

## Optimization of the working fluid in a Joule–Thomson cold stage

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### ABSTRACT

Vibration-free miniature Joule–Thomson (JT) coolers are of interest for cooling a wide variety of devices, including low-noise amplifiers, semiconducting and superconducting electronics, and small optical detectors used in space applications. For cooling such devices, coolers are needed which have operating temperatures within a wide temperature range of 2–250 K. In this paper, the optimization of the working fluid in JT cold stages is described that operate at different temperatures within that range. For each temperature, the most suitable working fluid is selected on the basis of the coefficient of performance of the cold stage, which is defined as the ratio of the gross cooling power to the change in Gibbs free energy of the fluid during compression. In addition, a figure of merit of the heat exchange in the counter-flow heat exchanger is evaluated that depends only on the properties of the working fluid.

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### 1. Introduction

Miniature Joule–Thomson (JT) coolers [1–3] are relevant for cooling a wide variety of applications, including low-noise amplifiers [4], semiconducting and superconducting electronics [5], and small optical detectors used in space applications [6–8]. The operating temperature needed depends on the device. For example, low-noise amplifiers are typically cooled to 150 K to improve the signal-to-noise ratio. Superconductors have to be cooled below their critical temperature, which is below 80 K for high-temperature superconductors and below 8 K for low-temperature superconductors [9]. The operating temperature of optical detectors depends strongly on the type of detector. Infrared detectors typically operate in a temperature range of 2–150 K, whereas detectors for the visible spectral range operate between 200 K and 300 K [10].

In this paper, the application of JT coolers in a wide temperature range of 2–250 K is investigated. For a JT cooler operating at a specific temperature, multiple working fluids can often be used. The most suitable working fluid is selected on the basis of two parameters, which both only depend on the thermodynamic properties of the fluid. In Section 2, these two parameters are derived. In Section 3, the constraints used in the optimization analysis are discussed and in Section 4, the results of this analysis are shown. For each specific temperature within the range 2–250 K, the most suitable working fluid is selected. The influence of the constraints on this selection is investigated in Section 5, after which the paper is closed with conclusions.

### 2. Theory

Basically, a JT cooler consists of a counter-flow heat exchanger (CFHX), a JT valve or restriction, an evaporator and a compressor (Fig. 1). The heat of compression is, usually, rejected to the environment in an aftercooler heat exchanger, which in Fig. 1 is assumed to be part of the compressor. In the JT cooling cycle, a warm high-pressure fluid flows through a CFHX exchanging heat with the colder low-pressure fluid that flows in the opposite direction. The high-pressure fluid thus cools and reaches the JT restriction at a lower temperature. There, it expands adiabatically and partly liquefies. A heat load, resulting from the device to be cooled, then evaporates the liquid, and the vapor flows back through the CFHX. In a closed cycle, a compressor pressurizes the low-pressure fluid back to the high pressure.

The gross cooling power of a JT cooler depends on the change in specific enthalpy of the fluid at the cold end. In an ideal CFHX, no heat is exchanged with the environment and a maximum amount of enthalpy is exchanged between the high-pressure fluid and the low-pressure fluid. In this case, the change in specific enthalpy of the fluid at the warm end ( $\Delta h_{51}$ ) equals that one at the cold end ( $\Delta h_{34}$ ) [11]. The gross cooling power ( $P_{cool}$ ) of a JT cooler can thus be determined by the change in enthalpy of the fluid at the compression temperature, i.e. after compression and re-cooling,

$$P_{cool} = \dot{m}\Delta h_{34} = -\dot{m}\Delta h_{51} \quad (1)$$

with  $\dot{m}$  the mass-flow rate.

The coefficient of performance (COP) of a JT cold stage is defined as the ratio of the gross cooling power of the cold stage to the change in Gibbs free energy of the fluid during compression ('available work'),

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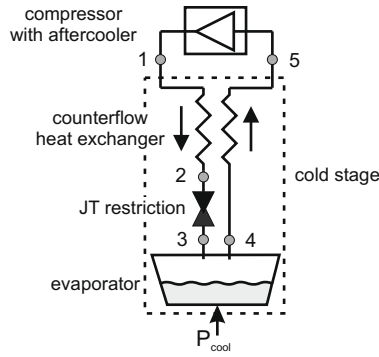


Fig. 1. Schematic representation of a JT cooler.

$$COP = \frac{-\Delta h_{51}}{\Delta h_{51} - T_h \Delta s_{51}} \quad (2)$$

Here,  $\Delta s_{51}$  is the change in specific entropy of the fluid during compression and re-cooling, and  $T_h$  is the temperature at which the fluid enters the warm end of the CFHX.

