed up during tutoring hours. The following categories are distinguished: insufficient prior knowledge, difficulties with the application of formal methods, and conceptual difficulties arising from the content of the course.

After a discussion of these difficulties together with problems in which they show up, support for concept formation is presented and a specific sequence of problems is proposed.

II. Insufficient Prior Knowledge

When students start the course they can calculate easily the resistance of three resistors in parallel, but most of them fail when asked to derive the rule they used from first principles and to apply this derivation to capacitors or inductances as in Problem I:

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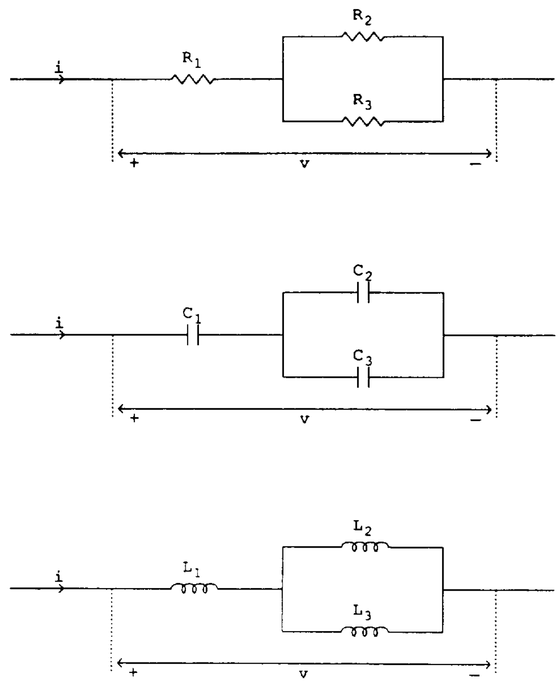

Fig. 1. Elements in parallel and in series (cf. Problem 1). The port carries the current i and the voltage v.

Problem 1: Insufficient Prior Knowledge

Calculate the relationship between the voltage v and the current i for the three ports in Fig. 1. What is the value of the equivalent single element?

Students can use some high-school knowledge (like $R_1 + R_2 = R$) to solve the first part of Problem I. This turns out to be not more than a memorized trick; they fail to find the equations that are needed (and were used in high school) to derive the "trick" and that now are needed to solve parts two and three. This kind of prior knowledge has to be reactivated first. Other difficulties concern analytical methods of calculation.

III. Difficulties in the Application of Concepts

Students show difficulties with formal methods. Some typical examples are given in this section.

Students find it difficult to derive one differential equation from a set of elementary equations and Kirchhoff's laws as in Problem 2.

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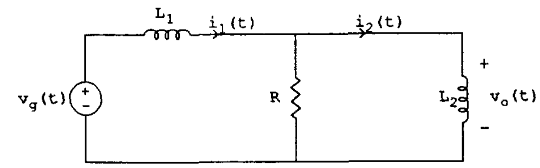

Fig. 2. An input-output system in a second-order network (cf. Problem 2). Input is voltage $v_g(t)$, output is voltage $v_o(t)$.

Problem 2: How to Get to One Differential Equation

A network is given as in Fig. 2.

a) What is the order of the network?

b) Derive the differential equations for the state variables.

c) Derive the differential equation for the voltage v_o.

Many students get the two equations of Problem 2(b) right, but fail to combine these equations with the resistor equation. Students get here a nontrivial example of a resulting differential equation being of order 1, while the network is of order 2. For the students the concept "differential equation" also is rather new.

Analytical methods in convolution integrals lead to errors. Students make many mistakes in the choice of limits of integration when calculating convolution with the impulse response as in Problem 3.

Problem 3: How to Simplify Boundaries of Integration

For the linear time-invariant input-output-system of Fig. 3 the output signal $v_o(t)$ can in general be calculated using the convolution integral.

a) How can the integral be simplified for an arbitrary *h*(*t*) and with $v_g(t)$ as given?

b) How can the integral be simplified for an arbitrary $v_g(t)$ and with *h*(*t*) as given?

c) How can the integral be simplified with $v_g(t)$ and *h*(*t*) as given (without calculating the result)?

