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Odd-frequency pairing state in superconducting junctions

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Abstract

Using a quasiclassical Green's function formalism, we show that odd-frequency pairing state can be ubiquitously induced in the superconducting junctions. In ballistic normal metal–superconductor (N/S) junctions where a superconductor has even-frequency symmetry in the bulk, we show that odd-frequency pairing state can be induced at the interface. Even in the *s*-wave superconductor junction, the amplitude of the odd-frequency component is enhanced in the normal metal. The appearance of the midgap Andreev resonant states due to the sign change of the anisotropic pair potential at the interface is reinterpreted in terms of the generation of the odd-frequency pair amplitude.

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Keywords: Odd-frequency pairing state; Proximity effect

1. Introduction

Exploring novel pairing states in the field of unconventional superconductivity has been a challenging issue in solid state physics. In accordance with Fermi Dirac statistics, the condensate wave function should be odd with respect to the permutation of electrons. Therefore, spin-singlet even-parity and spin-triplet odd-parity pairing states are realized when the pair amplitude is an even function of Matsubara frequency. The realization of spin-singlet odd-parity and spin-triplet even-parity pairing states is also possible when the pair amplitude is an odd function of the frequency.

Odd-frequency superconducting pairing state, characterized by pair amplitude which is an odd function of Matsubara frequency, was first predicted by Berezinskii [1]. Although there have been many theoretical proposals [2–4], the existence of odd-frequency pairing in bulk uniform systems has not been clarified yet. On the other hand, there is a number of proposals to realize it in superconducting junctions. The realization of the odd-frequency pairing state without bulk odd-frequency pair potential has been proposed by Bergeret et al. [5] in ferromagnet/superconductor heterostructures. Several works about odd-frequency pairing state in ferromagnet junctions have been published up to now [6,7].

Recently, it has been shown that the odd-frequency pairing state does not require magnetic ordering. It appears, for instance, in a diffusive normal metal attached to a spin-triplet superconductor [8]. The induced odd-frequency component results in anomalous proximity effect with enhanced zero energy peak of local density of state specific to unconventional superconductor junctions [9]. It has also been clarified that in a conventional ballistic normal metal/superconductor (N/S) system, which do not have spin-triplet ordering, the odd-frequency paring is possibly induced due to a spatial variation [14] of the pair potential near the interface [13]. It has been revealed that the magnitude of the induced odd-frequency component is enhanced in the presence of the midgap Andreev resonant state

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(MARS) [10–12] specific to unconventional superconductors with sign change of the anisotropic pair potential at the interface. Very recently, even in the vortex core of a superconductor, odd-frequency pairing state is induced [19]. It is very interesting that just at the center of the vortex core of conventional s-wave superconductor, only the odd-frequency spin-singlet chiral p-wave state exists [19].

In the present paper, we introduce the current status of the understanding of the odd-frequency paring state in superconducting junctions. We show the results of typical example of ballistic N/S junctions. After that, using a exactly solvable one-dimensional model, we show the relevance to the odd-frequency pairing state to the mid gap Andreev bound state.

2. Proximity effect in N/S junctions

In the following, we consider a N/S junction as the simplest example of non-uniform superconducting system without impurity scattering. Both the spin-triplet odd-parity and the spin-singlet even-parity states are considered in the superconductor. As regards the spin-triplet superconductor, we choose $S_z = 0$ for the simplicity. We assume a thin insulating barrier located at the N/S interface ($x = 0$) with N($x < 0$) and $S(x > 0)$ modeled by a delta function $H\delta(x)$, where H is the magnitude of the strength of the delta function potential. The reflection coefficient of the junction for the quasiparticle for the injection angle θ is given by $R = Z^2/(Z^2 + 4\cos^2\theta)$ with $Z = 2H/v_F$, where θ $(-\pi/2 < \theta < \pi/2)$ is measured from the normal to the interface and v_F is the Fermi velocity. The quasiclassical Green's function in a superconductor is parameterized as

$$\hat{g}_\pm = f_{1\pm}\hat{\tau}_1 + f_{2\pm}\hat{\tau}_2 + g_\pm\hat{\tau}_3, \quad \hat{g}_\pm^2 = \hat{1} \tag{1}$$

with Pauli matrices $\hat{\tau}_i (i = 1-3)$ and unit matrix $\hat{1}$. Here, the index $+ (-)$ denotes the left (right) going quasiparticles [16,17]. It is possible to express the above Green's function as $f_{1\pm} = \pm i(F_\pm + D_\pm)/(1 - D_\pm F_\pm)$, $f_{2\pm} = -(F_\pm - D_\pm)/(1 - D_\pm F_\pm)$, and $g_\pm = (1 + D_\pm F_\pm)/(1 - D_\pm F_\pm)$, where $D_\pm$ and $F_\pm$ satisfy the Eilenberger equations in the Riccati parameterization [15]

