The Philosophical Quarterly Vol. 58, No. 230 *January 2008*
ISSN 0031–8094 doi: 10.1111/j.1467-9213.2007.533.x

IDENTITY OVER TIME: OBJECTIVELY, SUBJECTIVELY

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In the philosophy of science, identity over time emerges as a central concern both as an ontological category in the interpretation of physical theories, and as an epistemological problem concerning the conditions of possibility of knowledge. In Reichenbach and subsequent writers on the problem of indistinguishable quantum particles we see the return of a contrast between Leibniz and Aquinas on the subject of individuation. The possibility of rejecting the principle of the identity of indiscernibles has certain logical difficulties, leading us inexorably from ontology into epistemology. For the epistemological problem we attend to the differences that emerged between the (neo-)Kantian and logical empiricist traditions, also saliently displayed in Reichenbach's writings. After examining the contrast between Kant's and Leibniz's conceptions of empirical knowledge, specifically with respect to the irreducibility of spatiotemporal determinations, we explore an application of a neo-Kantian view to the same problem of indistinguishable quantum particles.

Arguments and views concerning identity and individuation resurfaced in the twentieth century in philosophical discussion of quantum mechanics connected with discernibility. Much dealt mainly with the *conditions of possibility of being*, that is, the conditions under which named or described entities are identical or distinct. Beginning with Kant, however, traditional questions of metaphysics were transposed into a transcendental enquiry, that is, enquiry into the *conditions of possibility of knowledge*. Accordingly, we shall address this topic in two parts.

I. CONDITIONS OF THE POSSIBILITY OF BEING

In general, the conditions under which something can *be* the case are not the same as those under which it can *be known to be* the case. It does not follow that something can be the case even if there are no possible conditions under which it can be known to be so. But the distinction suffices to allow us to start with the former. In doing so, we may engage in realist metaphysics – or, on the contrary, bring realism's limits to light.

Published by Blackwell Publishing, 9600 Garsington Road, Oxford OX4 2DQ, UK, and 350 Main Street, Malden, MA 02148, USA

I.1. *The problem of individuation*

Under what conditions are entities distinct? The traditional answer to this 'problem of individuation' seems to have been 'If and only if they are different in some way': *no distinction without a difference*. If this is denied, one faces a new question: how can entities be distinct, if they do not differ in any way?

Historically, there have been four types of views on this. Two of these, which we shall describe as the '*ubi*' solution and the principle of identity of indiscernibles (henceforth PII), entail that distinct entities are discernible. The other two, which we shall call the 'haecceity' solution and 'weak discernibility', allow for distinct entities which have no discernible difference: whatever is true of the one is true of the other.

Aristotle's *Metaphysics* took up individuation for composite substances, such as humans, which have a form as well as matter. It is the matter that individuates them:

> There is in the composite, for example, in Callias or Socrates, such and such a form in their particular flesh and bones; and though they differ in matter, for each has his own, father and son may share one form, for the form is indivisible.[1]

One may wonder why the accidental features are not mentioned. Socrates has a snub nose but Callias does not. Accidental features can be shared. If that suffices to deny that accidental features can individuate, then it is implicitly allowed that two substances could both have the same form and the same accidents. Callias' and Socrates' different flesh and bones are describable parts. Their matter – in the sense indicated, hence considered separately from the form and from accidental features – is not differently describable. It could perhaps be indicated ostensively: Socrates has *this* matter, Callias has *that*.[2] This is how Aquinas seems to read it in *De Ente et Essentia*, explicitly using the word 'this'. The *Summa* also explicitly uses the indexical:

> For it is manifest that the reason why any singular thing is 'this particular thing' is because it cannot be communicated to many: since that whereby Socrates is a man, can be communicated to many; whereas, what makes him this particular man, is only communicable to one. Therefore, if Socrates were a man by what makes him to be this particular man, as there cannot be many Socrates, so there could not in that way be many men.[3]

[1] Aristotle, *Metaphysics* Z 1034a 5–8, tr. R. Hope (Univ. of Michigan Press, 1960).

[2] See further N. White, 'Identity, Modal Individuation, and Matter in Aristotle', *Midwest Studies in Philosophy*, 11 (1986), pp. 475–94.

[3] *Summa Theologica*, I *q* 11, *a* 3 (Benziger Bros edn, 1947).

But besides the 'this matter' solution for composite substances, Aquinas introduces two other forms of individuation. Angels, that is, immaterial intelligences, have no matter and never did have matter. Accordingly, no two angels have the same form: '... such things as agree in species but differ in number, agree in form, but are distinguished materially. If, therefore, the angels be not composed of matter and form, as was said above, it follows that it is impossible for two angels to be of one species' (*Summa*, I *q* 50, *a* 4). Souls on the other hand may differ solely in their *history*, that is, in that they were the souls of distinct human beings before death:

> ... the individuation of the soul depends on the body, in an occasional manner, as to its inception, for the soul does not acquire for itself individual existence unless in the body of which it is the act. But nevertheless, if we subtract the body, the individuation does not perish because, since the soul was made the form of a given body, the form has absolute existence from which it has acquired individuated existence, and this existence always remains individuated (*De Ente et Essentia*, ch. V).

In the different treatment of these cases, two options are explored by later scholastics. One is to insist on a difference that can be described in terms of properties, whether essential or accidental. The other is to insist on a distinction in number made on a different basis.

The appeal to the soul's history is of the first kind, as is the differentiation of angels by their forms. Also of the first kind is the proposal that there is a uniquely distinguishing accident, the *ubi*, that is, the quality something has *by virtue of being where it is*. This is a kind of accident that can vary with time but can never be shared by distinct individuals, and so can individuate. The second option is to introduce something that might neither be an accident nor yet belong to the form. This may be read into the salient use of indexicals. Scotus' *haecceitas*, the 'thisness', in contrast with the *quiddity* which consists of all the determinations of the entity apart from the haecceity, exemplifies this second option. Robert Adams insists that haecceity is a property.[4] But if this solution is to differ significantly from the *ubi* solution, then haecceity cannot be an accident, that is (in our terms), it is not a property or quality or attribute.

