CURRENT DISTRIBUTION AND AC LOSSES IN TWISTED MULTIFILAMENTARY AC SUPERCONDUCTORS

A.J.M. Roovers and L.J.M. van de Klundert

University of Twente, Department of Applied Physics,
P.O.Box 217, 7500 AE Enschede, The Netherlands

Abstract

The development of AC superconductors has led to multifilamentary wires having a highly resistive matrix material and a small twist pitch and filament diameter. In this paper a numerical one-dimensional model will be presented which enables us to calculate the current distribution as well as the magnetization and transport current loss in a current-carrying AC superconductor subjected to a transverse alternating magnetic field. The impact on wire design will also be discussed.

Introduction

In the last few years there has been a growing interest towards the application of superconductors under AC conditions[1]. This development also implies a need for a different approach in the conductor design. Besides the criteria, imposed by DC conductors (stability, high critical current density), the loss behaviour of the wire also becomes an important issue. In order to reduce the loss in multifilamentary superconductors, highly resistive matrix material, a small twist pitch and ultra-fine filaments are applied[2]. For this kind of conductor the transport current loss may become a substantial part of the total loss. The current distribution in the conductor affects the loss behaviour of the wire. It is therefore important to include the current distribution in the loss evaluation. In this paper, a numerical model is presented which enables us to calculate the loss in a current-carrying multifilamentary AC conductor subjected to a transverse alternating magnetic field.

Definition of the problem

Introduction

Consider an infinitely long straight round multifilamentary conductor having many small filaments. The current density and the electromagnetic field are considered as averaged over an area large compared to a filament section. In this case the anisotropic continuum model as proposed by Carr[3] can be applied. The filaments, having a twist pitch L_p, are located in a filamentary zone, $R_i < r < R_o$. The matrix material exchange between them. The critical current density is determined by the value of the applied magnetic field, B^a. The contribution due to the transport current, B^i, is neglected. The model is therefore only valid in the case $B^a \gg B^i$. Furthermore only fully penetrated filaments are considered, i.e. the magnetic field change is large compared to the penetration field of a filament: $\Delta B^a \gg \mu_0 J_c d_{fil}/\pi$. The last constraint is the invariancy of all physical quantities with respect to the wire axis ($\partial_z = 0$).

Manuscript received August 22, 1988.

Maxwell's equations

The starting point is Maxwell's equations:

$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}} \tag{1}$$

$$\nabla \times \mathbf{B} = \mu_0 \, \mathbf{j}. \tag{2}$$

Within the frame of our model this set of equations can be simplified by choosing a cylindrical coordinate system and considering the invariancy with respect to the wire axis ($\partial_z = 0$) and the non-conductivity of the matrix material ($j_r = 0$). Including these constraints yields:

$$\frac{1}{r}\,\partial_\vartheta\, E_z = -\,\partial_t\, B_r \tag{3}$$

$$\partial_r\, E_z = \partial_t\, B_\vartheta \tag{4}$$

$$\partial_r\, E_\vartheta - \frac{1}{r}\,\partial_\vartheta\, E_r + \frac{1}{r}\, E_\vartheta = -\,\partial_t\, B_z \tag{5}$$

and

$$\frac{1}{r}\,\partial_\vartheta\, B_z = 0 \tag{6}$$

$$\partial_r\, B_z = -\,\mu_0\, j_\vartheta \tag{7}$$

$$\partial_r\, B_\vartheta - \frac{1}{r}\,\partial_\vartheta\, B_r + \frac{1}{r}\, B_\vartheta = \mu_0\, j_z\,. \tag{8}$$