To determine the efficiency of the heat exchange in the CFHX, a figure of merit (*FOM*) can be defined as follows. The heat flow ( $\dot{Q}_w$ ) from the CFHX wall to the low-pressure fluid in a small section  $dx$  at position  $x$  of the CFHX (Fig. 2) can be expressed by [12]

$$\dot{Q}_w = hO[T_w(x) - T_f(x)]dx \quad (3)$$

where  $h$  is the local heat transfer coefficient,  $O$  is the wetted perimeter of the fluid channel,  $T_w(x)$  and  $T_f(x)$  are the local temperatures at position  $x$  of the wall and of the low-pressure fluid, respectively. The heat transfer coefficient ( $h$ ) can be defined as [12]

$$h = \frac{Nu\lambda_f}{D_h} \quad (4)$$

where  $\lambda_f$  is the local heat conduction coefficient of the fluid,  $Nu$  is the Nusselt number and  $D_h$  is the hydraulic diameter of the fluid channel. The latter can be calculated by  $D_h = 4A_c/O$ , where  $A_c$  is the cross-sectional area of the fluid channel. Both,  $\lambda_f$  and  $Nu$  depend on pressure and temperature. Using Eq. (4), the heat transfer between the wall and the low-pressure fluid becomes

$$\dot{Q}_w = \frac{4A_c}{D_h^2} Nu\lambda_f [T_w(x) - T_f(x)]dx \quad (5)$$

The change in enthalpy flow of the fluid ( $\Delta \dot{H}_f$ ) at constant pressure in a small section  $dx$  of the CFHX can be expressed by [12]

$$\Delta \dot{H}_f = \dot{m}c_{p,f}dT_f \quad (6)$$

where  $c_{p,f}$  is the local specific heat of the fluid at constant pressure, of which the value depends on temperature and pressure, and  $dT_f$  is the local temperature difference of the fluid over a length  $dx$ .

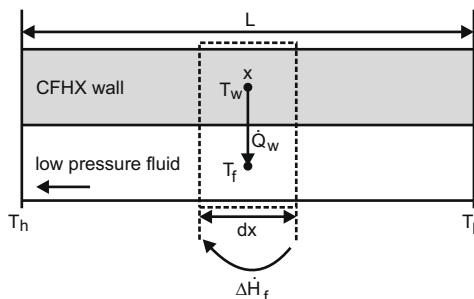


Fig. 2. Detail of the CFHX for calculating the *FOM*.

For an ideal CFHX, the change in enthalpy flow of the low-pressure fluid (Eq. (6)) equals the heat flow between the CFHX wall and the low-pressure fluid (Eq. (5)), thus

$$\dot{m}c_{p,f}dT_f = \frac{4A_c}{D_h^2} Nu\lambda_f [T_w(x) - T_f(x)]dx \quad (7)$$

By defining  $T_w(x) - T_f(x) = \Delta T_{wf}$  and by putting all temperature dependent terms on one side, integration over the length ( $L$ ) of the CFHX, under constant pressure, results in

$$\frac{1}{\Delta T_{wf}} \int_{T_l}^{T_h} \frac{c_{p,f}}{Nu\lambda_f} dT_f = \frac{4A_c}{\dot{m}D_h^2} \int_0^L dx = \frac{4A_c}{\dot{m}D_h^2} L \quad (8)$$

Here,  $T_h$  and  $T_l$  are the temperatures at the warm and cold ends of the CFHX, respectively. Now, by using Eq. (1), the length of the CFHX can be expressed as

$$L = \frac{-P_{cool}}{\Delta h_{51}} \frac{D_h^2}{4A_c} \frac{1}{\Delta T_{wf}} \int_{T_l}^{T_h} \frac{c_{p,f}}{Nu\lambda_f} dT_f \quad (9)$$