Leibniz's teenage work *Disputatio Metaphysica de Principio Individui* (1663) shows that he was well acquainted with the scholastic discussions; in his mature work, even if not without precedent, he takes the first option in a new way.[5] In his view, the complete concept of an individual contains all facts about the world it is in, and how it is situated there. He claims that it

[4] See R. Adams, 'Primitive Thisness and Primitive Identity', *Journal of Philosophy*, 76 (1979), pp. 5–26.

[5] A solution attributed to Porphyry and Boethius presaged the PII: see C. Leijenhorst, *The Mechanisation of Aristotelianism* (Leiden: Brill, 2002), p. 167.

follows from this 'that it is not true that two substances may be exactly alike and differ only numerically, *solo numero*, and that what St Thomas says on this point regarding angels and intelligences ... is true of all substances ...' (*Discourse on Metaphysics* IX). In fact this inference is not logically compulsory, except on the assumption that an individual's full concept is unique to it – which would beg the question if meant as a justificatory argument. Leibniz's PII is a *logically contingent* principle: there is no (numerical) distinction without a (discernible) difference (cf. *The Monadology*, §9).

The PII encounters putative counter-examples. One is Max Black's example of a universe consisting of two spheres which have the same volume and composition. Another is absolute space; assuming that points are entities, they seem no different one from the other. Stachel suggests in retort to such puzzles that an entity may acquire 'a certain measure of individuality ... from its position within a structure'.[6] The points of space have nothing intrinsic either to characterize or to differentiate them, but they bear spatial relations to each other: each occupies a position in a relational structure. However, since there are symmetries of the space enough to take any point into any other one, whatever we can say about any point, even about its relational position, we can say about each of them. Therefore this response works only for structures already reduced *modulo* all such symmetries, and not for space as ordinarily conceived.

A Leibnizian complete concept includes relational attributes, specifying how the individual is related to other elements having various other properties or standing in still further relations to further elements, and so forth. To be discernible, individuals cannot be alike both in the qualities they have and in their place in the relational structure in their world.[7] So if these examples, Black's spheres and points in absolute space, are admitted at all, they do violate the PII.

So we turn to the question how it is possible for individuals to be distinct if there is no discernible difference between them. There are two options: an individuating factor that does not imply discernibility (haecceity), and a condition explored by Simon Saunders.[8] To the remark that spatial points are not discernibly different, Saunders, following Quine, retorts that there

[6] J. Stachel, 'Structural Realism and Contextual Individuality', in Y. Ben-Menahem (ed.), *Hilary Putnam* (Cambridge UP, 2005), pp. 203–19.

[7] Kant discussed this critically (B327/A270); but see A. Jauernig, *Leibniz Freed From Every Flaw: a Kantian Reads Leibnizian Metaphysics* (Princeton University: PhD thesis, 2004), chs 3, 4. There have also been arguments to the effect that relations cannot provide the ground for individuality since they presuppose relata, which require prior individuation. This is not logically compelling as it stands. (Thanks to an anonymous referee for this point.)

[8] S. Saunders, 'Physics and Leibniz's Principle', in K. Brading and E. Castellani (eds), *Symmetries in Physics: Philosophical Reflections* (Cambridge UP, 2003), pp. 289–308, and 'Are Quantum Particles Objects?', *Analysis*, 66 (2006), pp. 52–63.

are *irreflexive relations* between them. Any point x is one metre from some point y, but not from itself. He therefore calls the points 'weakly discernible'.

The condition of weak discernibility certainly entails distinctness. But first, such a predicate as 'is one metre from some other point but not from itself' also applies to all points, and so does not express a difference between them. Secondly, the deduction that there are at least two Xs if some X bears an irreflexive relation to some X does not require the PII. It assumes only its converse, that is, substitutivity of identity. So although we shall stay with Saunders' terminology, we find it thoroughly misleading, for this use of the word 'discernible' is misplaced. Still, we have here a fascinating answer to the question how could there be more than one point, although they are not different in any way, even in the terms described in relations to other points. What Saunders, following Quine, effectively pointed out is a condition which entails that there is more than one entity of the given kind, without implying that there are specifiable differences that distinguish them.

The problem of individuation can therefore apparently be met in various ways, all of them displaying conditions which imply numerical multiplicity – some requiring discernible differences and some not.

I.2. *Identity and individuation in twentieth-century physics*

Particles characterized by the same constant features, such as mass or charge, are generally called 'identical particles'. They can still differ because each particle is capable of a range of states. If two particles are of the same kind, and have the same state, nothing in the quantum-mechanical description distinguishes them. *Yet this is possible.* We seem to have a dilemma: either this possibility violates the PII, or the quantum-mechanical description of nature is incomplete. We shall consider Margenau's and Reichenbach's arguments, which show both options.

A special complication: the elementary particles fall into two large classes, *fermions* and *bosons*. The former obey Pauli's exclusion principle, which is usually stated informally as *No two can be in the same state.* At first blush, then, the above dilemma would not apply to them. Pauli introduced the exclusion principle originally for electrons in a description of atomic structure. This was very fruitful: the shell structure of atoms could be deduced, and spectroscopic as well as magnetic properties understood. For example, each electron in an atom has its state completely characterized by its four quantum numbers. The qualitative statement of the principle amounts to the assertion that two such electrons cannot have identical sets of quantum numbers. So understood, it seems that the principle follows at once from the PII, given the *completeness assumption* that an electron is completely characterized by its constants plus the four quantum numbers.