By introducing the average electric and magnetic fields defined as:

$$\overline{A}(r) = \frac{1}{2\pi}\int_0^{2\pi} A(r,\vartheta)\, d\vartheta, \tag{9}$$

where A is a component of the electric or magnetic field, the following set of equations is obtained:

$$\partial_r\, \overline{E}_z = \partial_t\, \overline{B}_\vartheta = \partial_t\, B^i_\vartheta \tag{10}$$

$$\frac{1}{r}\,\partial_r\,(r\,\overline{E}_\vartheta) = -\,\partial_t\,\overline{B}_z = -\,\partial_t\, B^i_z \tag{11}$$

$$\partial_r\,\overline{B}_z = \partial_r\, B^i_z = -\,\mu_0\, j_\vartheta \tag{12}$$

$$\frac{1}{r}\,\partial_r\,(r\,\overline{B}_\vartheta) = \frac{1}{r}\,\partial_r\,(r\, B^i_\vartheta) = \mu_0\, j_z\,. \tag{13}$$

The average values of the radial components of the electric and magnetic fields are equal to zero. It should be noticed that the mathematical description according to Eq. 10 to 13 has become a one-dimensional problem. This implies a considerable reduction of computing time.

Constitutive equation

Besides Eq. 10 to 13 a constitutive equation describing the relation between $\mathbf{j}$ and $\overline{\mathbf{E}}$ is required. A fair approximation of this relation can be obtained by

0018-9464/89/0300-2127$01.00©1989 IEEE

considering a circular filament carrying shielding currents and a transport current in which the boundary, separating the current densities with opposite signs, is assumed to be a straight line[4]. This approach yields the following constitutive equation:

$$j_s = \begin{cases} j_c(B^a)\,\dfrac{\bar{E}_{\parallel}}{|\dot{\bar{B}}_{\perp}|R_f\frac{\pi}{4}} , & |\bar{E}_{\parallel}| < |\dot{\bar{B}}_{\perp}|R_f\frac{\pi}{4} \\ j_c(B^a) , & |\bar{E}_{\parallel}| \geq |\dot{\bar{B}}_{\perp}|R_f\frac{\pi}{4} \end{cases} \tag{14}$$

This equation relates the current through a filament with the component of the average electric field parallel to the filament. The dynamic resistance depends on the field change perpendicular to the filament as well as on the radius of the filament.

Calculation of the loss components

Solving Maxwell's equations combined with the constitutive equation yields the average values of the electromagnetic field and the current density. Knowing the electrical field and the current density suffices to calculate the transport current loss per cycle per unit length of wire:

$$Q_t = \int_0^T \iint_S \mathbf{j}\cdot\mathbf{E}\; dS\; dt = \int_0^T \iint_S j_s\, \bar{E}_{\parallel}\; dS\; dt . \tag{15}$$

In order to calculate the magnetization loss, besides the magnetic field, the magnetization is also required. In the loss calculations the following approximation for the magnetization vector will be used[4]:

$$M_{\perp} = \frac{-4}{3\pi}\, j_c\, R_f \left[1 - \left(\frac{j_s}{j_c} \right)^2 \right] \mathbf{e}_{\dot{B}_{\perp}} \tag{16}$$

The average magnetization can be applied in the evaluation of the hysteresis loss per cycle per unit length of wire:

$$Q_m = \int_0^T \iint_S -\mathbf{M}\cdot\dot{\mathbf{B}}\; dS\; dt \cong \int_0^T \iint_S |M_{\perp}|\; |\dot{B}_{\perp}|\; dS\; dt \tag{17}$$

In the case where the twist pitch becomes very small the component of the magnetization vector parallel to the filaments should be taken into account. Assuming the azimuthal and the longitudinal currents in the filaments do not interfere, the parallel component of the magnetization can be expressed as:

$$M_{\parallel} = -\frac{1}{3}\, j_{c,\vartheta}\, R_f \tag{16}$$

In order to calculate the magnetization loss the magnitude of the azimuthal critical current density has to be known. In literature, the ratio of the azimuthal and the longitudinal critical current density varies significantly. This component of the magnetization has not been taken into account in our calculations, which can be justified for most technical conductors. This component, however, has been mentioned for completeness and in order to be able to estimate its magnitude in relation to the perpendicular one.