$$v_{Fx}\partial_x D_\pm = -\overline{\Delta}_\pm(x)(1 - D_\pm^2) + 2\omega_n D_\pm \tag{2}$$

$$v_{Fx}\partial_x F_\pm = -\overline{\Delta}_\pm(x)(1 - F_\pm^2) - 2\omega_n F_\pm. \tag{3}$$

where v_{Fx} is the x component of the Fermi velocity, $\omega_n = 2\pi T(n + 1/2)$ is the Matsubara frequency, with temperature T. $\overline{\Delta}_+(x)(\overline{\Delta}_-(x))$ is the effective pair potential for left going (right going) quasiparticles. Since the interface is flat, $F_\pm = -RD_\mp$ holds at $x = 0$ [15]. Here we consider the situation without mixing of different symmetry channels for the pair potential. Then $\overline{\Delta}_\pm(x)$ is expressed by $\overline{\Delta}_\pm(x) = \Delta(x)\Phi_\pm(\theta)\Theta(x)$ with the form factor $\Phi_\pm(\theta)$ given by 1, $\pm\cos\theta$ for s and p_x-wave superconductors, respectively. $\Delta(x)$ is determined self-consistently. The condition in the bulk is $\Delta(\infty) = \Delta$. If we explicitly write $f_{1\pm} = f_{1\pm}(\omega_n, \theta)$, $f_{2\pm} = f_{2\pm}(\omega_n, \theta)$, we can show analytically that $f_{1\pm}(\omega_n, \theta) = -f_{1\pm}(-\omega_n, \theta)$ and $f_{2\pm}(\omega_n, \theta) = f_{2\pm}(-\omega_n, \theta)$ for any x. We note that the parity of the odd-frequency component $f_{1\pm}(\omega_n, \theta)$ is different from that of the bulk superconductor for all cases.

Let us now focus on the values of the pair amplitudes at the interface $x = 0$. We concentrate on two extreme cases with (1) $\Phi_+(\theta) = \Phi_-(\theta)$ and (2) $\Phi_+(\theta) = -\Phi_-(\theta)$. In the first case, MARS is absent since there is no sign change of the pair potential felt by the quasiparticle at the interface. Then $D_+ = D_-$ is satisfied. On the other hand, in the second case, MARS is generated near the interface due to the sign change of the pair potential. Then $D_+ = -D_-$ is satisfied [10]. At the interface, it is easy to show that $f_{1\pm} = \pm i(1 - R)D_+/(1 + RD_+^2)$ and $f_{2\pm} = (1 + R)D_+/(1 + RD_+^2)$ for the case 1 and $f_{1\pm} = i(1 + R)D_+/(1 - RD_+^2)$ and $f_{2\pm} = \pm(1 - R)D_+/(1 - RD_+^2)$ for the case 2, respectively, where the real number D_+ satisfies $\mid D_+ \mid < 1$ for $\omega_n \neq 0$. For the case 1, the magnitude of $f_{1\pm}$ is always smaller than that of $f_{2\pm}$. On the other hand, for case the 2, the situation is reversed. In the low transparent limit with $R \to 0$, only the $f_{1\pm}$ is nonzero.

In order to understand the angular dependence of the pair amplitude in a more detail, we define $\hat{f}_1$ and $\hat{f}_2$ for $-\pi/2 < \theta < 3\pi/2$ with $\hat{f}_{1(2)} = f_{1(2)+}(\theta)$ for $-\pi/2 < \theta < \pi/2$ and $\hat{f}_{1(2)} = f_{1(2)-}(\pi - \theta)$ for $\pi/2 < \theta < 3\pi/2$. We decompose $\hat{f}_{1(2)}$ into various angular momentum component as follows,

$$\hat{f}_{1(2)} = \sum_m S_m^{(1(2))}\sin[m\theta] + \sum_m C_m^{(1(2))}\cos[m\theta] \tag{4}$$

with $m = 2l + 1$ for odd-parity case and $m = 2l$ for even-parity case with integer $l \geqslant 0$, where l is the quantum number of the angular momentum.

We illustrate the above results by numerical calculations. As typical examples, we choose s-wave and p_x-wave pair potentials. Although both $\hat{f}_1$ and $\hat{f}_2$ have many components with different angular momenta, we focus on the lowest values of l. We denote $E_s(i\omega_n, x) = C_0^{(2)}$, $E_{px}(i\omega_n, x) = C_1^{(2)}$, $O_s(i\omega_n, x) = C_0^{(1)}$, and $O_{px}(i\omega_n, x) = C_1^{(1)}$ and choose $i\omega_n = i\pi T$ with temperature $T = 0.05T_C$. For the s-wave case, the pair potential is suppressed only for high transparent junctions (see Fig. 1a). The odd-frequency component $O_{px}(i\pi T, x)$ is enhanced for $Z = 0$ near the interface where the pair potential $\Delta(x)$ is suppressed, while the even-frequency component $E_s(i\pi T, x)$ remains almost constant in this case. For low transparent junctions, the magnitude of $O_{px}(i\pi T, x)$ is negligible (see Fig. 1b). For the p_x-wave junction with $Z = 0$, although the odd-frequency component $O_s(i\pi T, x)$ is enhanced near the interface, it is smaller than the even-frequency one $E_{px}(i\pi T, x)$ (see Fig. 1c). For the low transparent junction, the magnitude of $O_s(i\pi T, x)$ is strongly enhanced near the interface and becomes much larger than the magnitude of $E_{px}(i\pi T, x)$ (see Fig. 1d).