But it was precisely the fermions that were first cited as in violation of the PII. Henry Margenau pointed out that the informal statement of the principle does not fit well with its later mathematical formulation.[9] An ensemble of electrons is a composite system, which has a quantum state taken as a whole. The general principle of which Pauli's is a special case says that an ensemble of particles of the same kind must have a state which is 'symmetrized', that is, invariant under permutation. In this respect, the fermions are like the bosons: ensembles of particles of the same kind can only have a symmetrized state.

Margenau asks now whether there are any observables, pertaining to each particle individually, which have different values. The answer in both cases, for fermions as for bosons, is 'None'.[10] In fact, if we ask for the probability of the outcome of measuring an observable which pertains to a single particle, the answer we receive does not depend on that particle at all.[11] Margenau then (p. 202) adds as a consequence that the exclusion principle 'so far as it goes, contradicts Leibniz.... The two particles, as we have seen, differ in no observable respect. Nevertheless quantum mechanics would lead to entirely erroneous results if they were treated as *a single entity*. The particles, though they cannot be labelled individually, can be counted.' To be precise, the 'size' of an ensemble of particles (i.e., the number of its elements) is a quantum-mechanically representable quantity (the number observable). So we can say that there are *n* electrons, for example, but that they are all in the same state. This clearly asserts a distinction without a difference. Margenau continues (*ibid.*) 'But if number is an observable property, have we any right to say that two electrons in an atom do not differ in observable respects? Yes indeed, for the number 2 is not the property of each electron, but of the composite system.' This is a prime example of holism in the quantum world.

Simon Saunders suggests that the description of the ensemble should be read as

[9] H. Margenau, 'The Exclusion Principle and its Philosophical Importance', *Philosophy of Science*, 11 (1944), pp. 187–208. For a recent discussion see M. Massimi, 'Exclusion Principle and the Identity of Indiscernibles: a Response to Margenau's Argument', *British Journal for the Philosophy of Science*, 52 (2001), pp. 303–30, and fn. 11 below.

[10] As example, take a pure state of a pair of identical fermions, an equal superposition of two different states, the anti-symmetric tensor product state $(1/\sqrt{2})[(\phi\psi - \psi\phi)]$, where ϕ, ψ are orthogonal spin states. If we ask whether a state can be ascribed to any one of the fermions involved, the answer is 'Yes', by 'reduction of the density matrix'. The reduction assigns to each particle the mixed state $\frac{1}{2}P[\phi] + \frac{1}{2}P[\psi]$, where P[ ] is the projection on the indicated one-dimensional subspace. See further §II.3 below.

[11] The rationale for the calculation is this: the prediction probabilities based on this assignment, for observables pertaining to a single particle, will be the same as if calculated on the basis of the total state of the ensemble – as obviously it should be. We do not accept Massimi's critique of Margenau; but her conclusion about violation of the PII is the same.

1. There exist distinct particles x_1, ..., x_n whose ensemble is in a superposition of the states resulting from permutations of the state in which particle x_1 is in state ϕ_1, ..., particle x_n is in ϕ_n.

In the case of a fermion ensemble, unlike the case of bosons, these individual states ϕ_i must all be distinct. The first existential quantifier can be deleted, and then this can be read with 'particle x_1' as subject term as follows:

> Particle x_1 is a member of an ensemble in which there are n–1 other particles such that ... [here insert the rest of (1)].

The predicate applied to this subject term applies equally well if instead we delete the jth existential quantifier, and let x_j be the subject; so no differentiating feature appears. But as Saunders notes, the relation involved is irreflexive – you could not delete both the first and the jth existential quantifier, write y for both x_1 and x_j, and get something that could be true. Why not? Because y would appear in components in which it bears two states at once.

This is precisely similar to the above diagnosis of the multiplicity of points in absolute space, and applies also to Max Black's spheres. But once again it does not vindicate the PII. Rather, it is an explanation of how we can assert indiscernible entities to be distinct – indicating a condition that implies numerical multiplicity.

When Hans Reichenbach turns to the issue of identity over time he introduces the relation of *genidentity*.[12] Genidentity includes the *material* genidentity of, e.g., a water droplet, and the *functional* genidentity of the wave of which this droplet is a part. For a macroscopic physical object, *material genidentity* is characterized in terms of continuity of change, spatial exclusion and the verifiability of spatial position interchange. In the case of waves, there is continuity 'usually', but the other two criteria are not satisfied: waves have only *functional genidentity*. He does not offer an explicit definition for either.

The atomic hypothesis in classical form would ensure that cases of functional genidentity always supervene on a pattern in the materially genidentical atomic constituents. But in recent physics atoms too have internal structure, and the elementary particles of which they are composed appear to have none of the three characteristics criterial for material genidentity. The problem of individuation reappears.

Fermions and bosons display distinctive statistical behaviour. Bosons obey Bose–Einstein statistics, in contrast with the 'normal' or classical Maxwell–Boltzmann statistics applied to classically conceived atoms in gas theory.

[12] H. Reichenbach, *The Direction of Time* (California UP, 1956), p. 38.

Fermions, to which Pauli's exclusion principle applies, obey a third statistics, the Fermi–Dirac statistics. Tossing two dice offers a fanciful illustration. What is the probability that the results will add up to 11? The answers from the three, taken in the same order, are 1/21, 1/18, 1/15. Intuitively speaking, fermions favour combinations of distinct numbers, and bosons favour 'doubles'. The question of how to account for this difference in statistical behaviour, and the suggestion that it has to do with identity and individuation, has been recurrent since the beginning.

