Modeling of Spatial Lag in Bed-Load Transport Processes and Its Effect on Dune Morphology

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Abstract: In the present study, two bed-load transport models are introduced in an existing idealized dune model. These allow for the modeling of the spatial lag between the sediment transport rate and bed shear stress along dune surfaces. This lag is an important factor in determining transitions between bedform regimes. Results of the original dune model (using an equilibrium transport formula) are compared with (1) a new model version that directly models spatial lag with a relaxation equation and (2) a new model version including pick-up and deposition processes. Both bed-load models use mean particle step length as an important parameter, which is varied to assess which value is appropriate for the dune regime. Laboratory experiments are simulated with the model. This shows that the results are best with the pick-up and deposition model version, combined with a step length of 25 times the particle diameter. It is furthermore shown that in principle the model is also able to wash out fully grown dunes, by increasing the step length parameter. **DOI: 10.1061/(ASCE)HY.1943-7900.0001254.** *© 2016 American Society of Civil Engineers.*

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Introduction

In hydraulic models, roughness values play an important role in correctly determining water levels (Casas et al. 2006; Vidal et al. 2007; Morvan et al. 2008), which is important for flood management purposes. In many though not all rivers, a large part of the roughness is determined by dunes, which can form on river beds with sediment sizes ranging from silt to gravel (Kostaschuk 2000; Wilbers and Ten Brinke 2003; Best 2005; Jerolmack and Mohrig 2005; Kleinhans et al. 2007). River dunes increase the hydraulic roughness significantly, because their shape causes form drag. Water level forecasts therefore depend on accurate predictions of the presence and evolution of river dune dimensions, from the lower regime through the upper regime.

In the lower regime the riverbed is flat, and dunes can appear if the power the river exerts on the bed (flow power) or the shear stress increases (Simons and Richardson 1966). If flow power or shear stress keeps increasing, the dunes will grow and cause the hydraulic roughness to increase as well (Simons and Richardson 1966). Especially under high discharges, dunes can rapidly evolve, for example, during the 1988 flood in the Waal and Rhine River in the Netherlands (Julien and Klaassen 1995). At a certain point in the upper regime, the flow power will become so high that dunes are washed out completely. The washing out of dunes is linked to various factors, for example, a spatial lag between the bedform or flow field and bed-load sediment transport (Nakagawa and Tsujimoto 1980) or an increase in suspended sediment concentration (Smith and McLean 1977). The sudden disappearance of dunes causes hydraulic roughness to decrease, which causes water levels to decrease as well. Ideally, a dune model can therefore predict the full evolution of a dune from the lower-stage plane bed until the upper-stage plane bed.

Many approaches have been and are used to model dune dimensions, varying from equilibrium dune height predictors (e.g., Yalin 1964; Allen 1978; Van Rijn 1984) to different forms of stability analysis (e.g., Kennedy 1963; Engelund 1970; Fredsøe 1974; Yamaguchi and Izumi 2002). Recently, models have been developed that calculate the turbulent flow field over bedforms, in some cases in combination with morphological computations (e.g., Nelson et al. 2005; Tjerry and Fredsøe 2005; Shimizu et al. 2009; Paarlberg et al. 2009; Nabi et al. 2012, 2013a, b). These models are valuable to study detailed hydrodynamic processes, but can be computationally intensive.

Only a few of these relatively complex models are able to model the transition to an upper-stage plane bed in flume conditions. The Shimizu et al. (2009) model may have been the first model, which takes into account both turbulent flow over the bedform and morphological development. Shimizu et al. (2009) showed that their model predicts this transition in various numerical scenarios. Furthermore, the model is able to represent other important physical processes like hysteresis effects with regard to discharge and dune height. Shimizu et al. (2009) stated that the prediction of the transition to the upper-stage plane bed is enabled by the way they model the mean particle step length, which plays a key role in the bed-load transport model they use. This variable is the distance traveled from dislodgement to rest (i.e., during saltation) according to Einstein (1950).

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Sekine and Kikkawa (1992) used the experimental data of Francis (1973), Fernandez Luque and Van Beek (1976), and Sekine and Kikkawa (1984) to validate a numerical model of saltation of particles. The experiments were done to determine the dependency of bed-load transport and particle velocity, among others, on friction velocity, settling velocity, and grain diameter. The saltation model of Sekine and Kikkawa (1992) closely matches these experimental results. Furthermore, the predictions for the thickness of the saltating bed-load layer closely match the data of Sekine and Kikkawa (1984); the particles remain within a few grain diameters from the bed, as expected for bed-load transport.

Sekine and Kikkawa (1992) furthermore compared the step length values that follow from their model with the step length values derived from experiments by, among others, Nakagawa and Tsujimoto (1980). The *physical* experiments of Nakagawa and Tsujimoto (1980) showed a range of approximately 40–240 times the particle diameter. The suspension parameter u_*/w_s (the ratio of friction velocity to settling velocity) ranged from about 0.18 to 0.35. According to Van Rijn (1993), sediment is mainly transported as bed load when $u_*/w_s < 1$, so these experiments were clearly in the bed-load regime. The *numerical* experiments of Sekine and Kikkawa (1992) showed that the mean step length can vary between near 10 and about 250 times the particle, mostly related to friction velocity u_* (directly proportional) and settling velocity w_s (inversely). The suspension parameter u_*/w_s ranged from about 0.15 to 0.28. They found that all computed step length values are no more than two times larger or smaller than the observed values. To predict step length, Shimizu et al. (2009) used a conceptual model based on the flat-bed bed-load experiments of Nakagawa and Tsujimoto (1980) and the work of Engelund (1966). Shimizu et al. (2009) proposed that step length is constant in the dune regime at 50 times the particle diameter, increases linearly with the dimensionless grain shear stress in the transitional regime, and is again constant at 250 times the particle diameter in the upper-stage plane bed regime.

The Shimizu et al. (2009) dune evolution model uses the pick-up and deposition model of Nakagawa and Tsujimoto (1980) to calculate bed-load transport. The Nakagawa and Tsujimoto (1980) pick-up and deposition model inherently allows a phase-lag effect between bed elevation and bed elevation change. As Nakagawa and Tsujimoto (1980) argued, this lag is the principal cause of bed instability and thereby regime transitions. They identified two important sources of this lag. The first is attributable to the spatial distribution of bed shear stress, which can be taken into account by applying the transport formula to the local bed shear stress. The second is the sediment particle step length. This creates a phase-lag effect in nonuniform flow that they incorporated into their bed-load model by calculating the pick-up of sediment first and then determining the deposition of sediment away from the pick-up point with a distribution function that relies on the mean step length. This spatial lag is not taken into account in the often used bed-load formulation of Meyer-Peter and Müller (1948), which is meant for equilibrium conditions. The flow in the dune evolution model of Shimizu et al. (2009) is modeled with nonhydrostatic two-dimensional (2DV) flow equations, a free surface, and a nonlinear k-ε turbulence closure.

Besides the pick-up and deposition model of Nakagawa and Tsujimoto (1980), there are other ways to model the phase-lag effect caused by the probability distribution of sediment deposition. For example, Tsujimoto et al. (1990) derived a linear relaxation equation that also accounts for this phase-lag effect. This equation is based on the definition of sediment deposition and equilibrium bed-load transport of Einstein (1950), and the relaxation equation describing the response of bedform geometry to changes in flow presented by Allen (1974) and Nakagawa and Tsujimoto (1980). Furthermore it should be noted that suspended sediment transport can play a role in transitions to the upper-stage plane bed as well, through similar processes with regard to spatial lag. However, in the present study the focus is on the phase lag in bed-load processes. It is expected that phase lag of bed load alone may have a significant effect regarding bedform regimes and transitions [as shown by Shimizu et al. (2009)].