In the following, we discuss the odd-frequency paring state using an exact solution of the one-dimensional model [20,12]. In the low transparency limit of a p_x-wave junction,

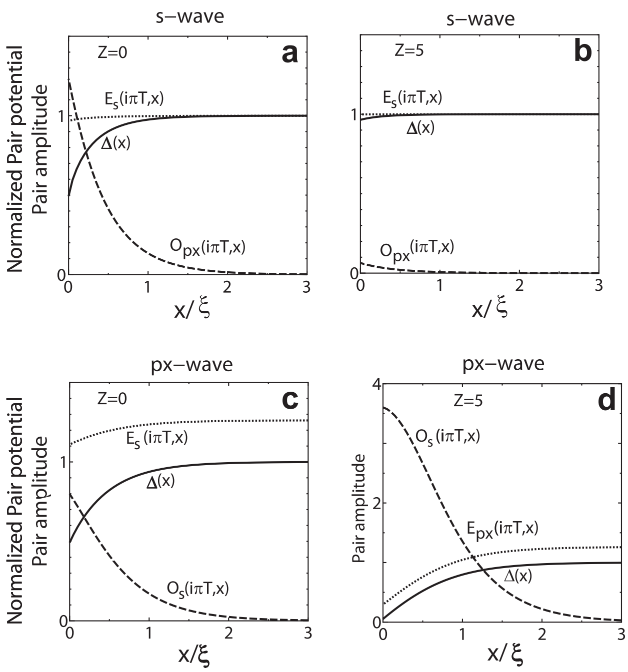

Fig. 1. Spatial dependence of the normalized pair potential (solid line) even-frequency pair amplitude (dotted line) and odd-frequency pair amplitude (dashed line). (a) and (c): fully transparent junctions with $Z = 0$. (b) and (d): low transparent junctions with $Z = 5$. $E_s(i\pi T, x)$ and $E_{p_x}(i\pi T, x)$ are the even-frequency components of the pair amplitude, $O_s(i\pi T, x)$ and $O_{px}(i\pi T, x)$ are the odd-frequency components. The distance x is normalized by the v_F/Δ. T_C is the transition temperature of the superconductor.

$\Delta(x)$ is absent at $x = 0$. Here, we assume that the spatial dependence of the pair potential is

$$\Delta(x) = \Delta \tanh(x/\xi) \quad (5)$$

with $\xi = v_F/\Delta$, and we only focus on $\theta = 0$, i.e. the trajectory perpendicular to the interface. After some algebra, we can get

$$g_+ = \frac{1}{\sqrt{\omega_n^2 + \Delta^2}}\left[\omega_n + \frac{\Delta^2}{2\omega_n}\mathrm{sech}^2(x/\xi)\right],$$
$$f_{2+} = \frac{1}{\sqrt{\omega_n^2 + \Delta^2}}\Delta \tanh(x/\xi) \quad (6)$$
$$f_{1+} = \frac{i}{\sqrt{\omega_n^2 + \Delta^2}}\frac{\Delta^2}{2\omega_n}\mathrm{sech}^2(x/\xi)$$

with $f_{1-} = -f_{1+}$, $f_{2-} = f_{2+}$ and $g_- = g_+$. As seen from above equations, it is evident that the odd-frequency pair amplitude is localized around $x = 0$ where the amplitude of the pair potential $\Delta(x)$ is suppressed. At the same time, the g_+ has a localized solution, and its amplitude is enhanced at $\omega_n = 0$. If we replace $i\omega_n$ with real frequency ε, the local density of states has a zero energy peak. As seen from these equations, the existence of the MARS just corresponds to the generation of the odd-frequency pairing state.

3. Conclusions

In summary, using the quasiclassical Green's function formalism, we have shown that the odd-frequency pairing state is ubiquitously generated in the normal metal/superconductor (N/S) ballistic junction system. We demonstrate the generation of the odd-frequency pairing state. In the one-dimensional limit, the analytical solution is obtained. We have shown that the appearance of the MARS is the manifestation of the odd-frequency pair amplitude. There are several interesting phase-coherent effects relevant to MARS [18]. These phenomena can be reinterpreted in terms of the odd-frequency paring state.

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