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A model for the contact behaviour of weakly orthotropic viscoelastic materials

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ABSTRACT

Fibre reinforced elastomers behave anisotropically as well as viscoelastically. Yang and Sun (1982) developed an elastic contact model for anisotropic materials which, in the present work, is extended to account for viscoelastic effects. The developed viscoelastic contact model uses the creep compliance function of the material in the direction of indentation. The results of the model agree with experimental results obtained on short fibre reinforced EPDM. Furthermore, a parameter study of the coefficients of the creep compliance function on the real contact area has been made. The results show that, at short time scales, the viscoelastic real area of contact can be significantly smaller than when assuming fully elastic behaviour. At long time scales the results of the viscoelastic contact model equal those of the elastic model of Yang and Sun.

1. Introduction

Fibre reinforced elastomers are used in a range of industrial applications such as tyres, transmission belts and seals. Similar to fibre reinforced polymers, the addition of fibres to an elastomeric matrix causes improved mechanical performance, albeit typical reinforcements for elastomers, especially those with short fibres, are rather low [1,2]. With the increasing use of short fibre reinforced elastomers, it has become important to characterize their contact behaviour as this influences their tribological performance. Fibre reinforced elastomers behave viscoelastically due to the elastomeric matrix and anisotropically as the result of the preferred orientation of the fibres. To determine the contact between a rigid spherical indenter and an anisotropic viscoelastic material we require a contact model that considers both effects.

Models describing the contact behaviour of viscoelastic materials are usually limited to isotropic material behaviour. In the present study, a viscoelastic anisotropic material with a low degree of anisotropy is considered. This means that the time dependent material properties, such as creep compliance in tensile and shear, have similar, but not necessarily equal values in each principal direction.

The effect of anisotropy in the elastic contact problem was studied theoretically by Willis [3], who considered a three dimensional elastic contact of full anisotropic bodies. Swanson [4] used the Willis [3] approach to calculate the Hertzian contact problem for elastic orthotropic materials. Therefore, if provided with nine different material properties (such as the elastic moduli, shear moduli and Poisson's ratios, in three directions) it is possible to calculate the area of contact for orthotropic materials. However, for fibre reinforced elastomers in the rubbery state, it is difficult to accurately obtain the material properties in that many directions and a model that describes the contact behaviour whilst requiring a smaller number of parameters is highly desirable.

Yang and Sun [6] and Tan and Sun [7] performed indentation tests in laminated composites, showing that the loading curve for these anisotropic materials follows a power law with the same index as would be expected from the Hertz theory. Consequently, for an anisotropic elastic material that is indented by a rigid sphere they proposed to approximate the deformation by using the elastic modulus of the anisotropic material in the direction of indentation only.

Chen [8] showed that for isotropic and anisotropic materials under pure normal loading, the normal displacements and the pressure distributions are identical; only the absolute values of the pressure may differ. Furthermore, according to Chen [8], the stress distribution inside an elastically deforming orthotropic body is symmetrical when one of the principal axes of the orthotropic material coincides with the direction of indentation.

This means that the approximation of Yang and Sun [6] and Tan and Sun [7] to model the anisotropic behaviour as a modification of the isotropic behaviour appears to be valid.

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Nomenclature

a	radius of the contact area, for elastic materials [mm]
$a(t)$	radius of the contact area, for viscoelastic materials [mm]
E'	equivalent elastic modulus [MPa]
E_z	elastic modulus in the direction of indentation [MPa]
E_x	elastic modulus perpendicular to the indentation direction [MPa]
F_N	applied normal force [N]
$F_N(t)$	time dependent applied normal force [N]
$H(t)$	heaviside function [–]
$p(r,t)$	pressure distribution at a viscoelastic contact [MPa]
r	spatial variable in the contact pressure [mm]
R	radius of the spherical indenter [mm]
S^*	time it takes to reach a stable state in viscoelastic contact [s]
t	time, temporal variable in the contact pressure [s]
$\psi_z(t)$	stress relaxation function, measured in the indentation direction [MPa^{-1}]
$\phi_z(t)$	creep compliance function, measured in the indentation direction [MPa^{-1}]
$\phi\ (t)$	creep compliance function of the material model [MPa^{-1}]
ϕ_r	relaxed creep compliance [MPa^{-1}]
λ_i	retardation time of the ith component [m s]

