AD QUANTA INTELLIGENDA CONDITA (DESIGNED FOR GRASPING QUANTITIES)

E.J. Dijksterhuis

[*Translator's note:* The address *Ad quanta intelligenda condita*, delivered by Dijksterhuis in 1955, is written in Dutch, whatever its title might otherwise suggest. It is held by many to be the finest piece Dijksterhuis ever wrote – which is no mean statement to make about the author of *The Mechanization of the World Picture*, of *Archimedes*, of *Simon Stevin*, and of many other books and articles, most of which have remained locked inside the Dutch language. In the address Dijksterhuis takes up in concentrated form a key topic that runs all across his work in the history of science: the mathematization of nature; what precisely is meant by that expression, and how the essence of the process can be grasped through the study of its manifestations in the past. Dijksterhuis seeks to penetrate to the core of the issue by asking how mathematics and science can be connected at all – how is it that a description of the empirical world can have mathematical precision as one key attribute.

This is not the place to celebrate Dijksterhuis' unique achievement in the history of science or to sketch his life and works. Professor K. van Berkel is currently preparing a biography (in Dutch), and I have a section on Dijksterhuis' conception of the Scientific Revolution in my forthcoming book *The Banquet of Truth*. So let me just give the briefest of outlines. Eduard Jan Dijksterhuis was born in 1892. He studied science and mathematics, which disciplines he taught to high school students until 1953. In that year he was appointed to a newly created chair at Utrecht University as 'extraordinary professor' in the history of science and mathematics. Two years later he was in addition appointed to a similar chair at Leyden University. *Ad quanta intelligenda condita* is the inaugural address he gave there on Friday, 21 January 1955. When, in 1960, Utrecht University converted his chair into an 'ordinary professorship' he gave up his Leyden chair, partly for reasons of health. He retired in 1963, and died two years later.

The bulk of the Dijksterhuis archival collection is located at the Museum Boerhaave in Leyden. A detailed inventory of this collection has been available since 1984.

The original text of *Ad quanta* has been fully translated here, with the sole exception of some ritual solemnities at the beginning and the end. I thank Mrs. J.K.E. Smit-Dijksterhuis and Meulenhoff Publishers for permission to prepare a

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translated version, and Steven Engelsman, Harry Leechburch, and Joella Yoder for their kind help. I hope very much that, besides providing a faithful rendition of Dijksterhuis' expressed meaning, my translation manages to convey some faint idea of the sculptured beauty of Dijksterhuis' literary prose, which in 1951 won him the highest Dutch literary prize.

H. Floris Cohen]

The history of the exact sciences, which I have been assigned to teach at this University, comprises the historical development of mathematics and of the exact sciences, that is to say, of those natural sciences which can be treated with the help of mathematics. In this definition we find mathematics and science juxtaposed, but in such a way that the latter concept is limited in its scope by the former. By itself, the juxtaposition might suggest a purely accidental link between the two – after all, it is not inconceivable for one teacher to teach two different, mutually independent disciplines. Yet the limitation of the one concept by the other makes it clear that an essential connection is intended here.

This gives rise to a sense of wonder. If mathematics is a free creation of the human mind, whereas science deals with a reality that is there without man's doing, then we should be amazed to find the two disciplines coupled in an essential way.

True, today this amazement is rarely experienced anymore. Accustomed as he is from early on to the combination of words 'science and mathematics', present-day man has undergone here, too, the dulling effect convention always exerts on our capacity to be amazed by things.

To inquire into the nature of the connection between mathematics and science is a matter of epistemology (in the sense of the theory of science). Thus, in principle, the question seems answerable by studying both disciplines in their present-day guises. However, experience teaches us the virtual impossibility of taking up a given science and studying it solely as a phenomenon of our own time. Again and again questions will arise about its historical origin, about the roots in the past of its problems, its methods, its concepts, its terminology. This is why in every epistemological inquiry the history of science has a contribution to make. Let us then ask, in the present hour, what message the history of science has in store concerning the relationship between mathematics and science.

For a response, we turn to where every inquiry in the history of science takes its inevitable point of departure – the civilization of ancient Greece. There we find, in the Pythagorean school, four *mathemata* – arithmetic, geometry, music, and astronomy. Here already we notice both an opposition between science and mathematics and a close connection. Arithmetic has been an independent science from an early day onward. It is no longer in need of appealing consciously to sense experience; therefore, it may already be called pure mathematics. Music and astronomy, which deal with musical tones heard and celestial motions seen, respectively, are equally indubitably natural sciences. With regard to geometry doubts are possible. Is it still that part of natural science that treats of the formal properties of solid bodies? Or is it already that part of mathematics which, surely, finds its psychological origin in empirical evidence taken from such bodies, but no longer uses such evidence in a conscious manner for its subsequent construction? Proklos' statement[1] that Pythagoras taught that geometry is studied *auloos kai noeroos*, that is to say, immaterially and noetically, through thought alone, indicates that already in the Pythagorean school geometry had ceased to be geodetics, its one remaining link with tradition being the preservation of those many terms, such as 'squaring', 'applying', and 'adding', that derive from its empirical origin and were ridiculed later by Plato.[2]