In the context of flood early-warning systems, a detailed hydrodynamic model is a drawback as it leads to computation times that are too large, especially when applied on the large spatial domain of a river segment. The objective of the present study is to increase the understanding of the effect of spatial lag on dune dimensions and to explore the potential of an existing idealized dune evolution model (Paarlberg et al. 2007, 2009) to represent a transition to the upper-stage plane bed. This computationally cheap dune evolution model works well in the dune regime, without needing to incorporate a very advanced turbulence model. The aforementioned bed-load models, which allow for the naturally occurring phenomenon of spatial lag because of the travel distance of sediment, will be implemented. This means that more physical processes are taken into account than with a bed-load formula like that of Meyer-Peter and Müller (1948), while it is expected that computational time still remains limited. The focus will be on sand beds and flume conditions.

In the model of Paarlberg et al. (2009), the flow separation zone is parametrized instead of using full hydrodynamic equations. Furthermore, the model employs a constant eddy viscosity as a very basic turbulence closure. The model is able to predict the evolution of dunes from small initial disturbances up to equilibrium dimensions with limited computational time and good accuracy (Paarlberg et al. 2009). In addition, this model has been coupled with an existing large-scale (depth-averaged) hydraulic model to form a *dynamic roughness model* that works efficiently on the river scale (Paarlberg et al. 2010). The coupled model clearly shows the expected hysteresis effects in dune roughness and water levels during flood waves, and different behavior of sharp-peaked versus broad-peaked flood waves within the dune regime (Paarlberg et al. 2010). However, the model is not able to model a transition to the upper-stage plane bed. Paarlberg et al. (2009) assumed that along a dune stoss-side shear stress and transport are directly coupled as they are in steady uniform conditions, so a transport formula like that of Meyer-Peter and Müller (1948) is used. This makes it impossible to model a transition to the upper-stage plane bed with their dune evolution model.

This paper will answer the following main research questions: (1) to what extent are the linear relaxation equation (Tsujimoto et al. 1990) and the pick-up and deposition sediment transport formulation (Nakagawa and Tsujimoto 1980) in the Paarlberg et al. (2009) model able to reproduce and/or improve the results of the MPM (Meyer-Peter and Müller 1948) formulation for describing dunes? and (2) what are the prospects for the model version that performs best in the dune regime to describe the transition of the dune regime to the upper-stage plane bed?

Dune Model

General Setup

The basis of the present model is the dune evolution model developed by Paarlberg et al. (2009). Paarlberg et al. (2009) modified the process-based morphodynamic sand wave model of Németh et al. (2006), which is based on the numerical model of Hulscher (1996),

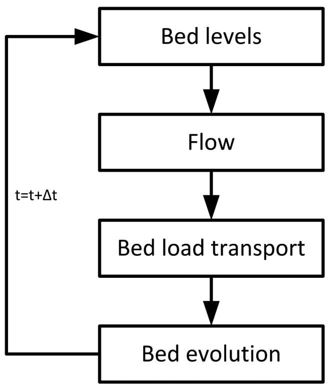

Fig. 1. Model process (adapted from Paarlberg et al. 2009)

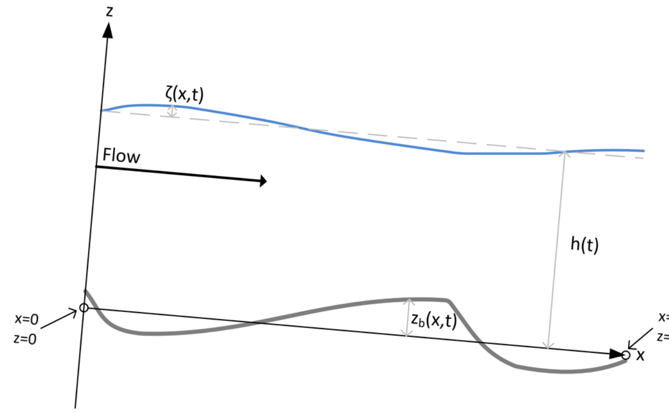

Fig. 2. Dune moving along a downward-sloping bed, with the computational domain superimposed

to enable simulation of finite amplitude river dune evolution in unidirectional flow.

The model consists of a flow module, a sediment transport module, and a bed evolution module that operate in a decoupled way. This means that, based on the bed levels at the start of a calculation step, first the flow is calculated. Then bed shear stress is derived from the flow and used to determine sediment transport along the domain. The gradient of sediment transport determines bed evolution, and so finally new bed levels for the next calculation step are calculated. See Fig. 1 for a schematic representation of these steps in the calculation, where $t = t + \Delta t$ signifies the moment when the calculation is advanced by one computational step Δt.

The model uses periodic boundary conditions, which means that the sediment transport and flow at the downstream boundary is used as input at the upstream boundary of the model. The model has a domain length that is equal to one dune length. Combined with the periodic boundary conditions, this implies that an (infinite) train of identical dunes passes through the domain. The dune length is selected by a numerical stability analysis, which is included in the model. This model calculates the growth rate of a series of small bed disturbances with a range of different wavelengths. The length of the fastest-growing disturbance is chosen as the dune length. If the water depth changes more than 5% compared to the value with which the dune length was determined, the linear stability analysis is done again to determine a new dune length. Paarlberg et al. (2009) showed that by using this method most predicted dune lengths are less than 25% larger or smaller than the observed dune lengths. Colombini and Stocchino (2012) also showed that linear stability analyses can predict the wave length of two-dimensional dunes well.

Flow Model

In the 2DV flow model, x is the streamwise coordinate and z is the coordinate perpendicular to x. The x-axis follows the average channel slope i, which is an input parameter for the model. A schematization of a dune moving along a downward-sloping bed is shown in Fig. 2. The coordinate system of the computational domain is superimposed on the dune to show that it is rotated according to channel slope i. i is generally much smaller than implied in Fig. 2: of the order of 10^{-3} for flumes and 10^{-4} for lowland rivers.

In Fig. 2, λ denotes the dune length, h is the domain-averaged water depth, and z_b is the bed level relative to the x-axis. The water surface elevation is defined as the deviation from the average water depth and is denoted by ζ.

Governing Equations

The flow in the model of Paarlberg et al. (2009) is described by the two-dimensional shallow water equations in a vertical plane (2DV), assuming hydrostatic pressure conditions. For small squared Froude numbers ($\mathsf{F}^2 \ll 1$) the momentum equation in vertical direction reduces to the hydrostatic pressure condition, and the time variations in the horizontal momentum equation can be dropped. The governing model equations that result from the analysis done by Paarlberg et al. (2009) are

$$u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -g\frac{\partial \zeta}{\partial x} + A_v\frac{\partial^2 u}{\partial z^2} + gi \quad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \quad (2)$$

where u and w = velocities in the x and z directions, respectively; the parameter g = acceleration attributable gravity; and A_v = eddy viscosity. The flow in the domain is forced by the term gi, which signifies the effect of the (additional) water level difference along the domain because of the channel slope.