Numerical results

Introduction

In order to solve the above-mentioned equations numerically a staggered grid has been used. The reason for choosing a numerical approach is the non-linearity of the problem. The non-linearity is caused by the field-dependence of the critical current density as well as by a possible saturation of a part of the conductor ($j = j_c$). In this section the influence of several wire parameters on the loss behaviour will be examined. As an example of an AC conductor a wire manufactured by Alsthom will be used. The wire characteristics are listed in Tab. 1.

Table 1. The characteristics of the wire used as an example of an AC conductor for numerical loss calculations.

Manufacturer	Alsthom
Wire radius	60 μm
Filament radius	0.29 μm
Inner radius filamentary zone	± 40 μm
Outer radius filamentary zone	± 60 μm
Twist pitch	0.8 mm
Number of filaments	14496
Superconductor	NbTi
matrix material	Cu + CuNi barriers
Cu : CuNi : NbTi	0.83 : 1.16 : 1

The calculations in the next sections are performed, if not mentioned, for an overall critical current density of $1\cdot10^9$ Am^{-2}, an ampltitude of the applied alternating magnetic field of 100 mT and an amplitude of the transport current of 5.65 A (0.5 I_c). The applied magnetic field and the transport current are in phase.

Location of the filamentary zone

For conductors having small filament diameters the major part of the transport current usually flows in the outer region of the conductor. The question arises whether it is energetically favourable to locate the filaments in a specific part of the conductor instead of distributing them homogeneously over the area. In order to examine this problem two cases have been considered. In the first case the loss contributions have been calculated for:

a: $R_i = \alpha R_w$; $R_o = R_w$, $0 < \alpha < 0.8$

b: $R_i = 0$; $R_o = \alpha R_w$, $0.2 < \alpha < 1.0$

The overall critical current density has been kept constant. The results are depicted in Fig. 1.

The magnetization loss which is not depicted in Fig. 1 hardly depends on the location of the filamentary zone. It only varies within a few percent. For larger amplitudes of the magnetic field the current distribution becomes more homogeneous. For this reason, and because the magnetization loss becomes dominant, the location of the filamentary zone under those conditions is not as important any more.

A second case is the one in which the area of the filamentary zone as well as the local average critical current density are kept constant and the inner and outer radius are varied. The results of the calculations for the second case are depicted in Fig. 2.

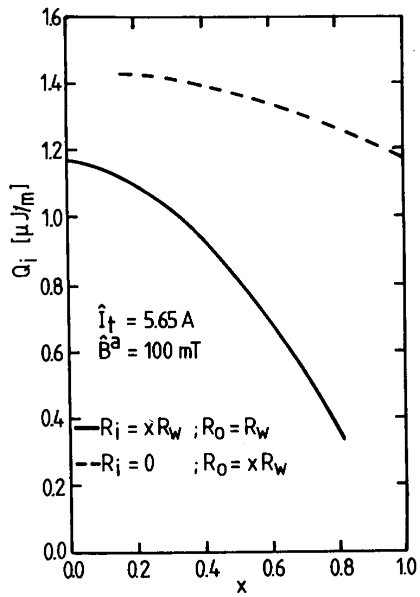

Figure 1. The transport current loss as a function of the inner radius (solid line) and the outer radius (- - -) of the filamentary zone while the outer and inner radii respectively are kept constant.

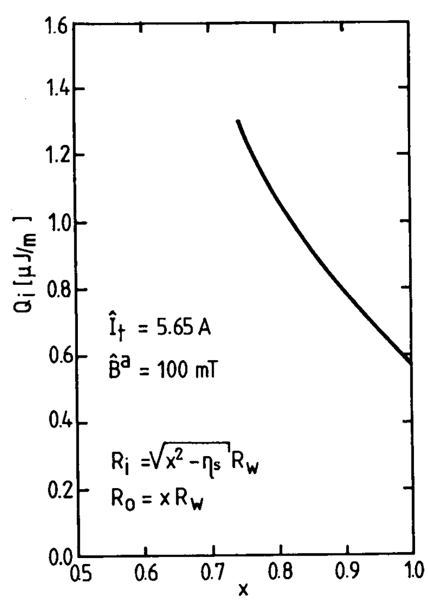

Figure 2. The loss behaviour as a function of the inner and outer radius of the filamentary zone (the area is kept constant).