Reichenbach suggests that the clue lies in the criterion of verifiability of spatial position interchange. This is satisfied for classical particles but not for bosons. Hence he concludes that at first blush, particles satisfying Bose–Einstein statistics do not have material genidentity. But then he notes that logically speaking, distinct possible arrangements could just have unequal probability. This would mean that the particles are materially genidentical, but subject to a causal anomaly – statistical correlation without causal explanation. He regards this as a choice for us: there is no empirical question involved – 'Neither interpretation is "more true" than the other; the two are equivalent descriptions' (p. 235). For Fermi–Dirac statistics, he first suggests that there is no similar problem, since all admissible arrangements are equiprobable. But again if we take the particles to be materially genidentical we have a statistical correlation without causal explanation. Reichenbach did not live to finish this book, and we can only speculate on what precisely he had in mind. Can we complete Reichenbach's discussion so as to find, contrary to Margenau's claims, an interpretation of quantum theory in which Leibniz's principle is not violated?

The answer is 'Yes', but only through an addition of 'empirically superfluous' hidden variables.[13] Suppose first that we regard fermions such as electrons as not enduring in time: the 'histories' of such particles are *really* sequences of events *not connected* by a genidentity relation. Yet when, at a given time in a given region or situation, there are *n* such events, then we *say* that *n* such particles are present. The weak discernibility pointed out by Saunders for a state of the ensemble *at any given time* suffices now to allow for a consistent addition of hidden characteristics to make these events mutually discernible.

For bosons, this will not work, since the state of a boson ensemble at a particular time does not satisfy the weak discernibility condition. Yet there too scientists speak intuitively of the presence of *n* particles. Reichenbach's discussion of genidentity provides the option of discernibility *over time*. Could not the required hidden parameter then be the relation of genidentity to

[13] Cf. B. van Fraassen, *Quantum Mechanics: an Empiricist View* (Oxford UP, 1991), ch. 11, §§10–12.

previous states? That would precisely echo Aquinas' solution for the individuation of souls.

So, logically speaking, the issue is once again unsettled. One option implies that *pace* Margenau the fermions are particles only in a 'conventional' or Pickwickian sense (there is no genidentity), but they do not violate Leibniz's principle. *Pace* Reichenbach, however, the PII can be saved for bosons also by postulating that they are really genidentical, though the criteria for neither material nor functional genidentity are satisfied. But it is crucial to such choices that empirically superfluous parameters are introduced, whether for the sake simply of interpretation, or as actual high metaphysics.

I.3. *Logical arguments for the PII*

If the PII were a logical truth, then the possibility of distinct but indiscernible entities, which we have taken so seriously, would make no sense at all.[14] But there are arguments to that effect.[15]

Argument 1. A set *s* with two indiscernible but distinct elements has four subsets, including two that have just one member each. But the elements are indiscernible, *so cannot belong to different sets*. Hence the only subsets of *s* are *s* itself and the empty set ∅. This is a contradiction: it is a theorem of set theory that sets have more subsets than members (Cantor). Should we change set theory, or admit that PII is a logical truth?

This argument can be countered by saying that unit sets of indiscernible but distinct elements are themselves distinct but indiscernible.

Argument 2. A set *s* has two members if and only if there exists a function *f* which maps *s* one–one onto the set {0,1}. But then there is one member in *s* which has the property of being $f^{-1}(0)$, and another member which does not. So the two members of *s* are discernible after all.

One change in set theory which has been suggested is to allow sets whose elements are indiscernible to have a cardinality but no ordinal. But given the definition of equicardinality by one–one mapping, it follows that if there is *any* well ordered set of cardinality *n*, then all sets of that cardinality admit a well ordering, and hence a differentiation of the above sort. So this argument is not so easily countered.

Argument 3. If two entities are indiscernible, then any predicate that applies to the one will also apply to the other. But to say that they are distinct, we have to use the identity symbol '=' (since 'distinct' means 'not identical'). Therefore we can make up a predicate using '=' which will apply to the one

[14] Thanks to Charles Daniels, who suggested the arguments for the PII in this section.

[15] As Jauernig points out, Leibniz himself did not take the PII to be a logical truth, but allowed violation by abstract entities.

and not to the other. If there are distinct indiscernible entities, it must be true for any and every definable predicate F that $(\exists x)(\exists y)(x \neq y \ \& \ [Fx \equiv Fy])$. Taking the predicate '$x$ = ...' for F, we obtain $(\exists x)(\exists y)(x \neq y \ \& \ [x{=}x \equiv x{=}y])$, which is a self-contradiction.

Argument 4. In second-order logic we can quantify over predicates, and so define identity: '$x{=}y$' is short for '$(\forall F)(Fx \equiv Fy)$', thus establishing the PII at once. To stop that move we can object that if the predicate variable ranges over a restricted set of properties only, then the defined relation is an equivalence relation that could assimilate distinct objects. Even without this objection, we have a right to reject the definition. However, we still have a problem: given that we still need a way to say that there are distinct entities, we then require '=' as a primitive symbol. From here on, the argument continues like the preceding.

Can we offer a diagnosis of these arguments which will save standard logic and set theory while rejecting the PII? There is just one option, it seems – to deny that every verbally definable predicate stands for a discernible difference. Kant's reaction to 'real' and 'exists' in the ontological proof of the existence of God was to argue that there is no difference between a hundred thalers and a hundred real thalers. No use saying that there is a difference because you can see only the real ones – as if there were unreal ones that are invisible!

Without a rationale, this reaction would render the PII effectively vacuous. Any rationale would have to specify plausibly what is and what is not to be taken seriously as a discernible difference. But where the problem arises in practice, as in quantum mechanics, we can clearly see what is and is not taken seriously in this regard. The electrons in the same atomic orbit are indiscernible because there is no observable (i.e., no measurable quantity) which will differentiate them individually. Any difference postulated between them would have to involve *empirically superfluous* parameters, and there is no compelling reason to believe in these. The classification as empirically superfluous, however, is in *epistemic* terms: they make no difference to probabilities in empirical predictions.

We do need a rationale for a distinction between genuine discernible differences and merely verbal differences. The only clue we have at this point, but a very noticeable one, is the way in which physics is distinguished from pure mathematics, namely, through its bearing on experience and experimentation. Hence this dialectic concerning the conditions of the possibility of being leads us inexorably into *the conditions of the possibility of knowledge*.