In the frequency domain there are also difficulties impeding insight. Students have problems with the representation of complex functions of the frequency variable by Bode diagrams (logarithmic plot), or a phasor diagram. For example, when asked where the frequency (dc) is on the Bode diagram, quite a few cannot give the correct answer. An example of difficulties with phasors is given in Problem 4.

Problem 4: Complex Representation in the Frequency Domain

A port consists of two resistors and an inductor in series, as in Fig. 4.

a) Calculate the complex impedance of the port.

b) If the impedance is represented as in Fig. 4, what is a?

c) Draw the complex representation of the admittance of the port. Indicate the values of the frequencies clearly.

Students find it difficult to express variables like a in terms of the port variables *R* and *L*, and many fail to understand the limit behavior for high and low frequencies. The admittance is derived from formulas, instead of from the impedance curve. Representations of complex functions as parametrized curves in the complex plane seem to be difficult to understand,

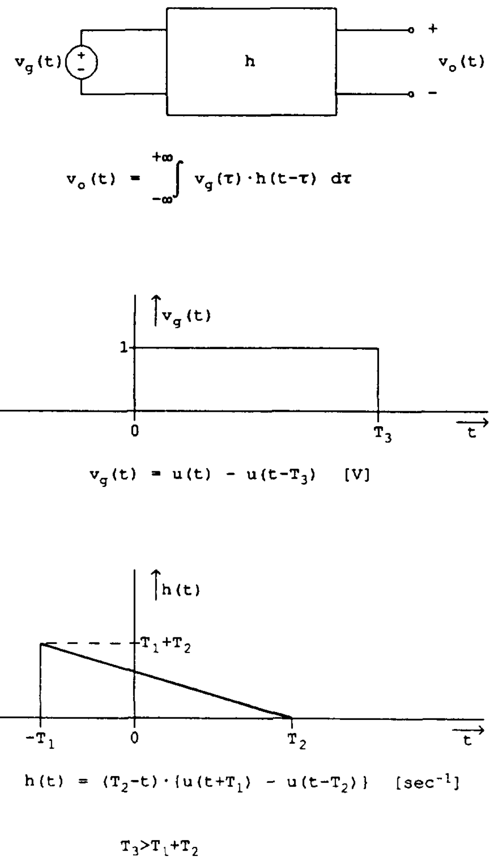

Fig. 3. An input-output system, with input signal and impulse response (cf. Problem 3). $u(t)$ represents the unit step signal.

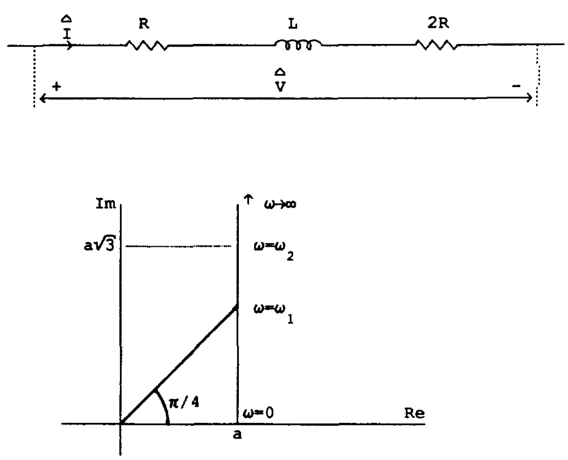

Fig. 4. A simple port and the polar diagram of its complex impedance (cf. Problem 4). The current and the voltage are in complex phasor notation.

although complex numbers are part of a first trimester course in calculus. Probably the representation of a complex-valued function is not an easy generalization from the representation of complex numbers.

Part of these difficulties have to do with a lack of experience or with forgetting prior knowledge, but some might be related to conceptual difficulties.