Boundary Conditions

The boundary conditions are defined at the water surface ($z = h + \zeta$) and at the bed ($z = z_b$). The boundary conditions at the water surface are that there can be no flow through the surface [Eq. (3)] and no shear stress at the surface [Eq. (4)]

$$u\frac{\partial \zeta}{\partial x}\bigg|_{z=h+\zeta} = w \quad (3)$$

$$\frac{\partial u}{\partial z}\bigg|_{z=h+\zeta} = 0 \quad (4)$$

The kinematic boundary condition at the bed [Eq. (5)] yields that there is no flow through the bed

$$u\frac{\partial z_b}{\partial x}\bigg|_{z=z_b} = w \quad (5)$$

As a basic turbulence closure, a time- and depth-independent eddy viscosity is assumed, which leads to a parabolic velocity profile (Engelund 1970; Hulscher 1996). In order to represent the bed shear stress correctly for a constant eddy viscosity, the partial slip condition at the bed presented in Eq. (6) is necessary

$$\tau_b = A_v \frac{\partial u}{\partial z}\bigg|_{z=z_b} = S u_b \tag{6}$$

where τ_b ($\mathrm{m^2/s^2}$) = volumetric bed shear stress; u_b (m/s) = flow velocity at the bed; and the resistance parameter S (m/s) controls the resistance at the bed. Engelund (1970) used a parameter similar to S to relate friction velocity and thereby bed shear stress to the flow velocity at the bed. Paarlberg et al. (2009) determined that $A_v = (1/6)\beta_1 \kappa u_* h$ and $S = \beta_2 u_*$, where β_1 and β_2 are calibration parameters, the Von Kármán constant $\kappa = 0.407$, and u_* is the friction velocity. The calibration results of Paarlberg et al. (2009) are used, who found that $\beta_1 = \beta_2 = 0.5$.

Solving the Flow Equations

To solve the flow equations the average water depth is needed as input. However, to be able to model flume situations, discharge is used as an input. This means that the average water depth has to be determined iteratively. The model starts with an initial value for h nd solves the flow equations described previously. The discharge that results from the flow field is compared with the discharge given as input, and h is adjusted if they are not equal. This process is repeated until they do match. For more details about the model equations and numerical solution procedure, refer to Paarlberg et al. (2009) and Van den Berg et al. (2012).

Bed-Load Sediment Transport Model

Three different bed-load models are used: (1) the Meyer-Peter and Müller (1948) formulation as used in Paarlberg et al. (2009); (2) a Meyer-Peter and Müller (1948) formulation with a spatial lag via a relaxation equation; and (3) the Nakagawa and Tsujimoto (1980) pick-up and deposition model.

Meyer-Peter and Müller Sediment Transport Model

In the original dune evolution model, a formula of the type of Meyer-Peter and Müller (1948), including gravitational bed slope effects, is used. Eq. (7) denotes this formula in dimensional form (as volumetric bed-load transport per unit width, $\mathrm{m^2/s}$)

$$q_{b;e}(x) = \begin{cases} \beta[\tau_b(x) - \tau_c(x)]^n \left(1 + \eta \frac{\partial z_b}{\partial x}\right)^{-1}, & \tau_b > \tau_c \\ 0, & \tau_b \le \tau_c \end{cases} \tag{7}$$

where $\tau_c(x)$ = local critical (volumetric) bed shear stress ($\mathrm{m^2/s^2}$); $n = 3/2$; and $\eta = \tan(\varphi)^{-1}$ with the angle of repose $\varphi = 30°$ for sand. The proportionality constant β ($\mathrm{s^2/m}$) describes how efficiently the sand particles are transported by the bed shear stress (Van Rijn 1993) and its value can be estimated with

$$\beta = m/(\Delta g) \tag{8}$$

where $\Delta = \rho_s/\rho - 1$ and the empirical coefficient $m = 4$ is based on analysis done by Wong and Parker (2006). The grain density of sand ρ_s s set to 2,650 $\mathrm{kg/m^3}$, and the density of water ρ is set to 1,000 $\mathrm{kg/m^3}$. The local, critical bed shear stress $\tau_c(x)$, corrected for bed slope effects, is given by the following equation from Paarlberg et al. (2009), which was adopted from Fredsøe and Deigaard (1992):

$$\tau_c(x) = \tau_{c0} \frac{1 + \eta \frac{\partial z_b}{\partial x}}{\sqrt{1 + \left(\frac{\partial z_b}{\partial x}\right)^2}} \tag{9}$$

where τ_{c0} = critical bed shear stress for a flat bed, defined by Eq. (10)

$$\tau_{c0} = \theta_{c0} g \Delta D_{50} \tag{10}$$

where θ_{c0} = critical Shields parameter for a flat bed and D_{50} = median grain size. The bed slope–corrected critical Shields parameter θ_c can be derived by combining Eqs. (9) and (10).

Meyer-Peter and Müller Sediment Transport Model Extended with Linear Relaxation

Tsujimoto et al. (1990) showed that sediment does not directly respond to changing flow conditions along the bed (i.e., using equilibrium transport introduces errors in that situation): the sediment transport only reaches its equilibrium value after a certain adaptation length. Nakagawa and Tsujimoto (1980) proposed a pick-up and deposition model to capture this relaxation process, which is presented in the next section. As an alternative Tsujimoto et al. (1990) derived a simple relation to model relaxation in sediment transport, namely

$$\frac{dq_b}{dx} = \frac{q_{b;e} - q_b}{\Lambda} \tag{11}$$

where $q_{b;e}$ = equilibrium sediment transport (following Meyer-Peter and Müller 1948) and Λ (m) = mean step length. Einstein (1950) stated that the mean step length is the average distance traveled by sediment particles (from where they were entrained to where they were deposited) under certain flow conditions and can be determined by

$$\Lambda = \alpha D_{50} \tag{12}$$

where α = nondimensional step length parameter that is often assumed to be 100 as originally defined by Einstein (1950). This parameter is further discussed later. Tsujimoto et al. (1990) showed that Eq. (11) follows from a more general linear approximation of the change of sediment transport over distance as the result of a difference between its local value and its equilibrium value. This depends on a spatial scale of relaxation, which in this case is the mean step length.

Eq. (11) needs a boundary condition at $x = 0$ for q_b. Only a periodic boundary condition is defined, which states that values at the start of the domain ($x = 0$) are equal to the values at the end of the domain ($x = \lambda$). This is not enough information as it does not directly define $q_b(x = 0)$. To come to a solution, $q_b(x = 0)$ is set to $q_{b;e}(x = 0)/2$ as a *first estimate*. The other values in the spatial domain can then be determined using Eq. (11) and a backward Euler scheme. The resulting value at $x = \lambda$ should be the same as the value at $x = 0$, to conform to the periodic boundary condition. If this is not the case, a new estimate is made for $q_b(x = 0)$. It is set to the average of the previous estimate of $q_b(x = 0)$ and $q_b(x = \lambda)$. This process is repeated until the periodic boundary condition is met, or until the difference between the value at $x = 0$ and $x = \lambda$ is smaller than 0.1%. Eq. (11) is then solved, and the model proceeds to the next step in the process (bed evolution). Different first estimates at $x = 0$ were tested (e.g., $q_b = 0$, $q_b = q_{b;e}$), but this did not have an effect on the final model results.

Pick-Up and Deposition Model

The pick-up and deposition model of Nakagawa and Tsujimoto (1980) uses the following formulas to determine bed-load transport. Pick-up of sediment (probability of a particle being picked up in $\mathrm{s^{-1}}$) is determined by

$$p_s(x) = F_0 \sqrt{\frac{\Delta g}{D_{50}}} \theta(x) \left[1 - \frac{\theta_c}{\theta(x)}\right]^3 \tag{13}$$