In the case of laminar flow, the Nusselt number is a constant, which is fully determined by the geometry of the CFHX [12]. Then, Eq. (9) indicates that, the higher the thermal conductivity of the fluid and the smaller its specific heat, the more effective is the heat exchange in the CFHX and, as a result, the shorter the CFHX can be. To investigate the impact of the specific heat and the thermal conductivity of a working fluid on the heat exchange in the CFHX, a figure of merit (*FOM*) in the case of laminar flow can be derived from Eq. (9),

$$FOM_{lam} = -\Delta h_{51} (T_h - T_l) \left[ \int_{T_l}^{T_h} \frac{c_{p,f}}{\lambda_f} dT_f \right]^{-1} \quad [W/m] \quad (10)$$

Except for a geometry factor, the reciprocal value  $FOM^{-1}$  represents the length needed for the CFHX in order to achieve a certain cooling power. To reduce the effect of the actual temperature interval  $T_l$  to  $T_h$ , this interval is included in Eq. (10).

In the case of turbulent flow, the Nusselt number does depend on fluid properties. Then, the Dittus–Boelter equation is often used to express the Nusselt number [12],

$$Nu = 0.023 Re^{0.8} Pr^n \quad (11)$$

where  $n$  is 0.4 when the fluid is being heated ( $T_w > T_f$ ), and 0.3 when the fluid is being cooled ( $T_w < T_f$ ). The Reynolds number ( $Re$ ) and the Prandtl number ( $Pr$ ) are defined as [12]

$$Re = \frac{\rho_f v_m D_h}{\mu_f}; \quad Pr = \frac{c_{p,f} \mu_f}{\lambda_f} \quad (12)$$

where  $\rho_f$  is the fluid density,  $\mu_f$  its dynamic viscosity, and  $v_m$  the mean fluid velocity. All fluid properties in Eq. (12) depend on pressure and temperature. By substituting Eq. (12) in Eq. (11), and by using  $\dot{m} = \rho_f v_m A_c$ , the Nusselt number can be written as

$$\begin{aligned} Nu &= 0.023 \left( \frac{\dot{m} D_h}{\mu_f A_c} \right)^{0.8} \left( \frac{c_{p,f} \mu_f}{\lambda_f} \right)^{0.35} \\ &= 0.023 \left( \frac{\dot{m} D_h}{A_c} \right)^{0.8} \mu_f^{-0.45} \left( \frac{c_{p,f}}{\lambda_f} \right)^{0.35} \end{aligned} \quad (13)$$

where the exponent  $n$  is set to 0.35. Now, substituting Eq. (13) in Eq. (9) gives

$$L = \frac{1}{0.023} D_h^{1.2} A_c^{-0.2} \dot{m}^{0.2} \frac{1}{4\Delta T_{wf}} \int_{T_l}^{T_h} \mu_f^{0.45} \left( \frac{c_{p,f}}{\lambda_f} \right)^{0.65} dT_f \quad (14)$$

Analogous to the laminar case, the *FOM* in case of turbulent flow is defined as

$$FOM_{turb} = (-\Delta h_{51})^{0.2} (T_h - T_l) \left[ \int_{T_l}^{T_h} \mu_f^{0.45} \left( \frac{c_{p,f}}{\lambda_f} \right)^{0.65} dT_f \right]^{-1} [(W/m)^{0.2}] \quad (15)$$

### 3. Method

The working fluid in a JT cold stage, at a specific operating temperature, is optimized on the basis of the COP as defined in Eq. (2). If the COP values for various working fluids are almost equal, the two FOMs (Eqs. (10) and (15)) are taken into account. In this way, the most suitable working fluid is selected on the basis of these parameters. The constraints used in the analysis are discussed below.

#### 3.1. Temperature at the warm end

It is assumed that the temperature at the warm end of the JT cold stage is 300 K. Only for helium, hydrogen and neon lower warm-end temperatures were selected, because those three fluids all have a maximum inversion temperature below 300 K [11]. In Section 5, the influence of decreasing the warm-end temperature on the selection of the most suitable working fluid is investigated.