So there were two mathematical and two scientific *mathemata*; however, the latter relied heavily on the former. The *mathema* 'music' consisted of a doctrine of intervals which sought the essence of what the sense of hearing observes as a difference in pitch between two distinct tones in a numerical ratio such that the addition of intervals meant the multiplication of ratios. In astronomy both geometry and arithmetic found their application, the former in devising systems of motion which made it possible to render account of the observed phenomena; the latter in the consideration of distances and times of revolution of celestial bodies, giving rise to cosmological speculations on the harmony of the spheres and on the periodicity of events in the world.

As geometry was increasingly treated noetically, more and more attention was inevitably drawn to the fact that the geometrical entities on which generally valid, exact, and apodictic judgments were being passed could not be identical to the objects of sense experience that carry the same names – the straight lines, circles, and spheres shown to us by experience are different from those called thus by geometry.

The question of what realm of being these mathematical entities may belong to is answered in Plato's philosophy[3] by the statement that they possess an ideal existence which shares with the mode of being of the ideas the properties of eternity and of immobility, being distinguished from the ideas only in that it lacks the property of unity. The relation between the realm of the ideas and the realm of geometrical form is elucidated[4] by comparing it to the relation between physical bodies and their shadows or mirror images. This relation, in its turn, is the same that obtains between geometrical forms and the physical bodies that carry the same

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1 *Procli Dadochi in primum Euclidis Elementorum librum Commentarii*, ed. G. Friedlein (Leipzig, 1873), p. 65, 18.

2 Plato, *Politeia* VII, 527A.

3 This is what Aristotle has to say about the *metaxu* position of mathematical entities in *Metaphysica* A 6, 987b16-18.

4 Plato, *Politeia* VI, 509D-510E. Comments by Proklos, edition cited in note 1, 10, 21-11, 9. See further E.J. Dijksterhuis, "Proklos' Commentaar op *Euclides* I vertaald," *Euclides* 25, 1949-1950, p. 47.

names, e.g., between a sphere and a bouncing-ball.

It is important to distinguish carefully between two elements in the Platonic conception of the relationship between the mathematical and the physical worlds. The first of these is the notion of a correspondence between the physical objects we observe by means of our senses, and the geometrical forms that carry the same names, whose properties are determined noetically. The second element is the conviction that the realm of geometry exists in its own right; that it is separate and independent from the realm of physical bodies, and that it is just as independent from the latter as the realm of ideas is from the realm of geometry, or as a physical body is from its shadow. Moreover, the sequence of idea – geometrical form – physical body – shadow also indicates an ongoing descent in degree of reality: a geometrical form surpasses, in its amount of reality, a physical body just as much as the latter does to its own shadow. It is important to insist on this distinction because the other great Greek thinker beside Plato to influence the making of the mental history of the Western world in a decisive manner, Aristotle, adopts the first of these two conceptions and supports it with his authority, but rejects the second. For him the realm of the ideas is not transcendent, nor, *a fortiori*, is the realm of mathematics. The properties treated by geometry are properties of physical bodies, provided these are considered from a particular, namely, a geometrical point of view, while barring for the moment all other possible modes of consideration.

For present purposes the primary thing is not that Aristotle does not accept Plato's mathematical ontologism, that is to say, the doctrine of the independent existence of geometrical forms. More important for our purpose is that his conception of a correspondence between exact geometrical forms and physical objects in which the blurred forms manifest themselves, concurs in practice with Plato's conception. If one wishes to emphasize the points of difference one may indeed oppose the former, as a theory of abstraction, to Plato's theory of idealization. However, one should realize that both lead to exactly the same result, namely, those very same ideal geometrical forms. Aristotle's abstraction is the means to achieve Plato's idealization. To say that we abstract from everything in respect to which the empirical forms deviate from the ideal, geometrical forms, implies the acknowledgment that those ideal forms do exist, whether in one mode or another. The remaining difference of opinion concerns the mode only: transcendent with Plato, immanent with Aristotle.[5]

There are more indications that these two conceptions are indeed quite close. The axiomatic structure of geometry which we find in Euclid may, on the one hand, easily be interpreted as the realization of certain epistemic ideals of Plato, whereas, on the other hand, it clearly serves as the prototype after which Aristotle modeled the reflections he devotes in the *Analytica Posteriora*[6] to the essence of demonstrative science.

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[5] Aristotle, *Metaphysica* M 2, 3.

[6] See for evidence H. Scholz, "Die Axiomatik der Alten," *Blätter für deutsche Philosophie* 4, 1930-1931.