II. CONDITIONS OF THE POSSIBILITY OF KNOWLEDGE

Kant wrote that we precipitate ourselves 'into darkness and contradictions' by seemingly unobjectionable steps that 'transcend the limits of experience, [and so] are no longer subject to any empirical test'.[16] The form of metaphysics he thus criticized can be avoided by turning instead to a transcendental enquiry, an enquiry into the conditions of the possibility of knowledge in a given subject area, with reference to both experience and understanding.

II.1. *The transcendental turn and logical empiricism*

For the sciences, this enquiry focuses on conditions for measurement and for obtaining given kinds of information from measurement, especially in order to bring hidden theoretical presuppositions to light. The neo-Kantian Marburg school concentrated on this, but in certain respects their enterprise was continued by the Vienna and Berlin Circles, despite the controversies between these two philosophical camps. When Reichenbach departed from the neo-Kantian tradition to forge his distinctive version of logical empiricism, he retained the conviction that nothing is meaningful unless it can be clearly co-ordinated with conditions of knowledge rooted in experience.

We have pointed out an example of this in his discussion (p. 225) of material genidentity, a relation which, though not defined by them, carries the hallmarks of the possibility of such co-ordination. When denying that elementary particles have material genidentity, he concentrates in his argument on what can be manifest in an experiment. Thus the conditions of possibility for knowledge, the characteristic focus of the Kantian tradition, remain crucial to *meaning* in Reichenbach's thought, even when he greatly modifies and weakens the infamous verification criterion for meaningfulness in his *Experience and Prediction*. It is in this spirit, without subscribing to a fixed or inflexible dogma concerning meaningfulness, that we shall now re-examine the issues pertaining to identity, individuation, and indiscernibility.

II.2. *Conception of knowledge: Leibniz to Kant*

Gilles Deleuze presents the PII as the key to a profound and indeed irreconcilable opposition between Leibniz and Kant.[17] What distinguishes

[16] Kant, *Critique of Pure Reason*, tr. N. Kemp Smith (Basingstoke: Palgrave Macmillan, 2003), Preface to the 1st edn.

[17] G. Deleuze, *Courses Given at Vincennes* 1980, http://www.webdeleuze.com/php/sommaire.html.

Kant from Leibniz here, he says, is a difference in how they conceive of what it is to know. For Leibniz, knowledge is gained by analysis: to know is to discover what is included in a concept. But for Kant, all analysis presupposes a synthesis. To affirm, as Kant does, the irreducibility of the space-time determination to the conceptual determination is, according to Deleuze, to introduce a new conception of space and time, no longer as an order of existence and of succession, reducible to what co-exists or follows one after another, but as *a priori* forms of exteriority and interiority.

Leibniz's principle implies that if a distinction is known, it must be known through a difference, and hence through a difference detectable through analysis of concepts. But for Kant, there is besides conceptual determination also spatiotemporal determination, which is a form of intuition, irreducible to the conceptual order. What we must seek now in the history of this problem of articulation between identity and indiscernibility are, first of all, the differences on the level of the conception of knowledge, and secondly, what it means to regard spatiotemporal determination as irreducible to conceptual determination.

Kant's first *Critique* begins with the famous 'There can be no doubt that all our knowledge begins with experience'. This rules out neither knowledge *a priori* nor empirical knowledge, which has its sources '*a posteriori*, that is, in experience'. But he introduces a further divergence: temporal determination is irreducible to conceptual determination: *the space of physics is not orientated, but perceptual space is*. Thus enters the indexical, as we can already clearly see in his early pre-Critical essay 'Von dem ersten Grunde des Unterschiedes der Gegenden ...':

(a) *the map*: 'a map of the heavens, if the direction were not specified relative to my hands, would not enable me ... to infer [where] to expect the sun to rise'
(b) *the compass*: 'No matter how well I may know the order of the compass points, I can only determine directions by reference to them if I know whether this order runs from right to left, or from left to right'
(c) *the two hands* (as later elaborated in *Prolegomena*, §13): 'the difference between similar and equal things, which are yet not congruent, cannot be made intelligible by any concept, but only by the relation to the right and the left hands which immediately refers to intuition'.

This last example is related to the other two through the surprising diagnosis Kant provides of it. The case of these hands contrasts with that of two arrows of the same shape and size. Even if the arrows are currently pointing in opposite directions, they can be considered geometrically alike in all respects, and either can be made to occupy the exact place of the other by means of a rigid motion (translation and rotation). The right hand and left hand are not superimposable in this way, although they are, as objects in Euclidean space, geometrically alike as well. They are what Kant called 'incongruent counterparts'. Contrary to how Leibniz conceived it, the noted difference between the pair of hands and the pair of differently orientated arrows need not, merely because it is knowable, be founded in 'intrinsic', that is conceptual, differences.

The symmetries which deprive the arrows of any individuality, of any specificity with respect to each other, are lacking in the case of the two hands. There are two sorts of Euclidean symmetries: 'rigid motions' and 'reflections'. In current geometric terms, mirror reflections are indeed symmetries of Euclidean 3-space, but they are not symmetries of the space in which we 'see' the objects of experience. For us, though they are located in the same Euclidean space, the absence of a rigid motion that turns one hand into the other signals a differentiation of a different kind. A description of a hand-shaped region in Euclidean space, given entirely in geometric terms, does not distinguish between left and right hands, precisely because that space is not an orientated space. Related by any symmetry, two spatial regions are geometrically, and hence conceptually, the same. But for us, for observers, there is a difference, which does not stem, as Kant claimed but Leibniz denied, from conceptual differences.