#### 3.2. Low-pressure range

The low pressure determines the evaporation temperature of a working fluid and thus the temperature of the cold stage. The temperature of the cold stage can only be varied within a limited range by varying this low pressure. If this pressure is set too low, the pressure drop along the low-pressure channel of the CFHX may have negative effects on the stability of the low pressure and thus on the stability of the cold-tip temperature. If the cold stage is driven by a sorption compressor, some limits are also imposed on the value of the low pressure [13]. As a result, the low pressure is selected to be between 0.2 bar and 10.0 bar.

#### 3.3. High-pressure range

The minimum high pressure is set at 1 bar, provided it remains higher than the low pressure of the cycle in every case. The maximum high pressure is set equal to the pressure of a standard  $5.0 \times 10^{-2} \text{ m}^3$  gas bottle, which is typically 200 bar.

#### 3.4. Counter-flow heat exchanger

It is assumed that the CFHX is ideal and thus Eqs. (2), (10), and (15) can be used to calculate the COP and the two FOMs, respectively. The fact that in practice the CFHX will not be ideal does not play a role in the selection of the most suitable working fluid. The COP yields the maximum attainable efficiency and the two FOMs indicate the minimum length of the CFHX required to achieve that efficiency.

#### 3.5. Cooling power

In this analysis, the cooling power of the JT cold stage is set at 100 mW. However, the chosen cooling power does not play an important role in the selection of the most suitable working fluid. For a given enthalpy change at the warm end of the system, the required cooling power determines the mass-flow rate. Since, in this ideal case, the input power to the cold stage scales linearly with the mass-flow rate, the COP is independent of flow and thus of the required cooling power. Also, the two FOMs are fully determined by fluid properties and by temperature and pressure boundary

conditions. It is thus concluded that the cooling power does not play an important part in the selection of the most suitable working fluid.

#### 3.6. Operating temperature range of working fluids

Because the operating temperature is determined by the low pressure that can only be varied within a limited range, a variety of working fluids will have to be used to cover the complete temperature range between 2 K and 250 K. The temperature range in which a fluid can be used depends on the selected low-pressure range and for the most common pure fluids it is shown in Fig. 3. The minimum temperature is calculated for a low pressure of 0.2 bar, except for those fluids that have a triple-point pressure above 0.2 bar. For such fluids, the triple-point pressure is used for evaluating the minimum attainable temperature. The maximum temperature corresponds to a pressure of 10 bar.

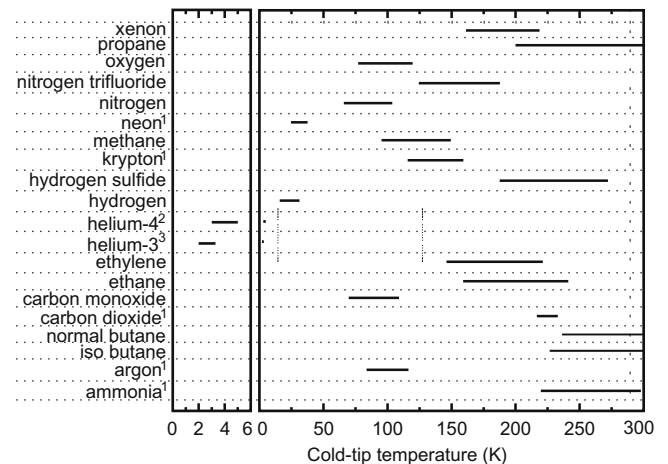
In Fig. 3, two temperature gaps can be distinguished in which no pure working fluid liquefies within the selected low-pressure range. These two gaps range from 5.2–15.9 K and 37.5–65.8 K, respectively. To cover these temperature ranges by JT cooling, for example, mixtures can be used [14].

### 4. Results

In this section, the optimization analysis of the working fluid in a JT cold stage is discussed. All fluid properties were taken from Refprop [15], except those of helium-3 which were taken from He3pak [16].