So far our discussion has remained fully inside the boundaries of mathematics, but is it not clear that it tends to overstep those limits and to enter the field of the natural sciences? If the world of quiescent, empirical forms stands in correspondence with a noetic realm of ideal, mathematical forms, are we not then very close to the suspicion that the changes that occur in the former stand in correspondence with processes in the latter that lend themselves equally to noetic treatment? The conception of a mathematical physics in the literal sense, that is to say, of a physics to be treated as mathematics, could not fail to press itself with equal force upon both Platonists and Aristotelians. The former could extend this conception by thinking that the mathematical system to be construed represented nature proper, of which the physical world experienced by our senses is no more than an imperfect, material imitation; they could find how, in his *Politeia*,[7] their master had emphatically defended such a conception in the cases of the Pythagorean *mathemata*, music and astronomy, and they could establish how fully the conception developed there agreed with the cosmology of the *Timaios*.

The shared, Platonic-Aristotelian view of the connection between mathematics and natural science is put into practice in exemplary fashion by Archimedes. In the first book of his work *On the Equilibrium of Planes*[8] he discusses the condition for equilibrium of a pivoted beam loaded with weights. However, this is not done by means of experiments and measurements, as might be expected with such an essentially physical problem. Rather, he handles it as if he were dealing here with a purely mathematical issue – he formulates certain self-evident axioms (for example, that with a symmetric load the beam remains in equilibrium) followed by deductive reasoning. In fully the same manner, namely, by starting from an axiom that expresses certain fundamental notions of Aristotelian physics, in his work *On Floating Bodies*[9] he derives through mathematical reasoning the theorem – still named after him – on the buoyancy undergone by a body immersed in a fluid.

Once more Custom, the great damper, stands in the way of a spontaneous realization of how odd these facts are. During our school years, when all of us pick up the basic principles of mathematics and science, and countless people reach the optimum level of their knowledge in these disciplines, we are not as a rule taught sufficiently to distinguish between the two. This blurring of an essential difference prevents a sense of wonder to arise over the apparent possibility of treating science in a deductive manner.

Does this possibility exist indeed? Do Archimedes' derivations really deal with true physical balances and with material bodies immersed in real fluids? One moment of critical inspection of the argument suffices to see at once that this is not the case. In *On the Equilibrium of Planes* (and we confine ourselves to this particular work here) we are not told about a material beam that is made to turn in the air around a material axis supported with friction, and from which material

[7] Plato, *Politeia* VII, 529D-530C; 530E-531C.

[8] Archimedes, "De planorum aequilibriis sive de centris gravitatis planorum," I, 6, in *Archimedis Opera Omnia cum Commentariis Eutocii*, ed. J.L. Heiberg, vol. 2 (Leipzig, 1913).

[9] Archimedes, "De corporibus fluitantibus," edition cited in note 8, vol. 2.

bodies are suspended, but rather about a mathematical line segment, assumed to be pivoted, on which plane figures are mounted, while to those figures certain numerical values are assigned which are proportional to their areas and are called their weights.

The same ideal sphere reigns here as we find ourselves in when we are doing geometry, and in this sphere alone the derived condition for equilibrium is fully valid. But – and this is the remarkable thing – the condition is happily applied to physical balances. In so doing, nothing but the precision with which the condition obtains appears to get lost – equilibrium now appears to manifest itself in a wider domain of values than those derived theoretically. That this is so, is ascribed to such circumstances as friction and resistance of the air, which disturb the ideal phenomenon. It appears to be possible to include these disturbing factors in the mathematical picture, but, in order to do so, these have to be idealized as well, and the situation is not principally altered thereby. It remains the case that we have depicted a natural phenomenon in a mathematical system with conclusions possessing validity in physical reality. But the validity is never more than approximate; the exactitude of the picture can never be found back in experience.

The application of mathematics to mechanics and hydrostatics is still far from giving a complete impression of the function mathematics can fulfill in science and, in antiquity, did fulfill already. Exploring the usage made of mathematics in Greek astronomy widens the picture further. Here, too, the observed phenomena were idealized. Replacing celestial bodies with mathematical points is an obvious example. However, here it was not possible to find out about the behavior of such points by deductive reasoning starting from self-evident axioms. To begin with, observed quantitative facts had to be taken into account. Moreover, one felt bound by certain *a priori* assumptions which rested on religious, aesthetic, or scientific grounds, and the facts had to be interpreted in their light. For example, it was accepted as an axiom that celestial bodies cannot carry out other than circular and uniform motions. Hence arose the task of reconciling the irregularities observed in the motions of the heavenly bodies with this so-called Platonic axiom.