If the space in which they are represented had an extra dimension, a right hand and a left hand could be superposed, just as two two-dimensional arrows represented in a three-dimensional space can be. But in a four-dimensional space there would also be examples of regions which are related by reflections and not by any rigid motion. So, even in general, the 'for us' is crucial here. Just as in the case of the map of the heavens, the information captured in geometric description must be supplemented by the act of relating the description to our own situation, that is, it must be instantiated perspectivally. The space-time determinations which complete the conceptual determinations of the hand or the chart, so as to make them objects of knowledge for us, are indexical.

So there is a difference between the non-orientated object of knowledge represented in a chart, geometric model or physical theory, and the orientated object of knowledge by which we can steer ourselves. How does it show up in the domain of scientific knowledge? To what could we today either oppose or assimilate the claim that space-time determinations are irreducible to conceptual determinations? Space-time determinations which can be called upon today by a neo-Kantian conception of knowledge are not limited, as they were for Kant, to 'natural' or individual forms of sensibility. What are they, then? How does the way in which we envisage the epistemic function of space-time determinations in knowledge affect the articulation between identity and indiscernibility?

Kant's way of accounting for incongruent counterparts changed from an appeal to absolute space to a transcendentalist perspective. Laying out the different options for accounting for incongruence, Oliver Pooley regards Kant's earlier position as externalist and substantivalist, where the incongruence is explained not in terms of an intrinsic difference but by appealing to something external, for instance an absolute, independent space.[18] He says that the later (transcendental idealist) Kant rejected this substantivalist interpretation, and claimed the incongruence to be not conceptually comprehensible but graspable only in perception. Pooley suggests (p. 260, fn. 19) that Kant must then have opted for explaining the incongruence in terms of intrinsic differences between left and right which 'cannot be made intelligible through concepts'. Yet Pooley recalls too that Kant called them counterparts because he thought they were 'perfectly identical in terms of the distances between their corresponding parts'. So what kind of *intrinsic* differences would be compatible with their being 'perfectly identical'? Pooley does not tell us, and his suggestion could be read as harking back to the metaphysics which Kant criticized, where any difference at all is reified. However one understands the evolution in Kant's position with respect to incongruence, it was always, in a critical perspective, using the case of incongruent counterparts as an argument against the necessary conceptual foundation of distinctness. What could be meant by intrinsic differences which are not conceptual, not a matter of instantiating different concepts? If one insists on seeing all accounts of incongruence as divided between substantivalism and relationalism, there is simply no room for the transcendentalist approach.

Moreover, to reject, as Kant did, the idea that any distinctness is necessarily founded in conceptual differences is not to say that people can have no conceptual understanding whatsoever of a distinction not founded in conceptual differences. We have a conceptual understanding of the non-conceptual foundation of the distinctness of incongruent counterparts, namely, that the distinction between left and right can only be made by ostension, that is, by an indexical judgement.

Pooley himself (p. 251) advocates the relationalist option according to which 'the difference between incongruent counterparts is grounded in their relations to each other and to other material objects'. Contrary to what Pooley sometimes suggests, transcendentalism does share with relationalism the idea that incongruence of counterparts is *not* to be accounted for by

[18] O. Pooley, 'Handedness, Parity Violation, and the Reality of Space', in Brading and Castellani (eds), *Symmetries in Physics*, pp. 250–80, at pp. 252–62.

intrinsic differences. But in other respects it is as far from relationalism as from substantivalism. Both the substantivalist and the relationalist look for a foundation of the distinction in things which have been distinguished which is indifferent to *how* they have been distinguished; this is where these theories differ from the transcendentalist approach.

The transcendentalist starts from the mere fact that in certain conditions people find themselves making a distinction, and asks for the conditions of the possibility of doing so. How do we come to make this distinction, under what sorts of circumstances, and what is involved in the making of this distinction? Can we imagine other circumstances in which this distinction could not have been made, and if so, are they conditions we could imagine ourselves living in? Is it contingent that we are not in these circumstances, or is the idea of being in such conditions incompatible with how we think of ourselves ?

The distinction between the left and right hand is understood, both by the relationalist and the transcendentalist, in terms of relations. But the transcendentalist does not begin with relations between things that can be conceptually formulated. For the transcendentalist, it is a matter of relations involved in the act of distinguishing, a matter of attending to how making a certain distinction depends on the conditions under which this distinction can actually be made.

The distinction which we make with the hands but do not make with the arrows has to be related to the fact that the hands are three-dimensional objects embedded in a three-dimensional orientated space. Geometrically, the incongruence of the hands is contingent on the dimensionality of the space in which they are embedded. But the dimension and orientation of perceptual space are a necessary condition of our perception of anything. We cannot conceive of a perceptual distinction independently of the fulfilment of these conditions; it is therefore an *a priori* condition of perception. This is overlooked if the distinctness is explained in terms of conceptual differences, or in terms of some relation between items distinguished, while no mention is made of the cognitive relation through which this distinction can be made. To postulate some conceptualizable 'intrinsic' difference graspable only in direct perception places the ground of the distinction between regions of space once again in the objects, and the restriction to perceptual access only serves to make it mysterious.

The need for such a postulate will be felt only by those who adhere, tacitly or explicitly, to some doctrine, along the lines of Leibniz's conception of knowledge, to the effect that any act of distinguishing requires some pre-existent difference conceptualizable without recourse to ostension or to indexical judgement. This is all the more regrettable because such recourse might be a key to how anyone can conceive of identity and individuation in the context of quantum mechanics.

II.3. *The world of physics*

As did the Marburg school of neo-Kantians, we aim to generalize on these reflections so as to arrive at an understanding of the physical sciences. Specifically, we aim to generalize from perception to measurement. Just as the non-orientated space of Newtonian physics must in practice be co-ordinated with the orientated perceptual space, so the general 'view from nowhere' in any scientific model needs in practice to be particularized to a situation relative to a measurement set-up. The significance and meaningfulness of (combinations of) factual judgements requires as a condition that they can in principle enter as conclusions in the context of a measurement of 'what they are about'. In quantum physics, the natural form of prediction is assignment of probabilities of outcomes *conditional on* measurement of an appropriate kind. (In some interpretations of QM these conditional probabilities are derived from absolute probabilities; but we take an orthodox line here.) That it is incumbent on observers to apply concepts only in ways that make sense experimentally and experientially does not, however, require them never to apply those concepts to situations in which there is no measurement. It requires only that conditions of possibility can be specified for these concepts, that is, that it can be shown how their application *could be* part of a judgement properly based in a well defined measurement context.