#### 4.1. JT cold stages for the range 65–250 K starting at 300 K

The most common pure working fluids that can be used in a JT cold stage to reach a temperature within the range 65–250 K are listed in Fig. 3. The COP is calculated as a function of the high pressure, and for the three cases of operating temperatures of 100 K, 180 K and 250 K the results are shown in Fig. 4a–c. At low operating temperatures, the COP of a JT cold stage is rather poor (Fig. 4a). In that case, the fluid is compressed at a temperature far above its

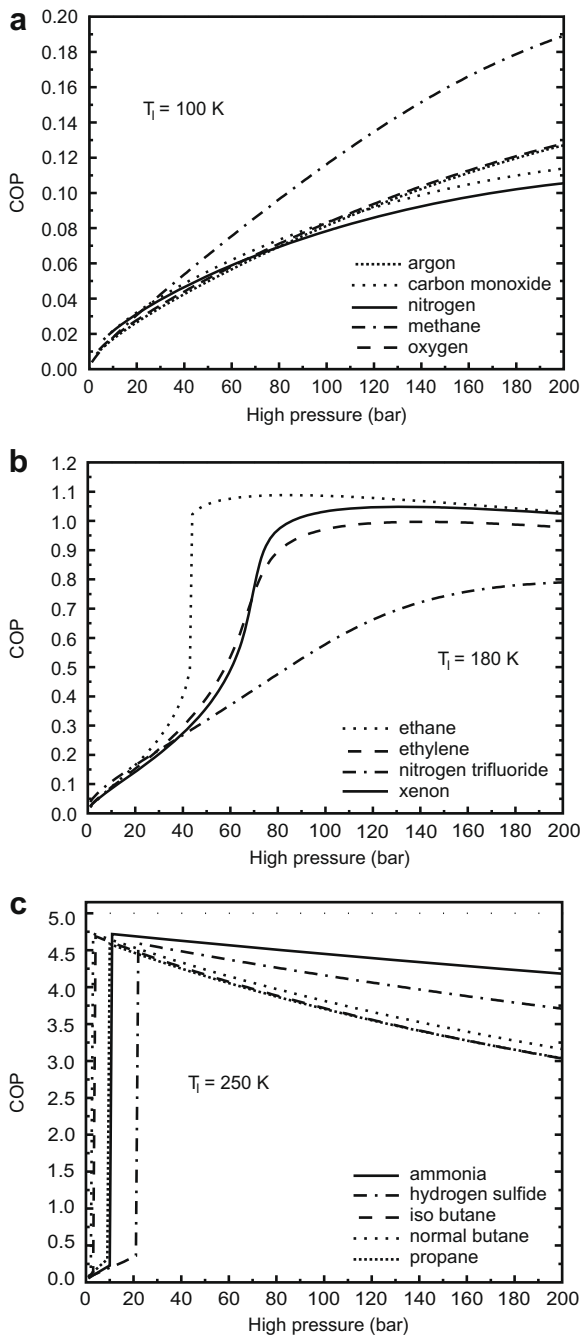


<sup>1</sup>Triple point pressure is used to calculate the minimum temperature

<sup>2</sup>Minimum temperature corresponds to 0.20 bar and maximum temperature to 2.25 bar.

<sup>3</sup>Minimum temperature corresponds to 0.20 bar and maximum temperature to 1.15 bar.

Fig. 3. Operating temperature range in which a fluid can be used for a JT cold stage, corresponding to a low-pressure range of 0.2–10.0 bar. For clarification, the temperature range 0–6 K is enlarged.



**Fig. 4.** COP as a function of the high pressure for a cold-tip temperature of (a) 100 K, (b) 180 K and (c) 250 K.

critical temperature, where it will behave as essentially an ideal gas. This means that only small changes in its specific enthalpy can be obtained during compression, which results in a rather low COP. A fluid at a temperature close to its critical temperature behaves highly non-ideal. Once in that region the fluid is compressed a much higher change in its specific enthalpy can be obtained. This explains the steep increase in COP of some fluids at a specific high pressure shown in Fig. 4b. For example, nitrogen trifluoride behaves as an ideal gas during compression, whereas the other three fluids behave highly non-ideal. Furthermore, fluids which have a critical temperature above 300 K may liquefy during compression, which causes an even steeper increase in COP. This is the case for all fluids shown in Fig. 4c.