This task, known as the Platonic problem, was carried out most completely by the astronomer, Klaudios Ptolemaios. For our purpose it is of importance to give a general formulation of the method he applied to achieve his aims. To do so, we consider how it worked in a simple case, namely, the motion of the sun. It was well-known that the seasons are not equally long, even though during each the sun passes through a 90° arc amidst the stars. In Ptolemaios' time the duration of spring was 94.5 days, and of the summer, 92.5 days. So as to reconcile these facts with the Platonic axiom Ptolemaios assumed that an observer on earth is not at the center of the uniform, circular orbit of the sun. By trigonometric means he determined the distance from the observer to the center of the orbit of the sun and also the position of the diameter on which it lies in such a way that indeed spring took 94.5 days, and summer 92.5. Since observations allowed him to derive as well where the sun had been at a given moment in the past, and through what arc it passes uniformly in a given period, he could calculate and predict for any arbitrary moment in the future where in the sky the eccentric observer will see the sun, and test the theory by comparing solar positions as computed with those observed in reality.

Now what has happened here? Using a certain amount of empirical material formulated in mathematical form (the duration of spring and summer), and taking into account certain general principles (the Platonic axiom), a hypothesis has been framed on how the observed phenomena truly fit together (the observer has an eccentric position). The hypothesis is construed in such a way that 1) the facts that gave occasion to framing it in the first place can be derived from it (obviously, this is the minimal requirement) and 2) it leads to predictions (solar positions at given moments in the future) which can be checked through new observations. If we employ a not fully satisfactory terminology, which already came up in antiquity and later came into general usage, we may divide the entire process into a part marked by resolution, which ends with the framing of the hypothesis, and one marked by composition, which consists of a twofold deduction from the hypothesis and of the empirical verification of the predicted facts.

It is important to note that in the two parts of the thought process the hypothesis performs a different function. In resolution it appears as a conjecture, in composition as the starting point for a deduction – as an axiom, that is. The former function corresponds to present-day terminology, the latter to Greek usage. To our ear, the word 'hypothesis' always contains a strong shade of uncertainty. In Greek, *hupothesis* (hypothesis) signifies the assumption taken as the foundation of an argument, the requirement for the assumption being that the audience accepts it. Thus it is simply synonymous with the word 'axiom', in which such a requirement is similarly expressed.

For our special purpose it is useful to point out those stages in the process where mathematics is of service. This happens 1) when, in resolution, empirical facts are formulated in mathematical form; 2) when the hypothesis framed has a content that can be expressed mathematically; 3) in the dual deduction to be found in composition.

It is clear that the truly creative element of the entire process consists of the framing of the hypothesis. Here, it should be stressed, one must not speak, as occasionally happens in slipshod discourse, of a deduction from phenomena. The point is not what follows from phenomena, but rather to find something from which they follow. One cannot possibly derive from the data about spring and summer that the observer has an eccentric position. While fully respecting the Platonic axiom one may equally well render account of the data by supposing the observer to be at the center of a circle orbited not by the sun but rather by the center of a smaller circle (a so-called epicycle) uniformly traversed by the sun.

Framing an hypothesis always contains an element that cannot be rationally accounted for. Only such terms as *nous*, *virtus intellectiva*, intuition, imagination, inspiration, may begin to define it. This is why, in the investigation of nature, not even interest, diligence, goodwill, accuracy, technical proclivities, and a talent for mathematics satisfy all requirements. A specific power is required that, in its highest form, is given only to a few blessed ones.

So far we have acquainted ourselves with the schema of the method of resolution and composition (better: of the hypothetico-deductive method) by means of mathematical astronomy as an example. But one can easily see that it is valid for all exact sciences, and that these are distinguished from one another only by differences in the relative scope and the relative importance of one or another part of the process. Thus, when geometry turned from a natural science into a branch of mathematics, the collection of theorems (whether acquired through measurement or through guesswork or in any other way) that the Greek mathematicians could take for known when their activities began, served as the empirical material from which to start when applying the method. The Euclidean system of definitions, postulates, and axioms similarly formed the hypothesis. Deductively, both the premises and new geometrical propositions were derived, and, because of the noetic character the discipline soon acquired, the Pythagoreans already ceased to feel any longer the need to check such propositions empirically. In the statical and hydrostatical investigations of Archimedes discussed by us, the starting material consisted of what, through experience of long standing, had been learned about the action of levers and the conduct of bodies immersed in fluids, which material was given shape in a few axioms felt to be self-evident. From the hypothesis new propositions were deduced – for example, numerous theorems concerning the stable floating positions of paraboloids of revolution. However, Archimedes apparently did not feel the need to verify these propositions empirically, even though, for us, it would not be amiss in a case like this to require such a verification.

It appears further that, over the course of time, the hypothetico-deductive method not only has been indeed continuously applied implicitly, but also has been explicitly investigated. In antiquity Galenos devotes attention to it; in the 13th century Robert Grosseteste and others display great interest.[10] Grosseteste in particular urges the necessity of checking the tenability of the hypothesis that has been established through empirical verification or falsification of its consequences. Naturally the *a priori* principles in light of which the hypothesis is conceived may differ from one case to the next. For Grosseteste and numerous later investigators of nature these principles include a belief in the uniformity of nature's modes of operation, together with the related *lex parsimoniae*, which states that nature does everything in the simplest manner possible, so that no more explanatory principles should be introduced than are strictly necessary. Still later, the method is extensively examined in the philosophical curriculum at the University of Padua, so that, when Galileo expounds the method explicitly in the early 17th century, he does nothing but carry on a tradition of long standing at the institution to which he belongs.