The above is valid for purely elastic materials. Viscoelastic materials show time dependent behaviour which affects the contact problem, as the boundary conditions change to become time dependent functions. This type of problem is more complex to solve than a time independent problem because methods such as the Laplace transform and the elastic–viscoelastic correspondence principle cannot be directly applied. A solution to the viscoelastic isotropic contact problem was found by Lee and Radok [9], who obtained the pressure distribution over the contact area for a non-decreasing contact area function. Graham [10] obtained viscoelastic analogues to the Hertz equations for a contact area function with a single maximum and, later, for a time dependent contact area with any number of maxima or minima [11]. Ting [12,13] expressed the viscoelastic solutions in terms of the solution to the elastic contact problem, thus solving the contact stresses between a viscoelastic solid and a rigid indenter for a contact area that follows an arbitrary time function.

The viscoelastic contact models discussed in [9–13] are for homogeneous and isotropic materials. In the present study, the method of Yang and Sun [6] and Tan and Sun [7] that approximates orthotropic behaviour by using the material properties in only one direction is extended to consider viscoelasticity. This is done by replacing the isotropic viscoelastic time function by the anisotropic viscoelastic time function in the direction of indentation.

2. Sun's anisotropic contact model

As discussed before, Yang and Sun [6] and Tan and Sun [7] proposed an approximation for the deformation of an anisotropic elastic material that is indented by a rigid sphere. Following this approximation, the contact area is a circle of radius given by

$$a=\left(\frac{3RF_N}{4E'_z}\right)^{1/3} \tag{1}$$

where E'_z is the reduced elastic modulus in the direction of indentation, z. A comparison of the contact areas calculated employing this unidirectional model with Willis's anisotropic contact model [3] has been made by Swanson [4]. He found that the anisotropic contact model only gives a 4% larger contact area than the unidirectional model, for a material that is twice as stiff in the plane of indentation (i.e. perpendicular to the indentation direction). Therefore the approximation of Yang and Sun [6] and Tan and Sun [7] can be considered valid for materials with a low degree of anisotropy. At high degrees of anisotropy, say when the difference in properties is more than 400%, the approximation proposed by Yang and Sun [6] and Tan and Sun [7] is not valid [4,5].

3. Extension to viscoelastic contact, based on Sun's model

A common approach in linear viscoelastic theory, see e.g. Lee and Radok [9] and Ting [12], is to express the viscoelastic solution of the isotropic contact problem in terms of the elastic solution. Following this thought, the stress distribution inside an orthotropic viscoelastic material can be described based on the orthotropic elastic solution. This means that, for an anisotropic viscoelastic material with a low ratio of reinforcement (i.e. $E_z/E_x<2$), the time dependent contact area can be calculated by combining the solution of the viscoelastic contact problem with the elastic model described in [6,7]. This means that the contact behaviour of the viscoelastic anisotropic material is characterised by time dependent material properties measured in the direction of indentation, such as a stress relaxation function $\psi_z(t)$ or a creep compliance function $\phi_z(t)$.

For the loading phase of the contact, i.e. for an increasing contact area, the pressure distribution in the contact area, p(r,t) is given by

$$p(r,t)=\frac{4}{\pi R}\int_0^t \psi_z(t-\tau)\frac{d}{dt}\left\{a^2(\tau)-r^2\right\}^{1/2}d\tau \tag{2}$$

where R is the radius of the spherical indenter, a is the radius of the contact area, r and t are the spatial and temporal variables, respectively and τ is the dummy variable from the convolution integral.

The total applied time dependent normal force, $F_N(t)$, is calculated by integrating the pressure distribution over the contact area. This results in

$$F_N(t)=\frac{8}{3R}\int_0^\infty \psi_z(t-\tau)\frac{\partial}{\partial\tau}(a^3(\tau))d\tau \tag{3}$$

The creep compliance $\phi_z(t)$ and stress relaxation $\psi_z(t)$ functions are related in the Laplace domain according to

$$\psi_z(s)\phi_z(s)=\frac{1}{s^2} \tag{4}$$

Assuming that the force is defined by a Heaviside function as $F_N\ (t)=H(t)\cdot F_N$, Eq. (3) can be inverted, giving the radius of the contact area:

$$a^3(t)=\frac{3}{8}R\cdot F_N\cdot\int_0^t \phi_z(t-\tau)\frac{d}{d\tau}H(\tau)d\tau \tag{5}$$

Solving the integral allows to express the radius of the viscoelastic contact area as

$$a(t)=\left(\frac{3R\cdot F_N}{8}\phi_z(t)\right)^{1/3} \tag{6}$$

Eq. (6) describes the contact patch between a smooth and rigid spherical indenter and a deforming viscoelastic anisotropic body as a circle with time dependent dimensions, characterized by the viscoelastic properties of the anisotropic material in the indentation direction only.