Both, the laminar and turbulent-case FOMs of each fluid are calculated for the optimum high pressure, defined as the pressure for which the COP is highest. The parameters at maximum COP are shown in Table 1a–c. Now, for each of the operating temperatures, the most suitable working fluid is selected on the basis of the COP and the two FOMs. For an operating temperature of 100 K, it is clear that methane can best be used as the working fluid. The COP and the two FOMs are highest for this fluid. At a cold-tip temperature of 180 K, the COP values of ethane, ethylene and xenon lie within 10% of each other. However, the laminar-case FOM of xenon is much lower than that of ethane and ethylene, and the turbulent-case FOM is also lower. Thus, xenon is not selected. The two FOMs of ethylene and ethane are almost equal. Besides that ethane has a slightly higher COP and FOM, for this fluid also a lower pressure is needed to reach the maximum COP, which is a big advantage in closed-cycle cooling. It is thus concluded that, in this case, ethane will be selected as the most suitable working fluid. In Fig. 4c it is shown that, for a JT cold stage operating at 250 K, the maximum COP of all working fluids is almost equal. However, the laminar-case FOM of ammonia is an order of magnitude higher than that of the other fluids. This is caused by the large heat of condensation of ammonia, which is an order higher than that of the other working fluids [15]. This means that  $\Delta h_{51}$  is much higher, resulting in a shorter length required for the CFHX (Eq. (10)). Ammonia also has a larger turbulent-case FOM and is thus selected as the most suitable working fluid at 250 K.

These calculations and considerations can be made for each arbitrary cold-tip temperature [17]. In this way, the wide temperature range 65–250 K can be divided into several small ranges, and in each range a specific working fluid can be selected. In Fig. 5, the maximum COP and laminar-case FOM are given as a function of the cold-tip temperature. In this figure, the most suitable working fluid for a specific temperature range is also shown. As could be expected, the maximum COP increases for an increasing cold-tip temperature. The laminar-case FOM also increases, but remarkable are the levels of almost constant laminar-case FOM within a small temperature range of the cold tip. In this temperature range, the laminar-case FOM is determined by a single working fluid. The ratio of thermal conductivity to specific heat is rather constant within that temperature range. As indicated by Eq. (10), this leads to a constant laminar-case FOM.

As shown in Fig. 5, in specific temperature ranges, carbon monoxide, nitrogen trifluoride, hydrogen sulfide and ammonia are se-

**Table 1**

Maximum-COP parameters for a JT cold stage which has a warm-end temperature of 300 K and operates at (a) 100 K, (b) 180 K and (c) 250 K.

Fluid	$P_{low}$ (bar)	$P_{high}$ (bar)	COP	$FOM_{lam}$ (W/m)	$FOM_{turb}$ (W/m) <sup>0.2</sup>
<i>(a) <math>T_i = 100</math> K</i>					
Argon	3.24	200	0.128	0.69	1.11
Carbon monoxide	5.44	200	0.114	0.59	1.03
Methane	0.34	200	0.189	1.63	1.26
Nitrogen	7.78	200	0.105	0.50	0.97
Oxygen	2.54	200	0.127	0.76	1.08
<i>(b) <math>T_i = 180</math> K</i>					
Ethane	0.79	85	1.09	3.06	1.37
Ethylene	1.82	141	1.00	3.08	1.38
Nitrogen trifluoride	7.25	200	0.79	1.65	1.30
Xenon	2.22	132	1.05	1.70	1.31
<i>(c) <math>T_i = 250</math> K</i>					
Ammonia	1.65	11	4.72	12.44	1.73
Iso butane	0.63	4	4.69	3.11	1.42
Normal butane	0.39	3	4.73	3.28	1.43
Hydrogen sulfide	4.89	22	4.59	6.59	1.64
Propane	2.18	10	4.58	3.48	1.44

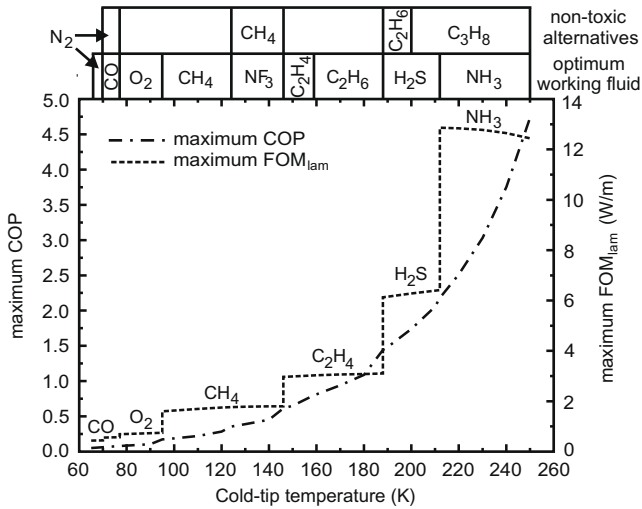


Fig. 5. Maximum COP and laminar-case FOM as a function of the cold-tip temperature.

lected as the most suitable working fluids. However, these fluids have the disadvantage that they are very toxic [18] and thus specific safety precautions has to be taken when using these fluids. Therefore, non-toxic alternatives to these fluids are also suggested in Fig. 5.