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where $F_0 = 0.03$; θ = Shields parameter; and θ_c = bed slope–corrected critical Shields parameter. The determination of deposition is done by applying the following formula (Nakagawa and Tsujimoto 1980):

$$p_d(x) = \int_0^\infty p_x(x-s)f(s)\mathrm{d}s \quad (14)$$

where the distribution $f(s)$ determines the probability that picked-up sediment is deposited a distance s away from the pick-up point (x–s). This means that in order to determine the deposition at a certain location x, the pick-up of sediment at the *upstream* locations needs to be known. The model first determines the pick-up along the domain and then for each cell distributes the picked-up sediment at that cell among the cells downstream, according to the distribution function. The pick-up at each cell in turn is simply the sum of the fractions of picked-up sediment it receives from upstream. The distribution function is defined by Nakagawa and Tsujimoto (1980) as follows:

$$f(s) = \frac{1}{\Lambda}\mathrm{e}^{-s/\Lambda} \quad (15)$$

The integral of this function is $F(s) = -\mathrm{e}^{-s/\Lambda}$. This means that the fraction of sediment picked up at a certain location that is deposited between that location and five times the step length in downstream direction equals $\mathrm{e}^0 - \mathrm{e}^{-5} = 99.3\%$. Because of this, Eq. (14) is applied from $s = 0$ to $s = 5\Lambda$ instead of applying it from $s = 0$ to $s = \infty$. The remainder of the sediment (0.7%) is deposited at the cell where $s = 5\Lambda$. Finally the transport gradient along the domain is determined as follows:

$$\frac{\partial q_b}{\partial x} = D_{50}[p_s(x) - p_d(x)] \quad (16)$$

Step Length

Based on the information on step length presented in the Introduction, the step length is varied between 25 and 300 times the particle diameter, which is consistent with bed-load motion as observed in experiments of Nakagawa and Tsujimoto (1980). This way it is possible to assess how sensitive the results are with respect to this parameter. Step length will be held constant along the dune, in line with the findings of Van Duin et al. (2012).

Bed Evolution

The bed evolution is modeled using the Exner equation given by Eq. (17), where the sediment transport rate is calculated with one of the three aforementioned bed-load models and ε_p is the bed porosity

$$(1-\varepsilon_p)\frac{\partial z_b}{\partial t} = -\frac{\partial q_b}{\partial x} \quad (17)$$

The equilibrium transport model is only applied outside the flow separation zone. See the next section for the procedure that is used inside the flow separation zone.

In case the linear relaxation model or the pick-up and deposition model is used, Eq. (17) is applied inside as well as outside the flow separation zone. When at a certain location the angle of the bed exceeds the angle of repose at the end of a calculation step, sediment is moved downward until the angle of repose is no longer exceeded anywhere. This means that in contrast to the original method as presented by Paarlberg et al. (2009), the avalanching procedure described in the next section is *never* applied. In this way picked-up sediment is allowed to deposit in the (nonexistent) separation zone.

Flow Separation in the Original Model

The method described in this section is only used with the original bed-load transport formulation described, and not with the two new formulations. Paarlberg et al. (2009) used a parametrization of flow separation, to enable simulation of finite amplitude river dune evolution. Flow separation is forced in the model when the lee-side slope exceeds 10°.

The flow separation streamline behind the dune is determined with a third-order polynomial based on experimental data of turbulent flow over two-dimensional subaqueous bedforms gathered by Paarlberg et al. (2007). Following the method of Kroy et al. (2002) for aeolian sand dunes, the flow is then computed with the flow separation streamline acting as the bed level in the flow separation zone. The flow and bed shear stress *within* the flow separation zone is assumed to be zero. The bed shear stress outside the flow separation zone is slightly adjusted to account for the presence of the flow separation zone (Paarlberg et al. 2009). In the separation zone the bed transport at the crest of the dune is deposited on the lee side of the dune under the angle of repose (i.e., avalanched). So, in this case an *integral form* of Eq. (17) is used for the lee slope of the dune (Paarlberg et al. 2009). Effectively, in the flow separation zone the crest moves and the rest of the bed remains undisturbed because the shear stress is zero there. Outside of the flow separation zone, the bed is normally active.

Test Cases

The reference case used for this study is Flow A of an experiment done by Venditti et al. (2005a, b). This flow will be used to assess which value of α best fits the bed-load regime. The model will then also be run for Flows B, C, D, and E of Venditti et al. (2005a, b). The experiment of Venditti et al. (2005a, b) was done in a recirculating flume 15.2 m long, 1 m wide, and 0.30 m deep. The suspension parameter u_*/w_s was between 0.3 and 0.4, which is in the bed-load regime. The median grain diameter D_{50} is 0.5 mm and a bed porosity $\varepsilon_p = 0.4$ is assumed (Van Rijn 1993). The initial parameters of the various flows of Venditti et al. (2005a, b) are presented in Table 1.

In this table h_i is initial water depth and q is discharge per unit width, which was constant during the experimental run. Starting from a flat bed, bedforms developed toward their equilibrium dimensions in 1.5 h for Flows A, B, and C. The bedforms for Flows D and E each grew from a single artificially made defect in the flume as opposed to the bedforms of Flows A, B, and C, which developed over the entire bed without interference (Venditti et al. 2005b). Bedform height, length, and migration rate were determined from measurements with echo sounders, and water depth with ultrasonic water level probes (Venditti et al. 2005b). Because bed-load measurements were too infrequent, Venditti et al. (2005a) calculated the

Table 1. Initial Parameters of the Experiments of Venditti et al. (2005a, b)

Parameter	Flow A	Flow B	Flow C	Flow D	Flow E
h_i (m)	0.152	0.152	0.153	0.153	0.153
i (10^{-4})	12	11	7	5.5	5.5
q ($\mathrm{m^2/s}$)	0.077	0.0723	0.0696	0.0611	0.0546
D_{50} (mm)	0.5	0.5	0.5	0.5	0.5

Table 2. Experimental Results of Venditti et al. (2005a, b)

Result	Flow A	Flow B	Flow C	Flow D	Flow E
Δ_e (m)	0.048	0.042	0.036	0.022	0.020
λ_e (m)	1.17	0.86	0.95	0.38	0.30
h_e (m)	0.17	0.17	0.17	0.17	0.17
R_e (m/s)	0.65	0.37	0.33	0.17	0.10
β_b	0.56	0.58	0.57	0.54	0.60
$Q_{s;e}$ (kg/h)	102.4	47.9	34.2	9.5	5.7

Note: h_e = equilibrium water depth; $Q_{s;e}$ = equilibrium sediment transport rate of the bedforms; R_e = bedform migration rate; β_b = bedform shape factor; Δ_e = equilibrium dune height; λ_e = equilibrium dune length.

volumetric dry sediment transport rate of the bedforms per unit width $Q_{s;v}$ with Eq. (18)

$$Q_{s;v} = \beta_b(1-\varepsilon_p)R_b\Delta_b \tag{18}$$

where Δ_b = bedform height; R_b = migration rate in m/s; and $\beta_b = A/(\Delta_b\lambda)$ is a bedform shape factor that depends on frontal bedform area A and the dune height and length. Venditti et al. (2005a) reported that the mean value of β_b was between 0.54 and 0.60 for the five different flows. Multiplying $Q_{s;v}$ with ρ_s gives the dry sediment transport rate of the bedforms in terms of mass per unit time per unit width. Venditti et al. (2005a) determined migration rate by measuring the time it took for each dune to migrate from one echo sounder to the next and by dividing the distance between the echo sounders by that time. The *equilibrium* values of the measured dune height from the experiments of Venditti et al. (2005a, b) can be found in Table 2.

Venditti et al. (2005a) reported a value of $Q_{s;e} = 102.4$ kg/h for Flow A, which is the mean of the estimates of sediment transport rate over a period of time in the equilibrium stage of the experiment. By using the *equilibrium* dune characteristics of Flow A as presented in Table 2, instead of the *full time series* as Venditti et al. (2005a) did, and by directly applying Eq. (18), the equilibrium sediment transport rate $Q_{s;e} = 100.0$ kg/h, which is consistent with the reported mean value. The same method will be used to determine sediment transport rates from the model results.

Model Results

The model starts with a flat bed and uses a θ_{c0} of 0.05. Discharge, slope, and grain diameter remain constant during the model run. The model was run using the three options for the sediment transport formulation described before. For the two bed-load models with a step length, α varied from 25 to 300, which is a slightly wider range than the one found by Nakagawa and Tsujimoto (1980). In Table 3 the measured equilibrium dune dimensions, water depth, bedform migration rate, and sediment transport rate of the bedforms are presented for Flow A, together with the corresponding model runs. The results from the runs in Table 3 are discussed in the following sections.