4. Results and discussion

The developed contact model has been used to calculate the time dependent contact area for two synthetic EPDM rubbers: an unreinforced one and a fibre reinforced one. The fibre reinforced EPDM contained 5 parts per hundred rubber (phr) of unidirectionally oriented Twaron® aramid fibres, supplied by Teijin Aramid B.V. The elastic and viscoelastic material properties of these materials were measured by employing a tensile tester and a Dynamic Mechanical Analysis (DMA), respectively. The elastic modulus of the unreinforced material is E = 4.39 MPa and the creep compliance is expressed as a series of discrete exponential terms [15] given by

$$\phi(t) = \phi_r - \sum_{i=1}^{3} \phi_i \exp\left(\frac{-t}{\lambda_i}\right) \tag{7}$$

in which ϕ_r represents the creep compliance at the fully relaxed state and λ_i the retardation times. For the unreinforced material a creep compliance in relaxed state of $\phi_r = 0.35\ \mathrm{MPa}^{-1}$ was measured, whilst for the reinforced material $\phi_r = 0.29\ \mathrm{MPa}^{-1}$. The remaining parameters in Eq. (7) are summarised in Table 1 for both materials.

In the following, first the results obtained using the viscoelastic contact model are compared with the purely elastic case, employing isotropic material properties of EPDM. Second, the anisotropic behaviour of the reinforced EPDM is analysed, while validating the model with experiments. Third, the influence of the creep compliance function $\phi(t)$, the relaxed creep compliance ϕ_r and the retardation times λ_i on the contact area is analysed. Due to the asymptotic behaviour of the quantities in Eq. (7), the modifications of the retardation times affect neither the final value of the creep compliance function nor the final size of the contact area. The goal of this analysis is to evaluate the effect at the small and intermediate time scales.

4.1. Elastic versus viscoelastic contact model

The results from the viscoelastic contact model are compared to those obtained with the purely elastic case for a contact composed of a rigid ball with a radius R = 12.5 mm against a deforming flat surface at a normal load of 2 N. Fig. 1 shows the calculated contact radius for EPDM, calculated for the purely elastic case, a, and when taking into account the viscoelastic behaviour, a(t). The difference between the two models occurs immediately after loading and decreases with increasing time; in this case, after 10 ms the difference between the viscoelastic and elastic contact area is about 17%, which reduces to less than 2% at 1 s. For contacts that occur at short time scales with respect to the relaxation time of the material, a viscoelastic contact model gives a more accurate description of the contact area. For long time scales, the contact areas calculated using a viscoelastic contact model or an elastic contact model are similar. Note that, as the EPDM material is isotropic, these results are equal to those obtained using isotropic contact models.

Table 1
Compliance coefficients and retardation times for EPDM and reinforced EPDM.

I	Unreinforced		Reinforced	
	ϕ_i [MPa^{-1}]	λ_i [ms]	ϕ_i [MPa^{-1}]	λ_i [ms]
1	4.69E−02	6.4	1.82E−01	7.3
2	4.90E−02	71.3	3.82E−02	6.6
3	8.23E−02	728.4	4.05E−02	71.1

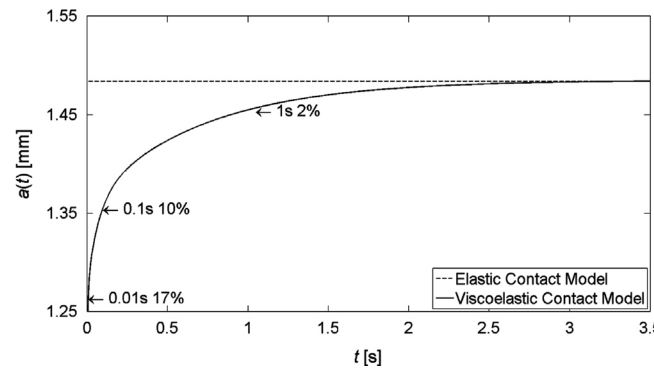

Fig. 1. Radius of the contact area, $a(t)$, calculated using the elastic contact model of Hertz (dashed line) and the viscoelastic contact model (solid line).