From Fig. 1 and 2 we can conclude that, in general, if the effects of the matrix can be neglected, it is favourable to locate the filaments in the outer region of the wire.

Twist pitch

A small twist pitch of the filaments is applied in order to lower the coupling loss in the matrix material. Due to filament twist a magnetic field component directed along the wire axis is present under current-carrying conditions. In an attempt to counteract the flux change a part of the transport current will be effected at the inner part of the conductor. In this section the effect of a different current distribution, due to the filament twist, on the loss behaviour will be examined. Figure 3 shows the calculated loss behaviour as a function of the twist pitch for the model wire. The value of the twist pitch in the range of practical twist pitches has little effect on the transport current loss. A slight decrease of the transport current loss for small twist pitches can be observed. For even smaller twist pitches the loss increases again. It should be noticed however, that for a certain transport current the ratio j_s to j_c in the filaments increases with a decreasing twist pitch. For our model wire a significant decrease of the maximum transport current occurs for a twist pitch smaller than 0.4 mm. For this twist pitch the critical current is 80 % of the critical current of an untwisted conductor. As a consequence, the transport current loss will increase rapidly for a constant transport current if the twist pitch becomes smaller than 0.4 mm due to the increasing ratio I_t/I_c. The magnetization loss is also hardly affected by the twist pitch for larger twist pitches.

The main reason for twisting a conductor therefore remains to reduce the coupling loss.

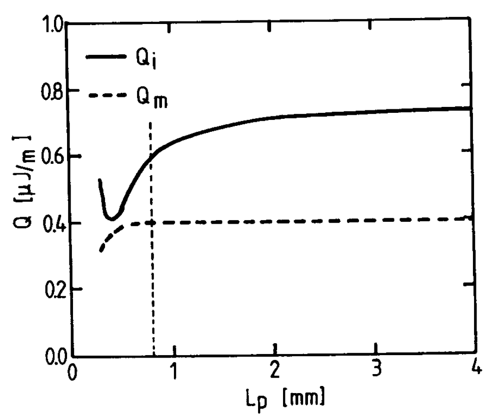

Figure 3. Calculated loss components as a function of the twist pitch. Magnetization loss (- - -), transport current loss (solid line).

Filament diameter

The initial reason for reducing the filament diameter of AC conductors was to decrease the hysteresis loss in the filaments. Filament diameters of the order of several nanometers (on laboratory scale) have been reported. Reducing the hysteresis loss however does not necessarily imply a decrease of the total loss. Other loss components may become dominant and thus the reason for further reduction of the filaments. Within the frame of our model it is possible to examine the influence of the filament diameter on the magnetization loss as well as on the transport current loss. Both loss components have been calculated for the model wire having different filament diameters. The results are shown in Fig. 4.

The magnetization loss decreases almost linearly with the filament diameter. The transport current loss however, becomes almost constant below a certain value of the filament diameter. In this region the current distribution resembles the one of a conductor under self-field conditions. In this case the loss is not determined by the dimensions of the filament but by the wire diameter. The self-field loss can be