The enduring interest displayed in the method has not at all times been able to effect an equally fruitful application. Before the sixteenth century the harvest of scientific results remains tiny, and its first accretions since come slowly. In principle, one knew quite well how to cultivate science, yet the practical application of

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[10] A.C. Crombie, *Robert Grosseteste and the Origins of Experimental Science* (Oxford, 1953).

the method met relatively rarely with success. The explanation of this remarkable phenomenon is still a primary problem in historiography. It belongs to a class of cognate questions, all of which come down to our desire to understand how it is that a certain development did not take place at a certain period of time when, so it seems to us in retrospect, it might very well have, and why at a later stage it suddenly did. The question may also be clothed in the form of asking whether there exists indeed a specific speed of growth for a specific science and, if so, by what it is determined.

Responses to such questions must undoubtedly take into account the cooperation of a great number of factors, part of which are of an internal-scientific nature, whereas others are socio-economic and political-historical. Whoever succeeds in pointing at some of these factors should never forget that he is always leaving a greater number out of consideration.

Keeping this restriction in mind, we shall enumerate here some circumstances which undoubtedly have slowed down the development, during the Middle Ages, of science treated in the mathematical-empirical way. To begin with, there was the underestimation, inherited from Greek thinkers, of the difficulty of the task confronted in the investigation of nature. This underestimation found expression in the establishment of hypotheses without possessing empirical material, acquired through observation and experiment, of sufficient scope and accuracy. Further, Aristotelian physics, which was generally accepted, was predominantly qualitative. It did not, to be sure, exclude the execution of physical measurements as a matter of principle (for attention was being paid to changes in the intensity of certain qualities), but it certainly did not elicit those, either. Next, there were the difficulties connected with the creation of a method of experimentation. But above all – and this is of importance for our particular topic – both algebra and practical computation had been developed only a little, and both still suffered from the consequences of the exclusively geometrical character Greek mathematics had acquired under Pythagorean-Platonic influence. Not until the 17th century did these two branches of mathematics reach a level that put them on a par with geometry, and even then it took a long time until it was learned how to profit from their further development.

Progress on the other points we have mentioned was doubtless faster and clearer. Even though it took much effort and much struggle, success was met, in the domains of physics and, later, of chemistry too, by liberation from the secular influence exerted by Aristotelian thought. A better awareness arose of the richness and the complication of the phenomena of nature. Between natural science and technology a connection began to be made of which both were to profit greatly in centuries to come.

These and similar causes led to a more rapid unfolding of science in the 16th and 17th centuries. It is remarkable how neatly, after an interim of so many centuries, this unfolding took up the developmental stage at which the Greeks had left science. As the first representative of the new science Copernicus resumes the cultivation of astronomy in fully the same spirit as the ancients. True, he alters the standpoint from which to consider the celestial phenomena, but the method applied remains the same. He even takes pride in a stricter compliance with the Platonic axiom than Ptolemaios had displayed. At the end of the 16th century Stevin ties his researches in statics and hydrostatics directly with Archimedes. He defines the 'weeghconst' ('art of weighing'), that is to say, the theory of equilibrium, as "a distinct, free branch of mathematics."[11] Thus the axiomatic treatment he provides of the statics of a solid with one point immobile runs entirely within the Archimedean, that is to say, the Euclidean framework. He appears fully aware of the ideal character of the system thus constructed. Repeatedly[12] he emphasizes the difference between what is valid 'when taken mathematically' and what is true 'when understood naturally', that is, in physical reality. The celebrated 'wreath of spheres' proof[13] of the law of equilibrium on an inclined plane is conceived so fully to take place in an ideal domain that not one word is devoted to the apparent impossibility of physically realizing the situation described. Now, just like his great predecessor from Syracuse, Stevin was not only given to reflection, but to action as well. Therefore one might expect to find him paying attention to the question of the extent to which physical reality is to deviate from ideal theory. Curiously enough, this expectation is hardly fulfilled. To be sure, Stevin supplements his 'Art of Weighing' with a 'Practice of Weighing',[14] where the theory explained in the preceding treatise is applied in practice. However, the machines discussed in the latter are equally kept in the domain of the ideal as is true of the windmill for which, in a separate work,[15] he develops a quantitative theory – the first one in history. True, Stevin repeatedly calls attention to the fact that, in dealing with real machines, things do not quite so nicely fit the theory taught in the 'Art of Weighing'. The impression gained, however, is that Stevin seriously underestimates the extent of the deviation.