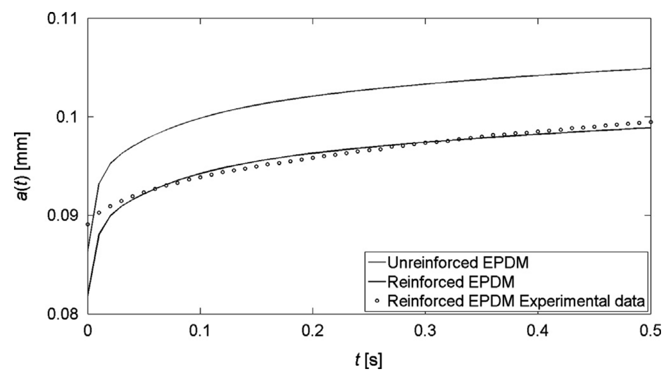

Fig. 2. Radius of the contact area from experimental data and calculated with the contact model.

4.2. Isotropic versus anisotropic contact

The anisotropic viscoelastic contact model can be validated using the fibre reinforced EPDM. Indentation tests were performed on this material employing an indenter with a radius of 1 mm at an applied load of 10 mN using a holding period of 500 ms. The contact area was obtained from the resulting load–displacement curve during this holding period, as described by Tweedie [16]. These experimental results are compared with the contact areas that are calculated using Eq. (6) and the creep compliance data from Table 1. The experimental and calculated contact areas are shown in Fig. 2.

It can be seen that the radius of the contact area obtained through micro-indentation is similar to the radius of the contact area calculated using the anisotropic contact model with material properties for the reinforced EPDM obtained using DMA.

4.3. Effect of different parameters of the creep compliance function

In a parameter study, the different parameters of the creep compliance function of EPDM have been modified to study their influence on the final creep compliance and therefore on the calculated contact behaviour.

4.3.1. *Creep compliance function $\phi(t)$*

The creep parameters as listed in Table 1 have been multiplied by factors of 2, 3, 4, 5 and 6, respectively. The resulting calculated contact radii as function of the time t are shown in Fig. 3. The solid black line marked ϕ(t) shows the radius of the contact as a function of time for the actual material whilst the dotted line represents the purely elastic solution. The various lines marked $2\cdot\phi$(t)–$6\cdot\phi$(t) show the radius of the contact area calculated at the various values of the creep compliance. As the creep compliance increases, the radius of the contact area also increases. This can be translated into a loss of stiffness of the material and therefore a higher indentation under the same loading conditions.

4.3.2. *Relaxed creep compliance ϕ_r*

The influence of the creep compliance at a fully relaxed state, ϕ_r, is analysed by multiplying ϕ_r with factors 2, 3, 4, 5 and 6. For relatively long time scales, the values obtained for the contact radius in each case will be equal to those shown in Fig. 3. At short time scales the results will differ. Fig. 4 shows the time it takes for the time dependent contact radius (a(t)) to reach a stable value (time S*, as defined in Fig. 3). S* represents the time after which the contact area calculated by the viscoelastic contact model deviates less than 1% from the contact area calculated as by the elastic contact model, $a(t=S^*)=0.99\cdot a(t=\infty)$.

Fig. 5 shows the relative size of the contact area at different instants of time (0.1 s, 0.7 s and 1.5 s). The multiplication factor 1 refers to the original creep function, a factor 2 indicates $2\cdot\phi_r$, meaning the relaxed creep compliance equals two times the original relaxed creep compliance, and so on. These curves show that an increase of only the relaxed creep compliance, whilst keeping the other creep parameters constant, does affect the rate of increase of the contact area, but not the final size of the contact in the relaxed state.

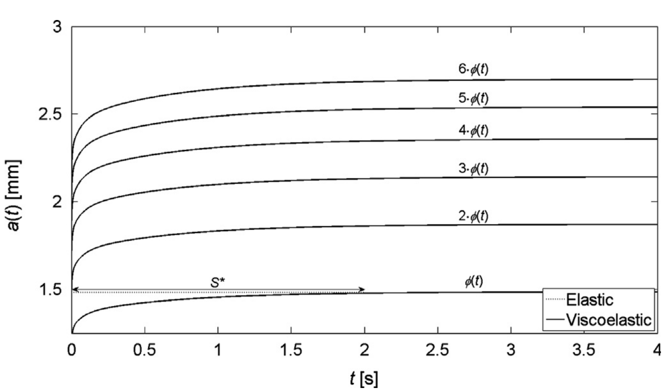

Fig. 3. Influence of the creep compliance function on the calculated radius of the contact area.