IV. Difficulties in Concepts

Students have several difficulties with the concepts involved that originate from the difficulty to explain and elucidate the concepts. The first one concerns linearity. Linear elements constitute linear circuits and linear input-output systems, but examples are given to show that nonlinear elements might also constitute a linear input-output system.

A source is not an input-output system in the usual sense in electrical engineering. A variable voltage source might be considered an input-output system, but then the input is the (often mechanical) change of the application point. In the last case the source may be considered linear or not, depending whether the voltage or the current is considered the output. Formally, the elementary equation of a voltage source $v_g(t) = E(t)$ doesn't represent a linear element because zero input doesn't give zero output. This poses problems to the lecturer in explaining linearity in a clear way.

The difficulties seem to find their origin in the principle of superposition. Very fundamental theorems, like Norton's or Thévénin's, are based on linearity and superposition. Active and passive elements play a different role here. Perhaps these conceptual difficulties can be solved by the use of the concept of a "complete model" of a network [2]. Linearity is defined here for the "dead network", i.e., the network without sources. Other difficulties are related to the first one.

A difficulty lies in the concept of impulse. What is an impulse and an impulse response? The Dirac delta function is seemingly easy for students to understand, as long as one uses the sifting property only. It is not a proper function, however, and therefore difficult to understand and use in other forms. For instance, why is an infinite current allowed in the concept of impulse, but not in other cases? What is the energy content of the impulse, and where does this energy go? These difficulties arise if an impulse response is derived by solving a differential equation.

A third type of difficulty lies in the concept of a complex signal, needed to introduce the system transfer function. A complex signal is much more difficult to represent than a complex number because of the time and frequency variables involved. After some time students become used to complex functions, but basically they don't understand the need for those functions.

Special educational measures were taken to meet these conceptual difficulties. These are outlined in the next sections.

V. Support of Concept Formation: Laboratory Sessions, Introduction, Connections

Some of the difficulties sketched above were anticipated at the time of construction of the course. To support concept formation, students have to attend a parallel course in laboratory exercises [3], [4]. To let students get an idea what linearity is, they have to discover — by measurement — the nonlinearity of a circuit that seems to be linear at first measurement (ratio of amplitudes of input and output signals is constant). To get an idea of impulse response, they have to measure the impulse response by observing the limit of block responses of stepwise smaller and larger input signals. Next, they have to measure Bode diagrams and polar figures of several circuits, and to compare these measurements with calculations. These exercises might help the students to construct a mental model of the concepts involved, in terms of actions [5]: they should be able to make the concepts both mentally and practically operational. The concepts and actions should not only be practiced, but also become related.

The relations are presented in the structural scheme (cf. [1]) that should help students to get an overview. This scheme is difficult to understand for a beginning student. Therefore, the main structure (cf. Fig. 3 of [1]) is presented separately along with an explanation that the student can understand. This introductory chapter (cf. Appendix, where the first part is reproduced) explains to the students the goals of the course and gives an outline of its contents. The chapter contains special questions to enable students to verify for themselves if they understand what is explained (cf. Problem 5).

Problem 5: Simple Appraisal Questions

Questions to be put after reading the first part of the introduction (Appendix).

1) An input-output system consists of a voltage source (voltage v_g) and two equal resistors (R) in series. The input of the system is voltage v_g, the output is the voltage v_o across one of the resistors. What are the impulse response, the transfer function and the differential equation for this input-output system?
2) What is the impulse response of an input-output system without a memory (i.e., a system that cannot store energy)?

The structure of the knowledge (how methods of the subject-matter are related), is explained first, before the student starts to study all methods in detail. First the map as a whole, then the roads separately. The map consists here of a "network" of methods, containing a conceptual network as an element. The introductory chapter coupled to the structural scheme of the methods should serve as a true, abstract advance organizer in the sense of Ausubel [6], [5].