Flow A with the Original Bed-Load Model

By using the original bed-load model (Meyer-Peter and Müller 1948), an equilibrium dune height of 0.064 m, dune length of 1.33 m, and water depth of 0.19 m are found. The dune length is predicted reasonably well with an overestimation of about 13% (the experimental result was 1.172 m), but the dune height is overestimated by about 33%. The migration rate is close to the experimental result, while the transport rate is overestimated by 62%. The resulting water depth is around 12% higher than the experimental result of 0.17 m. In Fig. 3 the evolution of the dune shape is shown.

The model actually simulated one dune length, but in Figs. 3–6, a train of four identical dune lengths is shown instead to make the results more clear. It can be seen that first low-angle dunes appear, which then evolve to high-angle dunes (triggering flow separation), which then become dunes with a lee side of 30° (because of the avalanching from the crest).

Flow A with Linear Relaxation

Extending the original bed-load model with linear relaxation leads to a strong suppression of the dune height and a limited suppression of the dune length. The height is reduced partly because with this bed-load model flow separation attributable to steep lee sides (and the resulting forced avalanching from the crest) is not used. With flow separation, all sand that reaches the crest would have remained in the flow separation zone, spreading out over the lee side and contributing solely to the height of the dune. However, opposite to the original model the lee side of the dune is still active; sediment is transported away from there toward the stoss side of the next dune. This means that sediment arriving at the lee side does not contribute as strongly to the height of the dune as it would in a situation with flow separation.

While dune height is more than 50% smaller than with the original bed-load model for all values of α, the water depth is only about 16% smaller. Compared with the experiment, the dune height is underestimated between 50 and 100%, while the water depth is underestimated by about 6%. With a nondimensional step length of 75 and greater, the smearing effect of the linear relaxation model is so strong that no more dune growth occurs at all. This is similar to what would occur when going toward an upper-stage plane bed, where the bed washes out. Because the dune height is small to nonexistent in that case, there is less hydraulic roughness so the water depth is smaller than in a regime with dunes. Although there is no dune height for values of $\alpha \geq 75$, a dune length is still reported. This value is used as the domain length, which follows from the

Table 3. Experimental Results of Flow A of Venditti et al. (2005a, b) and Model Results

			Linear relaxation with α =				Pick-up and deposition with α =							
Result	Experiment	MPM	25	50	75	100	25	50	75	100	150	200	250	300
Δ_e (m)	0.048	0.064	0.029	0.023	0	0	0.042	0.039	0.037	0.033	0.019	0	0	0
λ_e (m)	1.17	1.33	1.11	1.10	1.07	1.07	1.18	1.17	1.15	1.14	1.09	1.07	1.07	1.07
h_e (m)	0.17	0.19	0.16	0.16	0.15	0.15	0.17	0.17	0.16	0.16	0.16	0.15	0.15	0.15
R_e (mm/s)	0.65	0.59	1.20	1.30	—	—	0.29	0.58	0.88	1.19	1.94	—	—	—
$Q_{s;e}$ (kg/h)	102.4	162.5	179.5	155.6	—	—	58.1	111.8	161.2	201.2	188.4	—	—	—

Note: h_e = equilibrium water depth; $Q_{s;e}$ = equilibrium sediment transport rate of the bedforms; R_e = bedform migration rate; Δ_e = equilibrium dune height; λ_e = equilibrium dune length.

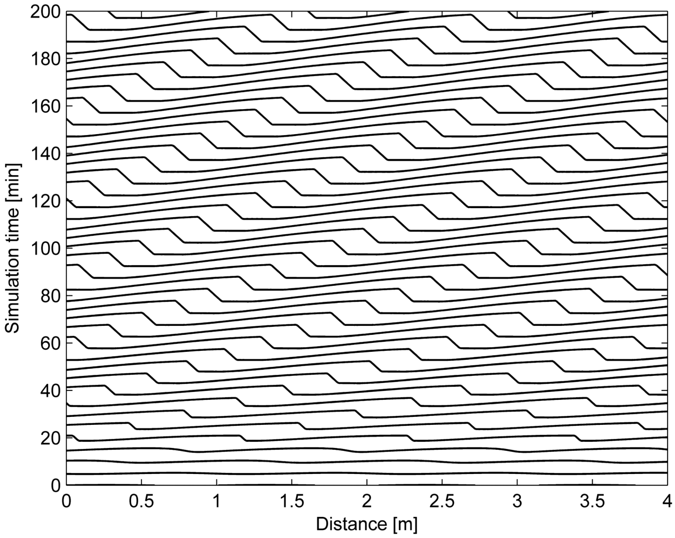

Fig. 3. Evolution of dune shape over time of the model run with the original bed-load formulation (flow left to right)

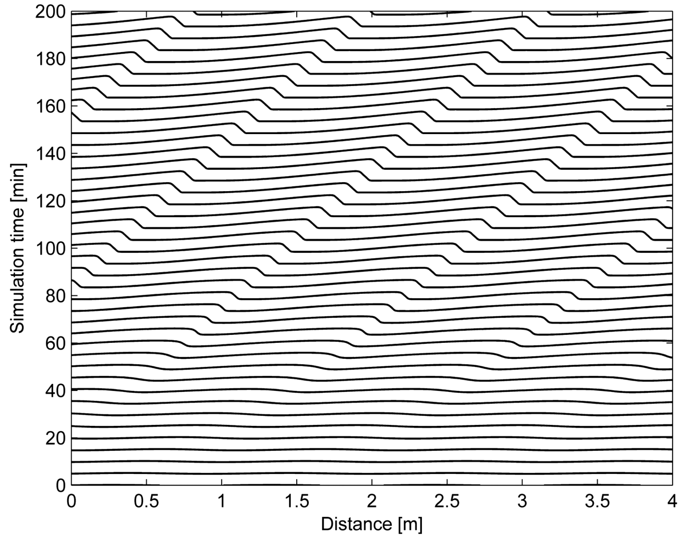

Fig. 6. Evolution of dune shape over time of the model run with the pick-up and deposition bed-load formulation, $\alpha = 25$ (flow left to right)

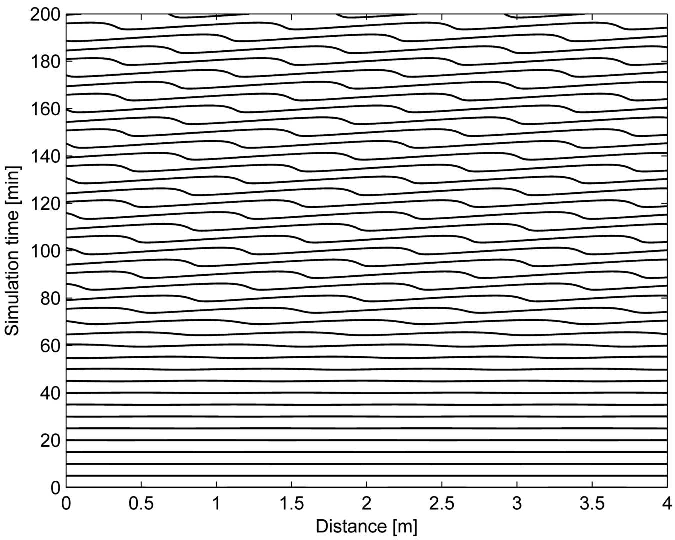

Fig. 4. Evolution of dune shape over time of the model run with the linear relaxation bed-load formulation, $\alpha = 25$ (flow left to right)

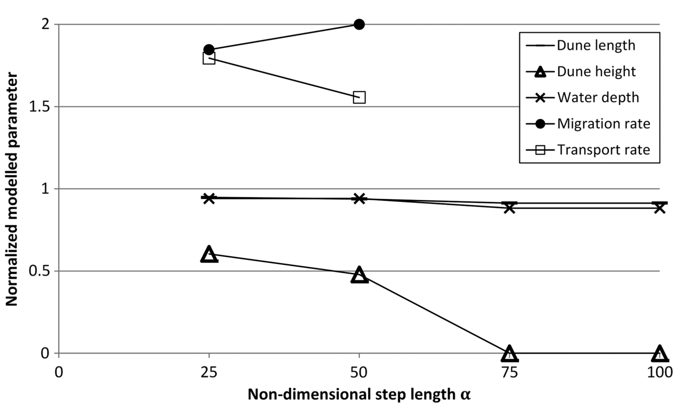

Fig. 5. Evolution of normalized modeled parameters with changing step length in the linear relaxation model

linear stability analysis described before. Furthermore, because dunes are absent for values of $\alpha \geq 75$ there is no meaningful migration rate and transport rate (which is derived from migration rate).