Galileo treats the phenomena of free fall and of projectile motion in the same manner as Stevin had treated statics.[16] Leaning on the *lex parsimoniae* he posits as an axiom that the speed of a body freely falling from rest is proportional to the time passed since motion began. From this axiom he derives mathematically a relation between distance and time. Another axiom enables him to treat fall along inclined planes as well. This puts him in a position to verify the law of fall empirically – a verification that is carried out in fact. However, from that point onwards the entire theory of fall and of projectile motion is further built up as a fully idealized construction. Experimental verification of the theory of projectile motion developed by him is not to be carried out until later, by the Accademia del Cimento. Barring this, though, one may say that Galileo, entirely aware all the time

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[11] Simon Stevin, *De Beghinselen der Weeghconst* (Leiden, 1586), *Anhang*, Hooftstick III: "Dat de Weeghconst een besonder vrie Wisconst is."

[12] Thus, for example, *Weeghconst* I, Vertooch 4, Voorstel 8.

[13] *Weeghconst* I, Vertooch 11, Voorstel 19.

[14] Simon Stevin, *De Weeghdaet* (Leiden, 1586).

[15] Simon Stevin, *Van de Molens* (Amsterdam, 1584).

[16] See for evidence E.J. Dijksterhuis, *Val en worp. Een bijdrage tot de ontwikkelingsgeschiedenis der Mechanica van Aristoteles tot Newton* (Groningen, 1924), chapter 4.

of what he is doing, applies the method of resolution and composition to the full.

Once again, however, the method manifests itself in its purest form when applied to the discipline that, from antiquity onwards, had been the exemplar of method in the investigation of nature – astronomy. This is particularly true of Kepler's researches into planetary motion. After years of trial he feels compelled to acknowledge that, when the empirical data assembled by Tycho are treated in accordance with the Platonic axiom and with the methods of Ptolemaean astronomy, a difference of eight minutes of arc remains between, on the one hand, the consequences of the hypothesis established and, on the other, certain measurements that had not previously gone into the making of the original hypothesis.[17] Having come that far, Kepler is the first to risk the rejection of the Platonic axiom and to introduce new mathematical tools for the representation of planetary motion. In the laws that bear his name he generalizes the Platonic circle into an ellipse, with constant linear speed being equally generalized into constant areal speed.

As the development of mathematical science progressed further, the question of the relation between physical reality and the mathematical picture that theory juxtaposes to it could be asked with ever more justification. In antiquity the question had concerned Euclid's geometry, Archimedes' mechanics, and Ptolemy's astronomy. Now the phenomena of fall and projectile motion, and soon those of circular motion, impact, heat, and optics, too, are involved as well. In the Middle Ages the old controversy over the relation between idea and physical reality had taken shape as the problem of universals. It is now revived as the question of the true meaning of the mathematization of natural science. And again, the three possible responses by means of which the problem of universals had by and large been resolved now presented themselves once more. The response that corresponds to the Platonic standpoint of the *universalia ante rem* ('universals prior to things') is the conception of nature as an imperfect realization of an ideal, and in this case a mathematical, world of thought. What corresponds to the Aristotelian standpoint of the *universalia in re* ('universals in things') is to conceive of an idealization that originates in the abstraction from experience. Finally, the counterpart to the nominalist standpoint of the *universalia post rem* ('universals posterior to things') is the conception of a practical tool that is of help in achieving an approximate description of reality. Note that the description leaves out everything that is as yet too difficult for thought to grasp in the given situation, but also whatever need not be taken into account in order to attain the aim one has set oneself. The key point is that no independent significance whatsoever may be attributed to the description.

It is unlikely for all those investigators who together, in the course of the 17th century, laid the foundations for the later flowering of science to have adopted a definite standpoint in face of these questions. One of the very greatest, however, Kepler, did give an unambiguous answer. Adopting and elaborating the key idea of Plato's *Timaios*, Kepler explains that God allowed Himself to be guided, in creating

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[17] Johannes Kepler, *Astronomia Nova* II, 19, in *Johannes Keplers Gesammelte Werke*, ed. Max Caspar, vol. 3 (München, 1937), p. 178.

the world, by mathematical considerations in that He kept in mind certain *logoi kosmopoietikoi*, that is, world-building ratios.[18] Furthermore, He created the human mind such as to enable it to discern quantitative relations. This is in fact its proper function. Just as the eye is geared to seeing colors, and the ear to hearing sounds, just so the human intellect is *ad quanta intelligenda condita*, that is to say, designed for grasping quantities.[19] Therefore, the mathematical system that we established with the observed phenomena as our point of departure not only contains everything in nature that it has been given to man to acquire knowledge of, but we may also rest assured that, in the very act of establishing the system, we may glimpse the divine plan of creation. The human intellect is God's spiritual image, just as the world is His material image. It is on this that man's ability to cultivate science rests.