Next the connections between the three methods are made easier. For instance, when introducing the frequency domain with complex transfer functions (after the time domain with its operator-like convolution integrals), the input-output relation is first derived for real-valued harmonies. The network is depicted as an operator on the input signal, changing both amplitude and phase. Then the transfer function is introduced as a mathematical operator doing the same. Finally, it is shown how easy It is to use complex harmonies from the beginning. Thus a contribution is made to insight in the relations between the various methods.

The structural scheme can help to see all the possibilities for solving a problem. The overview of the methods also provides for a check on the answer, because often more than one method of calculation can be applied, and both should

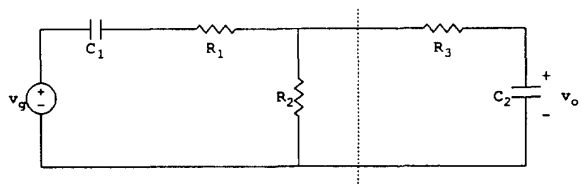

Fig. 5. An input-output system in a second-order network (cf. Problem 6). Input is v_g and output is v_o.

give the same answer. The presentation of the subject matter is based on the same principle, because a new method introduced is always shown to give the same results as an earlier method in simplified but rather general cases. The structural scheme can be of aid here, but learning an overview is not enough for solving problems. In order to help students solve problems, they should learn to travel the roads of the scheme and to make the right choices at the junctions.

VI. Problem Sequencing

A way to enable both good and average students to construct knowledge of the principles of network analysis is by presenting the subject matter in carefully designed portions. Pieces of instruction or information are presented together with problems, because it is thought that processing information in solving problems produces knowledge.

In examinations, and later in their study, students must not only be able to calculate currents and voltages along prescribed lines (the methods and procedures they learned), but they must be able to solve problems in a way they choose deliberately. The more strategic insight is attained, the easier new problems can be solved. However, it is very difficult to set problems in such a way that high level knowledge is produced. Problems that can be solved on a high level, where strategic insight and overview are products, can often be solved by using experience-based "tricks" that cannot be explained in any more detail, or by straightforward use of tedious calculations.

Such tedious calculations can be used to solve, for example, Problem 6. One can write network equations and element equations and try to solve these. For a more insight producing solution, it is necessary to see that application of Thévénin's theorem to the left of the dotted line in Fig. 5 leads to three complex impedances in series, providing the output voltage by voltage division. This insight is useful only if the student knows that the complexity of the problem is reduced. In other words, it is a prerequisite that the student can handle a (complex) voltage divider.

Problem 6: Solving Equations or Showing Insight

Derive the transfer function relating v_o to v_g for the network given in Fig. 5.

The sequence should therefore be to first give exercises in complex voltage divisions and second in application of Thévénin's theorem. This improves the possibilities to tackle problems on a higher level (application of Thévénin's theorem) thus producing more strategic insight (knowing the correct, easiest and most reliable ways to solve problems). Only then it makes sense to take a further step and to consider the conditions for application of Thévénin's theorem: nonharmonic signals or nonlinear circuits give wrong results (see Section IV).

Other kinds of questions are also designed to be an aid in the development of an overview. For instance, in Fig. 5, it is possible to ask questions concerning: (a) the order of the network; (b) the state variables; (c) differential equation relating input voltage v_g and output voltage v_o; (d) impulse response; (e) convolution of some $v_g(t)$ given to get $v_o(t)$; (f) transfer function; (g) Bode plot and polar plot of the transfer function; and (h) complex calculation of v_o for a given real harmonic v_g, with all questions based on the same diagram. More far reaching is to generate a problem that can be solved by several alternative methods, and to ask the students to find several methods and to compare the relative merits of these methods. Thus a discussion among the students can be enhanced.

Before overview problems can be tackled, procedural skills should have been developed by solving problems in which calculations are needed. Not all necessary information is usually given in the lecture notes. Separate tutoring notes can make the theory operational by giving hints or heuristics about the application of the concepts and calculations. For instance, in order to derive the differential equation for a network containing controlled sources, one has to preserve the control variables in the node or mesh equations ([7], pp. 102-103).