Paarlberg et al. (2009) showed that the dune length that follows from the numerical stability analysis is nearly linearly related to the water depth. This is also reflected in our results, as the decrease of water depth with about 16% corresponds to the decrease of dune length of about 16%. Because water depth and dune length were overestimated with the original model, they are actually closer to the experimental results with the linear relaxation model; an underestimation of about 5 and 6%, respectively, for the values of α where a dune is present. The resulting less steep and smoother dunes of limited height are shown in Fig. 4, presenting the bed morphology with a nondimensional step length parameter α of 25. Dune growth is slower than with the original model (see Fig. 3).

The computational results for Flow A are normalized by dividing the modeled parameter values by the measured value of that parameter. This is presented in Fig. 5, where the normalized values are plotted against the step length parameter α. This clearly shows that dune height decreases with step length, but step length does not have a great effect on water depth and dune length.

The migration rate of dunes is overestimated by 85 and 100% with $\alpha = 25$ and $\alpha = 50$, respectively. Regarding the sediment transport rate, this is partly compensated by the underestimation of the dune dimensions. The sediment transport is overestimated by 63 and 79%, respectively. Because the overall model performance is best with $\alpha = 25$, this value is chosen as the best-fit value for this model version.

Flow A with Pick-Up and Deposition

Using the pick-up and deposition model of Nakagawa and Tsujimoto (1980), the water depth and dune length are similar to the experimental results for values of α below 150. In general, the dune height is underestimated compared to the experimental results for all values of α. This underestimation is smaller than with the linear relaxation model. The pick-up and deposition model performs the best with $\alpha = 25$ (dune length is only 13% smaller, and dune height and water depth are almost exactly predicted as

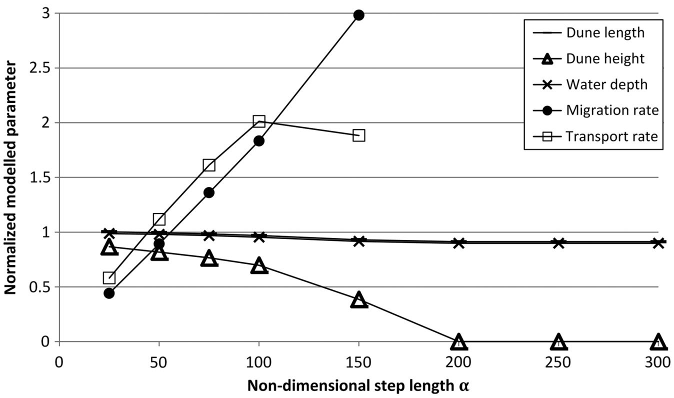

Fig. 7. Evolution of normalized modeled parameters with changing step length in the pick-up and deposition model

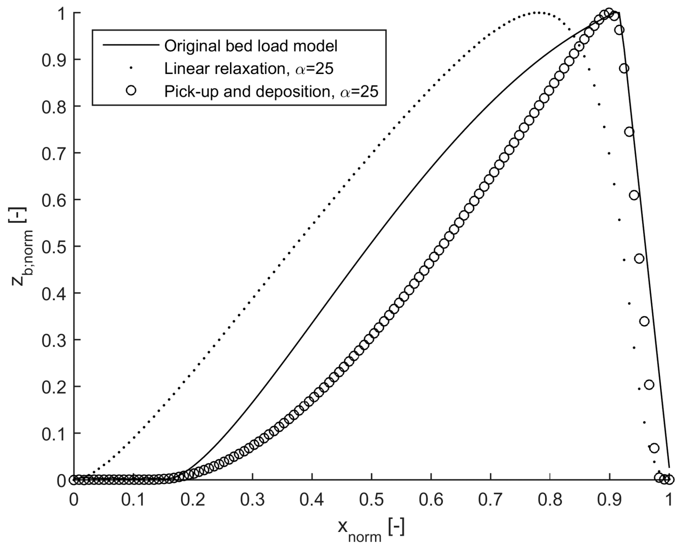

Fig. 8. Equilibrium dune shapes of the three model versions, with $\alpha = 25$ for linear relaxation and $\alpha = 25$ for the versions with pick-up and deposition (flow left to right)

measured), and better than with the original model. See Fig. 6 for the resulting dune evolution using $\alpha = 25$. Although the dunes are smoother than with the original model, they are still about as steep.

The results for Flow A are also presented in Fig. 7, where the normalized values are plotted against the step length parameter α. This again clearly shows that dune height decreases with step length, but step length does not have a great effect on water depth and dune length. Migration rate and transport rate seem to increase with step length.

For $\alpha = 25$, the migration rate and sediment transport rate are underestimated by 56 and 42%, respectively. For values of α of 50, 75, and 100, the model results deviate more and more from the experimental results regarding dune dimensions. Per increase in step length, the dune height decreases, with an underestimation of the experimental dune height of 30% for $\alpha = 100$. For the values of α higher than 100, dunes are strongly suppressed because of the smearing effect, which leads to an underestimation of the dune height of 61% for $\alpha = 150$. Starting at an α-value of 200, dunes do not grow anymore at all, i.e., the initial bed disturbance is completely washed away. The migration rate and sediment transport rate of the dunes increase until $\alpha = 100$, after which they decrease. When $\alpha \geq 200$, no more dunes are present so no meaningful migration rate and therefore no sediment transport rate can be determined. The migration rate and sediment transport rate are predicted best with $\alpha = 50$, with an underestimation of 10% and an overestimation of 12%, respectively.

Dune Shapes

The resulting equilibrium dune shapes of the three different model versions can be seen in Fig. 8, where $z_{b;\mathrm{norm}}$ is the normalized height along the dune and x_{norm} is the normalized distance along the dune. The normalized height is determined by shifting the bed-level height along each dune upward, so that the trough starts at zero, and then dividing the result by the crest height of that dune. The normalized distance is determined by dividing the distance from the trough along the dune with the dune length.

The normalized dune shapes of the three model versions show some significant differences, though they all have a general shape that is typical for dunes: a smoothed triangle, skewed toward the crest. The dune of the linear relaxation variant is quite smooth; it has gentle slopes compared to the other two and is not as strongly skewed toward the crest as the other two.

The original model with the Meyer-Peter and Müller (1948) bed-load formulation lets the lee side grow toward the angle of repose (30°) with an avalanching module that forces the lee side to migrate as one front, which causes a very sharp lee side. The pick-up and deposition version does not use this module, but the lee-side angle still grows toward the angle of repose. This is because although bed shear stress is nonzero in the trough of the dune, it is still very small. Virtually no pick-up occurs in the trough while deposition does occur, decreasing with distance from the crest. In the end this leads to a distribution of sediment in the separation zone similar to the original model. In this case the lee-side angle is almost 27°. Because the lee side develops naturally, it is somewhat smoother than the forced lee side of the original model. The results with the linear relaxation model show a gently sloping dune with a lee-side angle of about 16°.