Thus, just as for Plato, there exists for Kepler an ideal world which possesses a higher reality than the empirical world in which that reality is materially imitated. For Kepler, too, the function that experience fulfills in the investigation of nature is to give us occasion to reflect on a cognition already possessed by our thought in consequence of its divine origin.

In the 17th century the sciences achieved great progress through the application of the hypothetico-deductive method, that is to say, through a harmonic collaboration of empirical research and mathematical systematization. Given this, it is odd to find that, as an expression meant to characterize the method, the term 'experimental method' increasingly becomes the fashion, since only one of its two components is indicated thereby. This is probably a symptom of the powerful influence Francis Bacon's brilliantly formulated reflections on the method of science exerted on scientific thought in the 17th century and far beyond.[20] In those reflections the empirical element is emphasized in a quite one-sided manner, whereas the indispensable function of mathematics is ignored. The listing procedure he recommends for the systematic collection of empirical facts is not, accordingly, more than a most questionable impoverishment of the sole method that was to appear effective. Here not only the services of mathematics are disowned, but also the creative accomplishment expressed in establishing a fruitful hypothesis. Remarkably, a similarly incomplete nomenclature was adopted in the 19th century, when the philosophy of science once again displayed strongly Baconian features, and scientific method was frequently defined as empirical-inductive.

After the hypothetico-deductive method yielded the great gains to 17th-century science that are comprised in the works both of the investigators mentioned here and of many others, at the end of the century the method appeared susceptible of a considerable enlargement of its scope. Up to then a strict require-

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[18] Johannes Kepler, Letter of 19/29 August 1599 to Michael Mästlin. *Gesammelte Werke*, vol. 14 (München, 1949), p. 46. *Harmonice Mundi, passim*. *Gesammelte Werke*, vol. 6 (München, 1940).

[19] Johannes Kepler, Letter of 9 April 1597 to Michael Mästlin. *Gesammelte Werke*, vol. 13 (München, 1945), p. 113.

[20] E.J. Dijksterhuis, *De Mechanisering van het Wereldbeeld* (Amsterdam, 1950) (*The Mechanization of the World Picture*), sections 183-193.

ment for explanatory hypotheses in science had been that they should be as self-evident as possible. They should also lend themselves in any case to a visual picture in the sense that, minimally, they fit into the sphere of representation that is created by normal sense experience in daily life. In accordance with these requirements the two great corpuscular theories of the 17th century, those of Gassendi and of Descartes, allowed no other action of material particles upon one another than the action that is exerted in mutual contact, through pressure or impact, that is – the very type of action to be observed all the time in the macro-world. Such investigators as Robert Boyle and Christiaan Huygens display to us the full extent to which this corpuscularian conception – also designated as 'mechanical' – dominated the physicists' thought at the time. For them, this very trait established the decisive demarcation from medieval science. Descartes in particular defended such a conception in no uncertain terms. In his view a scientific theory should produce a mechanical model of physical reality, and such a model should, if need be, be susceptible to actual construction by skilful tinkering so as to demonstrate *ad oculos* the operations of nature.

Now Newton, in his theory of gravitation, introduced the supposition that each couple of material particles in the universe attracts one another with equal forces which counteract along the line that connects them, the magnitude of these forces being inversely proportional to the square of the distance of the two points. By means of this hypothesis he succeeded in conceiving of phenomena of both celestial and terrestrial motion as consequences of one and the same general law of nature. Despite the important results yielded by the theory it elicited resistance on the principal grounds that no visual picture could be made of how, all across empty space, a body here might act upon a body yonder.

Newton, however, had countered the objection by anticipation in declaring, in his *Principia*, that his gravitation should not be regarded as a *causa physica*, but rather as a *causa mathematica*.[21] His objective was no other than what astronomers all across time had always aimed at, namely, to provide a mathematical description of the course of things that is as simple as possible and that covers as many phenomena as possible. However, because of our human capacity to exert and undergo actions which we designate as forces, the language of 'forces' always raises a powerful illusion of visual picturability and, therefore, of comprehensibility. This is why Newton, in clothing his description in the garb of such picturesque force-language, created the impression that planetary motions, terrestrial tides and movements of falling or projected bodies had now been explained in the sense of making it clear how they are actually produced. In fact this was not the case at all. Terms like gravitation and attraction are no more capable of making anything intelligible than terms like substantial form or quality were in scholastic philosophy, or mechanical pressure and impact in the 17th century. Newton himself never for one moment lived under the illusion that they are. He explicitly called *actio in*

[21] Newton, *Philosophiae Naturalis Principia Mathematica*, Definitio VIII.

distans a physical absurdity,[22] but this did not prevent him in the least from making use of it in a mathematical theory of a natural phenomenon.