There is an analogy between incongruent counterparts and entangled states of identical particles. The state vector associated with the result of a preparation may have to be taken from the tensor product of two Hilbert spaces, so that the 'global system' is visualized as a thoroughly entangled pair of subsystems. This entanglement expresses a correlation between the possible outcomes of measurements of pairs of local observables. But the characterizations of these two subsystems within the entangled state are no different from each other, any more than were the concepts of the incongruent counterparts. So it becomes necessary to make a numerical distinction, with respect to the correlation, just as with respect to the incongruence; but it is impossible to state a difference in concepts pertaining to the distinguished entities or parts.

How can anyone make sense of correlation, which implies a numerical distinction, without being able to state an intrinsic difference, that is, a difference in the concepts of what are distinguished?

One option is to say that there are intrinsic differences, but they do not show up in the concepts of quantum theory, nor appear in the quantum state. Why not? Tellingly, a typical answer is that these differences make no empirical difference whatsoever: there is no means of grasping them empirically, and for that reason no way to describe them within the resources of the quantum theory. But these empirically superfluous 'hidden' variables are introduced to 'explain' an empirically attestable difference in the results of measurements which *can be made*. The explanation and its rationale do seem to imply that there is one kind of empirical evidence serving for numerical distinction and another kind for conceptual discernibility. There are here all the reasons needed to attend to the empirical conditions in which the tension between distinction and discernibility arises.

Another option is one we mentioned above, Saunders' contention that there is here not a case of distinction without discernibility, but rather a weak discernibility, to be stated in terms of a relation. This relation, symmetric but irreflexive, can be formulated in logical terms without any regard for empirical conditions. Can it, really?

Under the heading of weak discernibility come both the case of two identical spheres located at a certain distance from each other and the case of two identical particles in a superposition of pairs of opposite spin components. This assimilation overlooks an important difference, as has been carefully highlighted by Michel Bitbol (in an unpublished MS, 2005). The conditions of measurement under which (a) the two spheres are individually and differently located, or (b) found to be at a distance *d* from each other, are compatible. But in the case of a system of two electrons or photons, measurements which localize them individually are not compatible with a measurement which reveals a global property such as being in an entangled state. The required kind of compatibility is no longer available. In the context where an entangled state is attributed, there is no basis for the distinction other than the [anti-]correlation itself. Labels such as 'x_1' and 'x_2' used in describing the state seem misleadingly to stand for an ascription of individuality, but can actually do no more than express the blind, merely numerical, distinction.

Suppose we have a situation described in quantum-mechanical terms as *preparation* of systems in the symmetric tensor product state $(1/\surd 2)[(\phi\psi + \psi\phi)]$, where ϕ, ψ are orthogonal spin states – informally, spins 'up' and 'down' in a certain direction. If we calculate the predictions for the *counting observables* corresponding to *spin-up*, *spin-down*, and *spin-up or spin-down* (the latter being the observable with constant value 1, the 'identity' I) then we get the answers 1, 1, 2 with certainty, for the state is an eigenstate of each of these three observables. The measurements can be made without disturbance: the state is left intact. There are other such 'global' observables, of which the total system state is an eigenstate, for example, the sum of the two spins, which is a way to check whether this is a case of combining opposite spins.

Together, these judgements, which can thus be combined meaningfully with respect to measurement conditions, certainly yield a *judgement of distinctness*. In whatever way we want to phrase the facts about what we can predict with certainty here, we shall have to use the number 2.

The state described above can be ascribed to a boson pair, and is informally described as a system consisting of two bosons of the same type with opposite spins. But just as with Pauli's exclusion principle, as Margenau pointed out, such an informal description calls up a 'classical' picture: it would be just right only if the situation were *not* one of entanglement. The very conclusion that this is a system consisting of two distinct individuals is not one which can be satisfactorily linked to any similar non-disturbing measurement procedure which could bring it to light. Measurements which can be designed to support such a conclusion are necessarily ones which do perturb the state. Even the talk of 'disturbing' and 'perturbing' is unhappily laden with 'classical' imagery, and may be misleading. What we can say quite definitely is that such measurements cannot be regarded as doing anything to *reveal* (except relative to contentious interpretative principles added to the basic theory) what the situation was like before the measurement was made.

Suppose we try to design a measurement that would pertain to one particle rather than the other. There is a standard mathematical design for this. We can designate the system as $x+y$, with distinct labels x and y. To measure observable a that pertains to x is the same as measuring $a \otimes \mathrm{I}$ on this composite system, where I is the identity. (In quantum mechanics, states can be attributed to the components x, y individually, but the assignment is defined by the constraint *assign to x the state such that the expectation value of any pertinent observable a in that state is the same as the expectation value of a ⊗ I in the state of the whole.*) Even if a pertains also to the other, the expectation value of $a \otimes \mathrm{I}$ will in general not be the same as that of $\mathrm{I} \otimes a$. This is how the assertion that the system is composed of two different individuals is satisfactorily linked to feasible measurement possibilities.