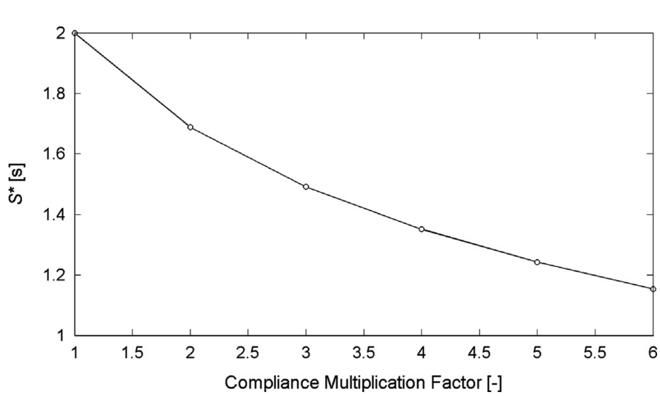

Fig. 4. Influence of the relaxed creep compliance ϕ_r on the time to reach a stable contact area.

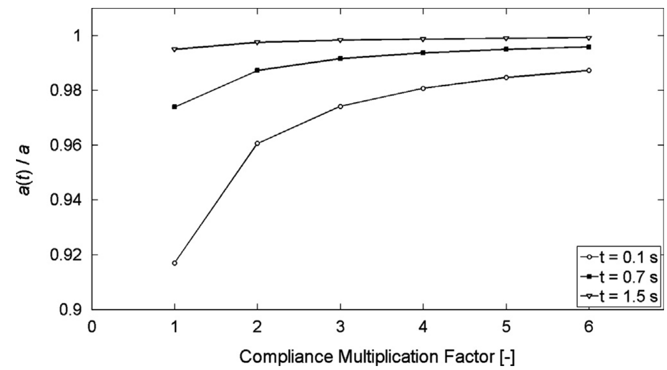

Fig. 5. Influence of the relaxed creep compliance ϕ_r on the radius of the contact area.

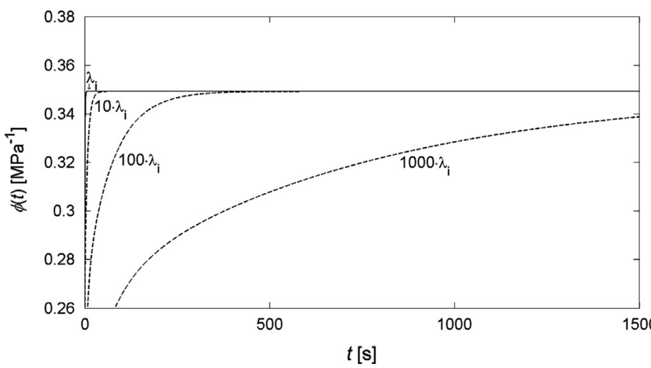

Fig. 6. Creep compliance function at different factors of the retardation times.

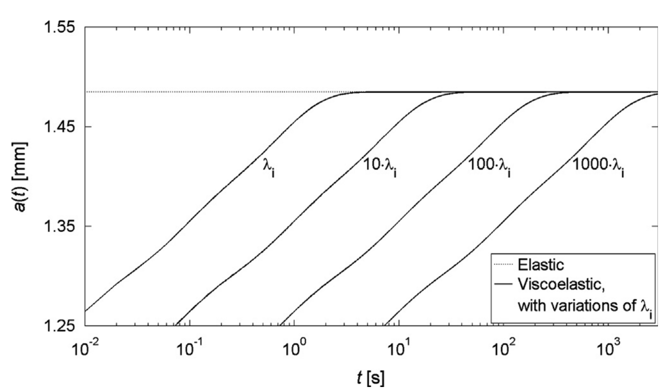

Fig. 7. Radius of the contact area, at different factors of the retardation times.

4.3.3. *Retardation times, λ_i*

Fig. 6 shows the influence of the retardation times on the creep compliance function; the retardation times, λ_i in Table 1, were multiplied by 10, 100 and 1000. Fig. 7 shows the resulting effects on the radius of contact. It can be seen that an increase of all the retardation times produces a decrease of the rate at which the creep compliance reaches the stable state.