4.2. A JT cold stage operating in the temperature range 2–65 K

Lower operating temperatures can be achieved by using the working fluids helium-3, helium-4, hydrogen or neon. Because these four fluids have maximum inversion temperatures below 300 K [11], precooling of the JT cold stage to below this maximum inversion temperature is essential to obtain cooling. In Fig. 3, the cold-tip temperatures for which these fluids can be used are shown. Helium-3 can be used in the range 2.0–3.3 K and helium-4 in the range 2.9–5.2 K. Both fluids can therefore be used in the temperature range 2.9–3.3 K. Furthermore, hydrogen can be used in the range 15.9–31.4 K and neon in the range 24.6–37.5 K, and both can be used between 24.6 K and 31.4 K.

To investigate the use of hydrogen and neon in the range 24.6–31.4 K, the COP and the two FOMs are calculated for different cold-tip and warm-end temperatures. In Fig. 6, the COP as a function of

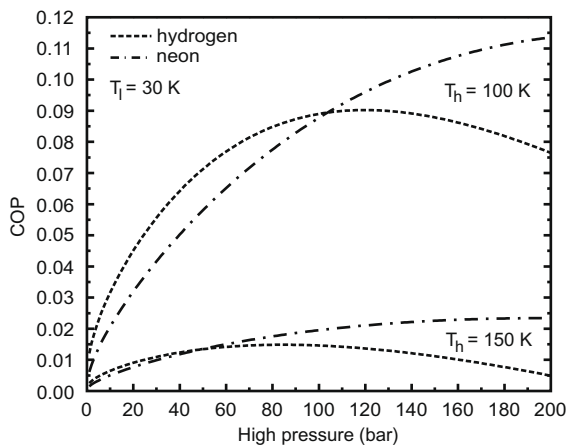


Fig. 6. COP as a function of the high pressure for a cold-tip temperature of 30 K and a warm-end temperature of 150 K (bottom two curves) and 100 K (top two curves).

Table 2

Maximum-COP parameters for a JT cold stage operating at 30 K and having a warm-end temperature of (a) 150 K, (b) 100 K and (c) 50 K.

Fluid	$P_{low}$ (bar)	$P_{high}$ (bar)	COP	$FOM_{lam}$ (W/m)	$FOM_{turb}$ (W/m) <sup>0.2</sup>
(a) $T_h = 150$ K					
Hydrogen	8.04	84	0.015	0.103	0.742
neon	2.23	200	0.023	0.112	0.777
(b) $T_h = 100$ K					
Hydrogen	8.04	120	0.090	0.382	0.949
Neon	2.23	200	0.114	0.284	0.928
(c) $T_h = 50$ K					
Hydrogen	8.04	62	0.729	0.612	0.970
Neon	2.23	82	0.958	0.512	1.018

Table 3

Maximum-COP parameters for a JT cold stage operating at 3 K and having a warm-end temperature of (a) 20 K, (b) 10 K and (c) 5 K.

Fluid	$P_{low}$ (bar)	$P_{high}$ (bar)	COP	$FOM_{lam}$ (W/m)	$FOM_{turb}$ (W/m) <sup>0.2</sup>
(a) $T_h = 20$ K					
Helium-3	0.82	8	0.034	0.0136	0.493
Helium-4	0.24	24	0.050	0.0269	0.575
(b) $T_h = 10$ K					
Helium-3	0.82	5	0.147	0.0051	0.382
Helium-4	0.24	11	0.234	0.0346	0.597
(c) $T_h = 5$ K					
Helium-3	0.82	3	0.695	0.0009	0.234
Helium-4	0.24	2	1.218	0.0323	0.565