The foregoing also applies to prior knowledge. Students must know what they are supposed to know already. For instance, exercises with complex numbers are given before complex valued functions are introduced (see Section III). Such exercises should preferably add to the knowledge of the students by giving unexpected conceptual difficulties or dilemmas.

VII. Reconciling Contradictions

Along with the sequence of appraisal questions, procedural problems, strategic or overview problems, other problems are distinguished that raise understanding by their own nature. Such problems often contain "stumbling blocks." The design of these problems is such that solving them by formal methods will lead to contradictions or to obviously wrong results as compared to other knowledge or other lines of thought. Solving such a problem helps raising the level of understanding because the two roads of thinking need reflective thinking and discussion in order to be reconciled.

These contradictions can be found on all levels of problems and for all kinds of representational means: diagrams, symbols, words, sentences. Problems in which these contradictions arise can be found in the literature or can be designed easily. An example about diagrams is presented in Problem 7.

Problem 7: Beware of Short-Circuits

Derive the relation between voltage v and current for the one-port in Fig. 6.

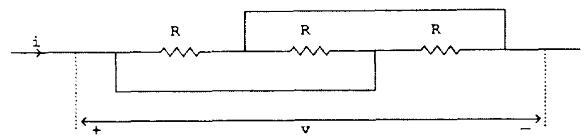

Fig. 6. A port consisting of resistors (cf. Problem 7).

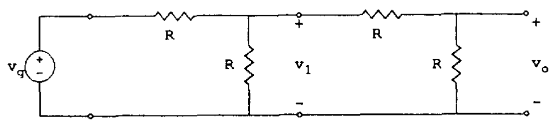

Fig. 7. A ladder network consisting of voltage dividers (cf. Problem 8).

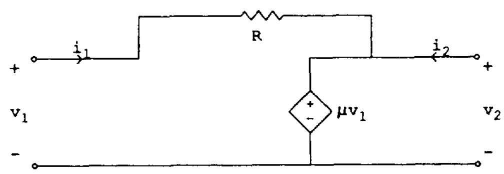

Fig. 8. A two-port with voltage-driven source (cf. Problem 10).

Students often do not see short-circuits. Sometimes the pictorial representation of networks, as in a diagram, already introduces difficulties. Formal application of algebraic methods in Problem 7 is tedious but redrawing Fig. 6 gives a one-port with three resistors in parallel, resulting in a better idea of 'what happens' in this case.

Other examples of such problems do in fact suggest counter-examples of procedures or formulas (matching of examples [8]). Problem 8 gives an example in which a product of factors is suggested but not applicable. In Problem 9 an example is given of a formula that is allowed if considered in a certain sense, but considered otherwise not.

Problem 8: Is a View Valid?

Two voltage dividers can be put in series as in Fig. 7. What is the ratio of the output voltage v_o, to the input voltage v_g?

Problem 9: Is an Expression Valid?

Is it allowed to write cos $\omega t+\phi=2$?

Another example of this kind of problems concerns the basis of the subject matter as in Problem 10. Some concepts look axiomatic that can be understood only by methods outside the proper domain of the subject matter.

Problem 10: Some Problems Cannot Be Solved by Formal Methods.

Is the two-port presented in Fig. 8 a passive or an active two-port?

Problem 10 cannot be solved by simply writing the equations pertaining to the definition of "active," but only by adding a load on the right-hand side and considering the energy input and the energy output on the left, or vice versa. This cognitive strategy is not explained in greater detail at this moment. It is something to "know," and later the student will understand the point. At the present moment the student will meet some limits to the formal methods presented.

Reconciling these "errors" in view or interpretation improves insight. Problems designed along these lines have already been used with good results in remedial sessions with a small group of students wanting extra help.