Fig. 7 shows that an increase in the retardation times slows down the increase of the contact area. Comparing the contact behaviour of a ‘fast’ viscoelastic material with that of a ‘slow’ one, i.e. two materials that differ only in the retardation times, will result in the same contact area at the long time scale, but the indentation occurs more slowly with higher retardation times.

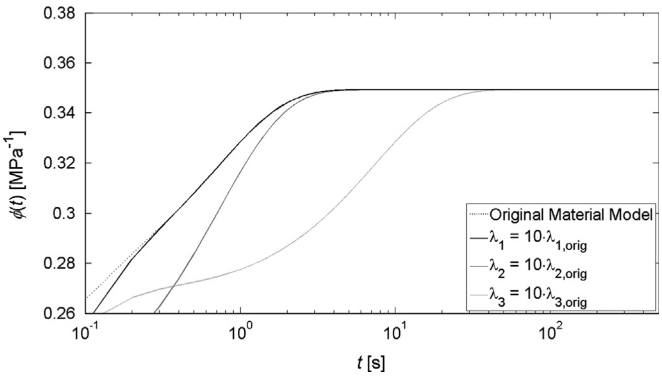

Fig. 8. Influence of individual retardation times on the creep compliance function.

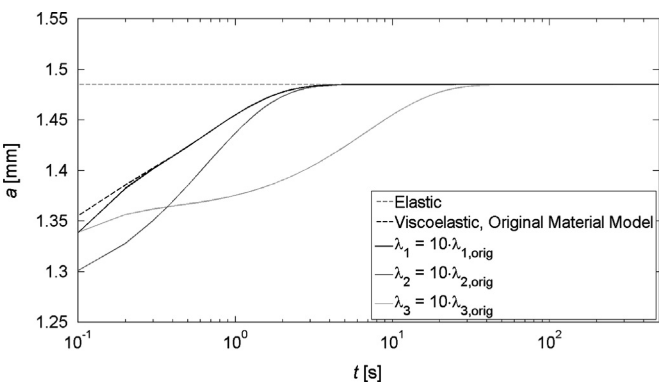

Fig. 9. Influence of individual retardation times on the radius of the contact area.

The influence of the separate retardation times λ_1, λ_2 and λ_3 is studied by independently multiplying each retardation time by a factor of 10. As the creep compliance coefficients are all of the same order of magnitude, it is possible to compare the influence of each retardation time. It should be noted that this is a hypothetical case, because multiplying λ_1 by 10 results in a value that is quite close to λ_2, meaning that in the calculation, the material is actually characterized by only two time responses. The resulting creep compliance curves can be seen in Fig. 8, where the dotted line shows the creep compliance function calculated for the actual material and the various grey lines show the creep compliance calculated from the modifications of the retardation times.

The resulting values for the radius of contact are shown in Fig. 9. The dashed black line represents the contact radius of the actual material. The various grey solid lines show the radius of the contact area calculated with the creep compliance resulting from modifications of the retardation times. At very short time scales ($\sim 10^{-1}$ s) the contact area with the modified smallest retardation time (λ_1) approaches the original contact area. At long time scales ($> 10^1$ s) a modification of the largest retardation time causes larger differences because the contact area approaches its stable value only very slowly.

5. Conclusions

A model that describes the contact behaviour of viscoelastic weakly anisotropic materials has been developed. The contact model is based on the fact that, under weak anisotropy, the contact behaviour can be appropriately described by taking only the properties of the material in the normal or loading direction. Similar to common practice in viscoelasticity, the solution in orthotropic viscoelasticity can be based on the orthotropic elastic case. A comparison between the contact areas calculated using an elastic contact model and a viscoelastic contact model shows that results coincide for long time scales. Experimental data of a short fibre reinforced EPDM was compared with the contact area calculated with the anisotropic viscoelastic contact model, showing good agreement. From the parameter study on the creep behaviour it can be concluded that an increase in the creep compliance function results in an increase of the size of the contact area. Furthermore, an increase in the retardation times inserted in the model results in a slower stabilisation of the size of the contact area.

The developed model is suitable for describing the contact behaviour of short-fibre reinforced elastomers. These have a rather low degree of anisotropy due to the non-perfect alignment of the short fibres and are often applied in rolling contact configurations, where the typical contact times in the order of milliseconds. The results of the model show that at short time scales the real area of contact as calculated using the viscoelastic material model can be significantly smaller than when assuming fully elastic behaviour.

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