Best Fitting Model Settings

The dune evolution model predicted the dune dimensions of Flow A best with the pick-up and deposition bed-load formulation and $\alpha = 25$ compared to the other two bed-load formulations and different values of α. With regard to the migration rate and transport rate, the pick-up and deposition bed-load formulation with $\alpha = 25$ did not give the best fit, as the results with $\alpha = 50$ were better. Because correctly predicting dune dimensions and especially dune height is more important in the context of predicting hydraulic roughness, the choice is made to use $\alpha = 25$ to model the other flows of Table 1 as well. These results are presented in Table 4.

Table 4. Model Results Using the Pick-Up and Deposition Model with $\alpha = 25$ for Flows A–E

Result	Flow A	Flow B	Flow C	Flow D	Flow E
Δ_e (m)	0.042	0.039	0.036	0.022	0.023
λ_e (m)	1.18	1.15	1.26	1.10	1.13
h_e (m)	0.17	0.16	0.18	0.16	0.16
R_e (mm/s)	0.29	0.26	0.14	0.08	0.09
$Q_{s;e}$ (kg/h)	58.1	49.9	23.3	9.4	9.9

Note: h_e = equilibrium water depth; $Q_{s;e}$ = equilibrium sediment transport rate of the bedforms; R_e = bedform migration rate; λ_e = equilibrium dune length; Δ_e = equilibrium dune height.

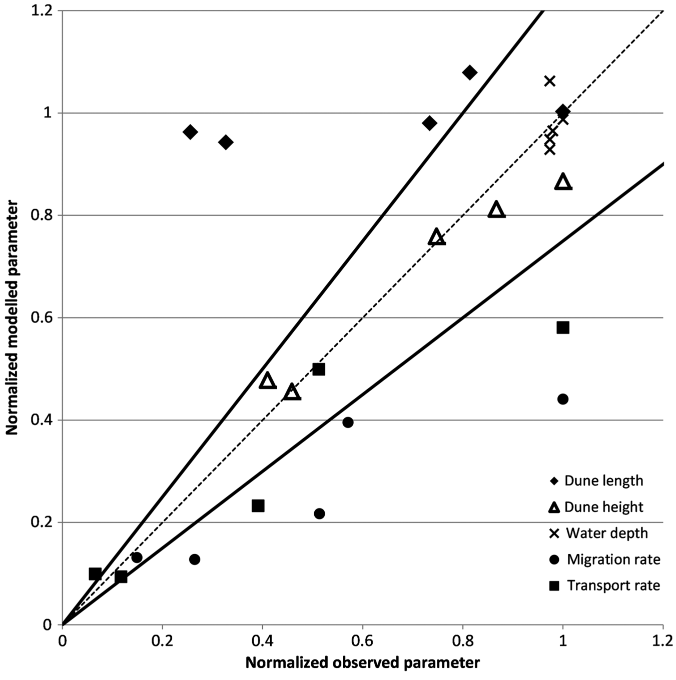

Fig. 9. Comparison of normalized modeled and observed parameters

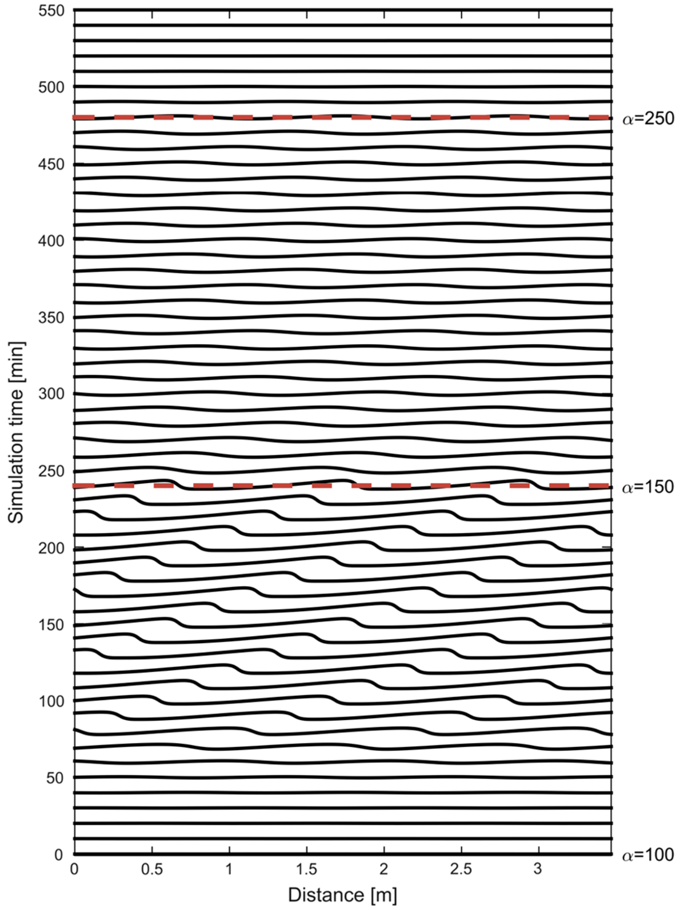

Fig. 10. Dune evolution with pick-up and deposition, starting with $\alpha = 100$, then $\alpha = 150$, and then $\alpha = 250$

The normalized experimental results are plotted against the normalized modeled results in Fig. 9, where the dashed line represents a perfect match between modeled and observed, and the area between the black lines represents modeled values within 25% of the observed value. Dune heights resulting from the model generally agree well with the experimental results. All modeled dune heights are within 0.5–17% of the experimental results. Dune length is represented less well for the flows the model has not been calibrated for, especially for Flows D and E where there are very large errors. It seems that the new model cannot well reproduce the lengths of the dunes that grew from artificially made defects (Venditti et al. 2005b), which is to be expected as the model of the present study is meant for naturally occurring dunes. For Flows A, B, and C the dune lengths are overestimated by 0, 34, and 33%, respectively. Water depth is represented well; all model results are within 10% of the experimental results. Migration rates are all underestimated, between 12 and 58%. This is part of the reason why the sediment transport rate is underestimated between 12 and 45%, except for Flow B where the sediment transport rate is overestimated by 4%.

Potential for Prediction of Upper-Stage Plane Bed

To investigate the potential of the new pick-up and deposition model for the washing out of grown dunes (i.e., a transition to the upper-stage plane bed), the model is now run with step-wise increasing α, without starting from a flat bed for each α as before. For each subsequent step length, the model is run until the bed is in equilibrium. This enables dunes to grow first, before being exposed to high step lengths. In the model computation, the two subsequent stepwise increases of step length are carried out without changing the flow boundary conditions (discharge, depth, and slope). This is not completely realistic, e.g., Sekine and Kikkawa (1992) and Shimizu et al. (2009), who showed that step length increases with increasing flow strength. The computation should therefore be considered as an indicative exercise to investigate whether and, if so, how fully grown dunes (instead of small disturbances of a flat bed) are affected by increasing bed-load lags.

The reference case is again Flow A of Venditti et al. (2005a, b). The pick-up and deposition version of the model is used, because it performs best in the dune regime (Table 4). The model run starts with $\alpha = 100$, for which dunes are expected. After equilibrium is reached, α is increased to 150, and the dunes develop to new equilibrium dimensions to become smaller and smaller with less steep lee sides. In the final step α is increased further to 250, for which the dunes should disappear completely. The results of this model run are presented in Fig. 10.