VIII. Conclusion and Discussion

The usual review questions are often considered too easy by the students because the answer can be written by inspection. Each chapter should preferably be accompanied by appraisal questions (cf. Problem 6) that can be answered if the student combines several paragraphs or applies the concepts in a new situation. Special questions presenting contradictions that have to be reconciled and lead to discussion are advised in order to improve understanding. Prior knowledge questions should reactivate old skills which the students need constantly. After these questions students can exercise new skills they have to develop, like solving differential equations, calculating transfer functions, and working out convolution integrals.

Only after the students have had some experience in these calculations, strategical insight problems or overview problems can be given. Here lies a big difficulty in teaching. To read information or to see problems being solved is only part of learning. Students build knowledge by solving problems themselves. It is very difficult to get students to work hard before the examination time in order to be able to tackle insight problems. Possible strategies are to set strategic insight problems as soon as possible in the course in order to confront students with the examination requirements. Possibly these problems might count for the final grade.

This would add to the external motivation for the students to study more continuously by the partial examination system [1]. Internal motivation, based on a gradually increasing notion that one learns step-by-step to understand new concepts and to use the related skills, can be fostered by a careful sequencing of instruction and problems. Internal motivation might be further stimulated by discussing real-life problems for which the methods of network analysis are developed and the outcome of calculations are of help and make sense.

Often students do not make exercises and other homework tasks. Sometimes it is thought that students have to be motivated to make the exercises. However, often the subject matter is too abstract and too complex for that. We are of the opinion that students are motivated when they find out that they are able to make the exercises (cf. [9]). Thus, introducing higher level problems step-by-step will facilitate problem solving and help the students to see possibilities to fulfill the tasks given and to pass the examinations. This raises a need for a clear goal from the beginning, and a sequence of learning problems and instruction that provide students with the cognitive instruments necessary to reach that goal.

APPENDIX
INTRODUCTION TO NETWORK ANALYSIS

Network Analysis is about the properties of electrical input-output systems. An electrical network is an idealized electrical circuit. Defining an input (variable source) and an output (voltage or current) provides us with an input-output system. Such an abstract system in an imaginary circuit can be viewed in several ways. Here three views are treated: the operator-on-signal view, the what-does-not change (eigenfunction) view, and the equilibrium approach.

1. Impulse Response

The first view implies that the system is characterized by the way it "sounds" after it is "hit." This can be compared to the way a bell sounds after a stroke or a sound system after a tap on the microphone. The ideal tap on the input is called an impulse: in one moment a certain amount of energy is transferred to the system. Afterwards one looks at the output how the output signal develops in time. This is called the impulse response. The impulse is (momentarily) infinite, the impulse response often remains finite.

The response to other input signals can be calculated from the impulse response by a technique called convolution. The convolution integral, containing the impulse response function, can be considered as an operator acting on the input signal.

2. Transfer Function

The system can be viewed in a different way when input signals that appear rather unchanged at the output are sought. Some signals come out, e.g., with the same shape but possibly a different amplitude. In case of linear, time invariant systems these signals turn out to be harmonic functions of time. The system can now be characterized by specification of the change in amplitude for each of those signals.

We can compare this with determination of the frequency response of an amplifier system. For each frequency the amplitude change and the change in phase are measured. These changes can be represented by one complex multiplicative factor. All these factors together represent the (complex) transfer function, a function of frequency. The frequency here is the only parameter of the input that is important for the transfer.

Time-invariant systems therefore do not distort harmonic input signals, but multiply them in a complex representation with the complex value of the transfer function. A harmonic input signal is an ideal signal that always repeats itself after one period, in the future as well as in the past. The amplitude is constant. (Compare this with the impulse above.) Every period a constant amount of energy enters the system, and every period a constant amount of energy leaves the system.

3. Differential Equation

The third view is based on the dynamical equilibrium between the energy-changes in the system. The system is no longer considered as a system into which something enters and something comes out, but as an equilibrium system between the input signal and the output signal both at the outside. This system is described by a differential equation. The solving of the differential equation must be done for each input signal anew. Differential operators usually act on both input signal and output signal.

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