Thus Newton abandoned the requirement of visual representation. He subjected a hypothesis to the sole condition that from it one can deduce both the facts already known before and new, empirically testable conclusions. In so doing Newton considerably extended the possibilities provided by the empirical-mathematical method of the investigation of nature. He also clarified a great deal our understanding of its essence. In the 17th century explanations by means of the contact action of material corpuscles had appeared persuasive because the phenomena of pressure and impact, with which daily life makes us familiar, were regarded as sufficiently intelligible. In the hands of a genius like Christiaan Huygens, among others, this conception had yielded important results. However, for the further development of physics it was of eminent significance that Newton abolished the principle of the necessary connection with what is macroscopically familiar.

Curiously, it did not take long before the very same concept of gravitation that no lesser minds than those of Leibniz and Huygens had branded as an essentially non-mechanical principle of explanation signifying a recourse to the occult qualities of Scholasticism, appeared so familiar and well-known to investigators of nature that they began to declare it a fundamental principle and to aim at turning it into the cornerstone of as many domains of science as possible – they began to consider it as a typically mechanical principle. After the major alterations in physical thought in the 20th century had taken their course a certain nostalgia for the visual representations of classical physics could even be observed on occasion, this nostalgia being felt in particular for Newtonian conceptions that, when pronounced for the first time, had been considered to lack picturability in the highest degree! This is how far judgments on the acceptability and manageability of a scientific concept appear to depend on the extent to which we have become familiar with it, which is determined in its turn by the environment in which we undergo our scientific education. Again and again a younger generation learns to handle effortlessly the concepts and conceptions that their great predecessors had introduced with much hesitation and in face of the opposition of their contemporaries, and then the new generation goes on to develop these further in an equally laborious process. It is this collective growth of human thought that makes progress in science possible.

Thus the expansion of the mathematical-empirical method accomplished by Newton was of a principal nature and significance in that the aim of a scientific theory could now be defined as providing a mathematical description of the course of a natural phenomenon, under the naturally added clause that the description must give rise to empirically verifiable consequences. Saying this should not be misunderstood to mean that such a conception of scientific method has since been universally accepted. To the contrary, the Cartesian desire for so-called mechanical

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[22] See, for example, the third letter to Bentley, quoted in L. Trenchard More, *Isaac Newton* (New York/London, 1934), p. 379; also in René Dugas, *La Mécanique au XVIIe siècle* (Neuchâtel, 1954), p. 433.

models of physical phenomena and events appears to have lived on unabated among many investigators of nature in the 19th century, some of whom were among the very greatest. The history of aether theories provides a striking example. However, it looks as if in the 20th century the Newtonian conception has gained the upper hand more and more.

On the other hand, already at the beginning of the 19th century a possibility had been opened to pursue further the road taken by Newton. When, in framing a scientific theory, we are no longer bound to physical picturability, geometrical picturability does not of course bind us either. With the construction of non-Euclidean and more-dimensional geometries an expansion of the mathematical means of description of physical processes became given at least in principle. Not until the 20th century, however, were these new means to be employed in fact. As we see so often happen in history, progress was achieved once more through the escape from the grasp of certain limitations the Greek founders had imposed upon mathematical-physical thinking. It is not impossible that the emancipation from our strong bond to Hellenic thought must be pursued further in order to warrant a continuing flowering of science.[23]

But this is no longer an historical statement, and thus it appears that, at least for the present hour, the history of science has come to the end of what this particular discipline has to say about the application of mathematics in science. No doubt a treatment in greater depth would bring to light many more data about their collaboration. Such a treatment might also demonstrate how richly mathematics has been rewarded for the indispensable services rendered to science in that the tasks it was assigned to fulfill had an inspiring effect upon its own development. It is not likely, though, that data of a principally new nature, fit to help resolve the problem put at the beginning of this address, would be acquired thereby.

Will the philosophy of science itself succeed in fully clearing up the function mathematics fulfills in science? Will it be able to explain how it is that we can cultivate science with the help of mathematics?

That depends on what we mean by 'explanation'. Presumably the answer will be of the type made fun of by Nietzsche in discussing Kant's response to the question of how synthetic judgments *a priori* are possible: *vermöge eines Vermögens* ('enabled by an ability').[24] The human mind – so the conclusion may well run – if first opening itself to what lessons unprejudiced observation has to offer, is capable of depicting the operations of nature in a mathematical system that enables us to predict what is going to happen under certain, consciously selected circumstances. In essence, this is the very conviction we heard Kepler commit himself to in the garb of theological language: In cultivating science with the help of mathematics the human mind, *ad quanta intelligenda condita*, shall understand of creation what it has been given man to understand of it, that is, *geometriae vestigia in mundo expressa*, the vestiges of geometry that stand expressed in the world.

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[23] Erwin Schrödinger, *Nature and the Greeks* (Cambridge, 1954).

[24] Friedrich Nietzsche, *Jenseits von Gut und Böse*, in *Werke* (Taschen-Ausgabe, Kröner), 8, 20.