The results in Fig. 10 show how the transition to the upper-stage plane bed can occur. First dunes arise, and then they become low-angle dunes before finally washing out. The results indicate that the model has the potential to simulate the washing out of existing dunes, and not just the small initial disturbances of before. Again, the flow conditions were not changed and they correspond to the dune regime of Flow A as presented by Venditti et al. (2005a, b), so the washing out of the dunes is solely caused by the increased step length (and thereby spatial lag). The computation shows that pick-up and deposition processes in general can directly contribute to the washing out of fully grown dunes.

Discussion

Both the model with linear relaxation and that with pick-up and deposition have the potential to simulate a transition to the upper-stage plane bed. With both new bed-load formulations, the model is able to completely wash out the small initial disturbance with certain constant step lengths. It was furthermore shown that pick-up and deposition processes are able to wash out *fully grown* dunes, by increasing the step length.

However, to model a transition in a more realistic way, it may be necessary to vary the step length with changing flow conditions automatically. For example, this can be done with the conceptual model of Shimizu et al. (2009) or the step length model for bed load over a plane bed of Sekine and Kikkawa (1992). The Sekine and Kikkawa (1992) model depends on friction velocity u_*, settling velocity w_s and the critical friction velocity u_{*c}. Therefore it depends inherently on the flow strength and the sediment diameter as well. The model is based on numerical and physical experiments regarding bed-load movement over a plane bed. This means that the total friction is caused by the sediment particles themselves, as there is no bedform to cause form drag. The step length model of Sekine and Kikkawa (1992) is presented in Eq. (19), where α_2 equals 3,000

$$\alpha = \frac{\Lambda}{D_{50}} = \alpha_2 \left(\frac{u_*}{w_s}\right)^{3/2} \left(1 - \frac{u_{*c}/w_s}{u_*/w_s}\right) \qquad (19)$$

Shimizu et al. (2009) used the minimum and maximum value of nondimensional step length α measured by Nakagawa and Tsujimoto (1980) to derive a relation between α and dimensionless grain shear stress θ'. Engelund and Fredsøe (1974) defined the grain shear stress as the part of the shear stress that is attributable to friction caused by the sediment particles themselves, and therefore not attributable to form drag. The values of θ' that determine the transitions between the various regimes are derived from the work of Engelund (1966). For values of θ' between 0 and 0.5 (the dune regime), α is constant at the minimum value of 50. For values of θ' above 0.8 (the upper-stage plane bed regime), α is constant at the maximum value of 250. For values of θ' from 0.5 to 0.8 (the transitional regime), α is linearly interpolated between 50 and 250 based on θ'. There is no further dependency on sediment parameters. In the present study, the step length parameter is lower at the regime transitions shown in the model results: for $\alpha = 50$ dunes are already (slightly) lower than their maximum value (at $\alpha = 25$), and they are not able to grow at all for $\alpha = 200$. This means that the relation between step length and regime transitions in the step length model of Shimizu et al. (2009) is reflected by the dune model of the present study to some extent.

Both methods, i.e., Shimizu et al. (2009) and Sekine and Kikkawa (1992), are consistent in the sense that step length increases with flow strength (friction velocity, grain shear stress). However, the methods model the behavior of nondimensional step length with regard to flow parameters in significantly different ways and it is not obvious which of the two works best for dune conditions. The Shimizu et al. (2009) step length model works well within their dune evolution model, but because of its conceptual nature care should be taken when applying it in other morphological models. The Sekine and Kikkawa (1992) step length model was validated with experimental results, but describes sediment motion along a plane bed, which is considerably different from sediment motion along a dune. It would be valuable to determine which of these methods, or at least which concepts of each method, can be generally applied with good results in the context of (idealized) dune evolution modeling.

A question in that regard is whether α should be varied only because of the changing flow regime, or along the dune as well because of local variation in shear stress. From experimental results, Van Duin et al. (2012) found that mean step lengths in the trough of a dune may be very similar to mean step lengths at the crest of a dune, which suggests that variation along the dune is very limited. A possible explanation is that although the turbulence-averaged bed shear stress in the dune trough is lower, the extreme turbulent events (e.g., attributable to flow reattachment) are much stronger. The mean step lengths therefore become more or less the same along the dune, which implies that it is probably adequate to vary step length as a result of the changing flow regime but to keep it constant along the dune. The effect of a variable α is most pronounced under changing discharge, so the effect of using different possible models for step length is most important in that context and requires further study.

High values of α will lead to the washing out of dunes within the model of Paarlberg et al. (2009), with the two newly implemented bed-load models. As mentioned before, this was achieved for hydraulic conditions where dunes were actually present in the corresponding experiment of Venditti et al. (2005a, b). This shows that spatial lag in bed-load processes by itself can directly trigger a transition to the upper-stage plane bed. However, further study is required to determine if the dune evolution model can also model an actual transition to the upper-stage plane bed. Firstly, the model will need to be able to (correctly) determine the appropriate step length for the varying hydraulic conditions as mentioned before. Secondly, it is unknown if in reality the spatial lag attributable to bed-load processes is indeed the driving factor of such a transition. Shimizu et al. (2009) stated that the modeling of that spatial lag was essential for their model to be able to model a transition to the upper-stage plane bed, and the effect of that spatial lag was very pronounced in the present study. This makes it likely that spatial lag in bed-load processes at least has a significant role in the transition to the upper-stage plane bed.

Another process contributing to the transition to the upper-stage plane bed is suspended transport, as it also causes lag between shear stress and the transport rate. This was not taken into account in the present study, as the modeled step lengths all fall in the bed-load range: they are based on experiments that only considered bed-load transport (Nakagawa and Tsujimoto 1980). Suspended sediment can make larger steps than those found for bed load. Based on the obtained experience with regard to incorporating the effects of spatial lag in bed-load processes, it is suggested to further investigate the use of a similar model concept for suspended transport. Because it does not require a complex hydrodynamic model and has a relatively small computation time, this method provides an efficient way to incorporate the effects of spatial lag in suspended transport processes as well. Using a relaxation distance variant for the suspended transport model or using a combination of pick-up-as-bed-load, pick-up-as-suspension, and deposition models would be an interesting approach. With such a method, it is important to be able to determine the appropriate range of step lengths for each mode of transportation.

Conclusion

The dune evolution model of Paarlberg et al. (2009) was chosen for this study because it fits the criteria for application within a flood management modeling framework: it is computationally cheap and works well within the dune regime. With this model three bed-load models were tested: (1) a formulation like that of Meyer-Peter and Müller (1948) as in the original dune evolution model, (2) a formulation like that of Meyer-Peter and Müller (1948) with a simple linear relaxation based on the step length [as proposed by Tsujimoto et al. (1990)], and (3) the Nakagawa and Tsujimoto (1980) pick-up and deposition model.

The first research question was how the two new model versions compared to the original. It was shown that the resulting dune morphology (dune height, length, and general shape) significantly depends on the bed-load transport formulation used. The dune shapes with all bed-load models differ, but all have the typical shape of dunes, namely a smoothed triangle, skewed toward the crest. The new models differ with respect to the original in terms of reduced lee-side angle and increased smoothness (especially for the linear relaxation variant). The version with the pick-up and deposition model with step length parameter $\alpha = 25$ gives the best agreement with a series of measured dune dimensions in the bed-load regime (Venditti et al. 2005a, b). The second research question was what the prospects of modeling a transition to upper-stage plane bed are with the two new bed-load models. Both new models show their potential to simulate the washing out process of small initial bed disturbances. The model version with the pick-up and deposition model was chosen to show that it is capable of washing out fully grown dunes as well by increasing the step length.

Further research and model development is needed to simulate the transition to the upper-stage plane bed during flood waves. The time dependence of the step length parameter α with varying flow strength should be further investigated, as well as the influence of suspended transport lag processes.

Acknowledgments

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