If we now try to do the same with the permutation invariant state of a pair of particles, we *cannot* get a difference in measurement predictions or expectation values for $\mathrm{I} \otimes a$ and $a \otimes \mathrm{I}$. We can illustrate this starting with Aspect's famous experiment to detect violations of the Bell inequalities. Intuitively, a pair of entangled photons are travelling in opposite directions away from a source. If the pair is in the boson state $(1/\sqrt{2})[(\phi\phi + \psi\psi)]$, their states are perfectly correlated. Results of polarization measurements with the two polarizers aligned in parallel are 100% correlated. That is, each photon may be found randomly either in channel + or – of the corresponding polarizer, but in this case, when one photon is found positively polarized, then so is its twin companion.[19] (Light polarization and photon spin are not the same, though they are intimately related. The photon's spin axis is parallel to the direction of travel, whereas the polarization axis of a wave of light is perpendicular to the direction of travel. But the effect of polarization on the behaviour of photons is formally similar to the effect of spin on the behaviour of massive particles, and the difference does not affect our arguments here.)

We can write the state in this more illustrative form: it is a superposition of |vert>⊗ |vert> and |horiz>⊗ |horiz>. If we do assign a state to the individual subsystems by the previously mentioned calculation ('reduction of the density matrix'), what we get is that each of the two component photons is in a 50/50 mixture of |horiz> and |vert>. That is why on each side we see the filter being passed 50% of the time. The pure state of the pair taken as a whole, however, is what accounts for the perfect correlation observed.

Especially important here is that the local measurements cannot be taken as revealing polarization *before* the measurements are made. Indeed, if we assume that the photons were already definitely polarized beforehand, the Bell inequalities can be deduced, in contradiction with the results obtained by Aspect. Therefore the measurement outcomes do not reveal that there was already a discernible difference between parts of the total system.

There certainly are ways to 'disentangle' such an entanglement, *but they are violent*. To begin with, on the basis that such an entangled state was prepared, we have a prediction of perfect correlation in the outcomes of two separated local measurements, and hence a conditional certainty for prediction. Finding an outcome on the left, we can predict with certainty the outcome that will be found, afterwards, on the right *if the same angle is chosen* for the polarizer there. Once the measurements are made, however, the situation is very different: the total system is no longer in an entangled state. If we contemplate subsequent measurements on the two photons, the result on one side will no longer be, independently of any knowledge of what was done or has happened on the other side, symptomatic of what will happen there whenever the same orientations are chosen.

Therefore judgements of distinctness do not bring with them any implication of discernibility. For judgements in which we discern two localized particles or events require a measurement set-up incompatible with maintaining a certain kind of state, entanglement, for which judgements of distinctness are perfectly possible and meaningful.

We conclude that judgements of *numerical distinctness* are perfectly possible and meaningful in certain states which *cannot be maintained* in the

[19] See A. Aspect, 'Bell's Inequality Test: More Ideal than Ever', *Nature*, 398 (1999), pp. 189–90, for a schematic account of his own and later experiments.

measurement set-up required for judgements in which two localized particles are discerned. *Judgements of distinctness do not bring discernibility along with them.*

In the case of incongruent counterparts, we mentioned Kant as calling for spatial determination in terms of the *a priori* conditions of perception. Transcendental attention to the conditions of possibility of any distinction must rethink its target: we need now to take into account the pragmatic reading of scientific knowledge which has become current in epistemology since Kant's time. We need to enlarge the idea of 'sensibility' to embrace conditions of experimental measurement.

To see an experimental set-up such as the above as discerning the two particles by ascription of different individual characteristics, related to the spin component, is to be oblivious to the experimental conditions of either asserting a correlation or ascribing a spin component. The correlation associated with the entanglement of the state vectors is tied to a certain experimental set-up. 'The entanglement of state vectors, far from denoting a substantial relation, expresses but a potentiality of a correlation if a certain kind of experimentation is performed in the following of a first measurement which stands then as the preparation for it', that is, for this second measurement (Bitbol). The correlation is just in the probabilities for outcomes of *measurements that could be made or will be made.* Moreover, as Bitbol says, the experimental set-up in which the correlation statement makes sense is not compatible with the experimental set-up in which measurement related to local observables, such as the spin component, are performed:

> the experimental set-up which defines a correlation and allows the obtaining of information about it is generally exclusive of an experimental set-up defining local observables and allowing the obtaining of maximal information about them.

In transition from the first kind of set-up to the second, no continuity condition for identity is satisfied which would enable us to say that what has *this* or *that* spin component now *is what* the correlation was about before. The now and the before refer to different preparations, and there is then no experimental basis for saying that the correlation concerns two distinct entities.

It is important to see how the Kantian case of the incongruent counterparts and the quantum case of the correlated particles diverge from each other, as well as how they are similar. In both we have a relation (incongruence, correlation) which is not founded in attributes of the relata, and for which the conditions of possibility of knowledge are tied to specific experimental contexts. But in the case of the counterparts, the *compatibility* between the two experimental contexts – that in which one hand is individuated as right or left, and that in which the relation of incongruence holds – makes it possible for the hands to be the relata of the relation. In the case of quantum particles, the incompatibility of the corresponding experimental contexts makes this impossible, and highlights, by contrast, the non-givenness of any route which could relate numerical distinction to discernibility. Thus conceptual discernibility cannot, even logically, be a foundation for knowledge of numerical distinctness, but is a further stage of knowledge which has to involve a cognitive procedure, of individuation, which requires particular conditions of possibility. This procedure is so 'naturally' conducted in ordinary contexts that one is oblivious to it and to the conditions it requires – but of these one is then forcefully reminded by their absence in the quantum context. The very fact that one calls this an absence betrays the reference-role still given to situations in which the possibility of this procedure is present. The transcendental perspective leads therefore to a more uniform, less preferential consideration of the different experimental and experiential conditions of knowledge.[20]

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[20] Appendices to this paper are available from the authors. We thank Michel Bitbol, Anja Jauernig, Fred Muller, Simon Saunders and Paul Teller for helpful conversation and correspondence. Our conclusions, though reached differently, are similar to those of Teller's 'The Ins and Outs of Counterfactual Switching', *Noûs*, 35 (2001), pp. 365–93.