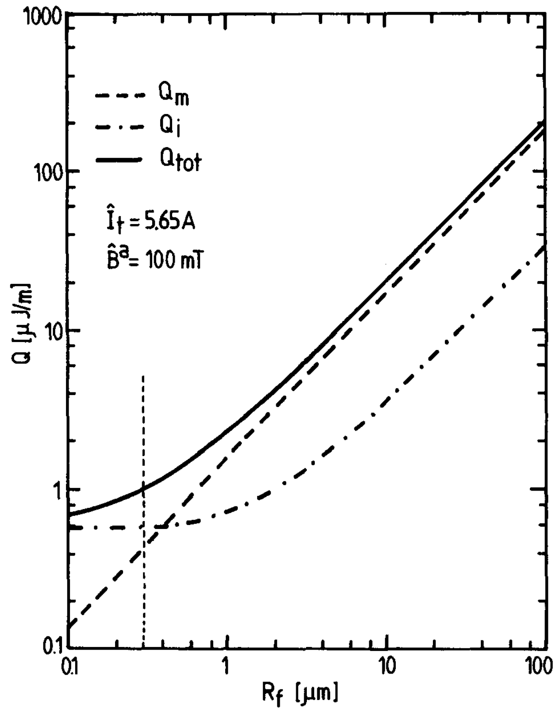

Figure 4. Calculated loss components as a function of the filament radius. Magnetization loss (- - -), transport current loss (-·-·-) and the total loss (solid line).

calculated by applying the following equation given by London[6]:

$$Q_{sf} = \frac{\mu_0\, I_c^2}{\pi} \left[\left(1-\frac{I_t}{I_c}\right) \ln\left(1-\frac{I_t}{I_c}\right) + \frac{I_t}{I_c} - \frac{1}{2}\left(\frac{I_t}{I_c}\right)^2 \right] \quad (17)$$

In our case this yields a self-field loss of 7.0 10^{-7} Jm^{-1}, which is approximately 20 % larger than the loss calculated numerically for small filaments (Fig. 4). The self-field loss however is slightly reduced due to the filament twist. A further reduction of the filament diameter is ineffective under those conditions. An optimization of the filament diameter in general is not possible. It depends on the conditions under which it operates.

A second point of attention concerning filament reduction, although not included in this model, is the occurence of the proximity effect. The spacing between the filaments is limited if coupling is not desired. The minimum distance allowed can be enlarged by applying highly resistive matrix material, like CuNi, or by adding ions which carry localized magnetic moments, like Mn, Fe or Cr[5].

Conclusions

A one-dimensional numerical model has been presented which enables us to calculate the current distribution as well as the transport current loss and the magnetization loss of current-carrying multi-filamentary AC conductors. In this paper the model has been used to investigate the influence of the location of the filamentary zone as well as the influence of the twist pitch and the filament diameter on the loss behaviour.

From these calculations we can conclude that if the transport current loss is a substantial part of the total loss, the filaments should be located at the outer region of the conductor.

The twist pitch has little effect on the loss when the ohmic loss in the matrix material does not contribute significantly to the total loss

Reduction of the filament diameter decreases the loss when the magnetization loss is dominant. If the the total loss is determined by the transport current loss lowering the loss by reducing the filament diameter can only be obtained as long as most of the current-carrying filaments are not saturated.

References

[1] A. Février and Y. Laumond, "Prospective uses of superconductors for 50/60 Hz applications", Proc. ICEC-11, 1986, pp. 139-152.

[2] I. Hlásnik, S. Takács, "Progress and problems in development of low loss very fine filament superconducting composites", Proc. MT-9, 1985, pp. 481-488.

[3] W.J. Carr, jr., "Electromagnetic theory for multifilamentary superconductors", Phys. Rev. B 11, 1975, pp. 1547-1554.

[4] P.C. Rem, Numerical models for AC superconductors, Thesis University of Twente, The Netherlands, 1986.

[5] T.S. Kreilick, E. Gregory and J. Wong, "Influence of filament spacing and matrix material on the attainment of high quality, uncoupled NbTi fine filaments", IEEE Trans. on Magn., MAG-24, 1988, pp. 1033-1036.

[6] H. London, "Alternating current losses in superconductors of the second kind", Phys. Lett. 6, 1963, pp. 162-165.