the high pressure is shown for a cold-tip temperature of 30 K. In this figure, a warm end temperature of 150 K is chosen for the bottom two curves and 100 K for the top two curves. Both, the laminar and turbulent-case FOMs are calculated for the optimum high pressure and the maximum-COP parameters are shown in Table 2a–c. The parameters for a warm-end temperature of 50 K are also included. It is concluded that the COP of neon is highest in all cases. However, that of hydrogen is highest below a certain high pressure (Fig. 6). Furthermore, the laminar-case FOM of neon is highest at a warm-end temperature of 150 K, but highest for hydrogen at 100 K and 50 K. So, in this case, it is not clear which working fluid can best be used. This depends on the warm-end temperature and the maximum attainable high pressure, and therefore this choice has to be made by the user.

In addition, an analysis is performed for a cold-tip temperature of 3 K, for which both helium-3 and helium-4 can be used, and different warm-end temperatures (5 K, 10 K and 20 K). The results are given in Table 3a–c. For all cases, it is shown that helium-4 has the highest COP and also the highest laminar and turbulent-case FOMs. Therefore, it is concluded that helium-4 is the most suitable working fluid.

5. Discussion

In this section, the influence of the constraints (defined in Section 3) on the selection of the most suitable working fluid is discussed.

5.1. Laminar and turbulent-case FOMs

In Section 2, FOMs were derived for the cases of laminar and turbulent flows. A study is performed, to investigate if the most suitable working fluid can be determined by using only the laminar-case FOM. The calculated values of both FOMs in Tables 1–3 show that, if the laminar-case FOM of a working fluid is higher than that of another fluid, also the turbulent-case FOM is higher. It is found

that this holds for almost all working fluids. The only exception found is between the fluids argon and oxygen. The laminar-case *FOM* of oxygen is higher, whereas the turbulent-case *FOM* of argon is higher. It is thus concluded that, in the selection of the most suitable working fluid, in general only the laminar-case *FOM* (Eq. (10)) needs to be taken into account.

## 5.2. Warm-end temperature

Precooling of a JT cold stage can be beneficial, because then  $\Delta h_{51}$  increases and thus also the *COP* (Eq. (2)) [19]. Precooling is useful when the reduction of the input power of the compressor is greater than the input power required for the precooler. To investigate the effect of decreasing the warm-end temperature on the selection of the working fluid, the maximum *COP* and laminar-case *FOM* are calculated as a function of this temperature. To facilitate a better comparison between the curves, the *COP* is normalized to the Carnot efficiency ( $COP_{carnot} = T_l / (T_h - T_l)$ ). Fig. 7 shows the *COP* and the laminar-case *FOM* as a function of the warm-end temperature for cold-tip temperatures of 100 K and 180 K, respectively. From this figure, it is concluded that the maximum *COP* increases for a decreasing warm-end temperature, which is as expected. In general, the laminar-case *FOM* also increases with a decreasing warm-end temperature. Furthermore, it is shown that, although the values of the *COP* and the laminar-case *FOM* change, the selection of the most suitable working fluid is not affected by the warm-end temperature in the temperature range 65–250 K. However, in the range 2–65 K, the choice between

the working fluids is affected by the warm-end temperature, as was discussed in Section 4.2.

## 5.3. High pressure

In this analysis, the maximum high pressure was set at 200 bar. It is very difficult to produce such high pressures by means of closed-cycle compression. This means that the maximum attainable high pressure will be lower. This especially affects the maximum *COP* of a JT cold stage operating at a temperature below 200 K. For such temperatures, the maximum attainable *COP* decreases. However, this will not change the selection of working fluid, because the *COP* of the selected working fluid is, in most cases, highest for the complete high-pressure range. Exceptions are hydrogen and neon, which were discussed in Section 4.2.

## 5.4. Low pressure

The temperature range within which a working fluid can be used depends on the selected low pressure. In this analysis, a low-pressure range of 0.2–10 bar is selected. If a higher upper limit is selected, the maximum operating temperature for which a working fluid can be used will increase. However, in none of the cases is a fluid most suitable all the way up to the top of its operating range. At higher values of the low pressure, another fluid operating at a lower value of the low-pressure appears to perform better. So, choosing the low pressure to be above 10 bar will not affect the fluid selection. Changing the lower boundary of the low pressure

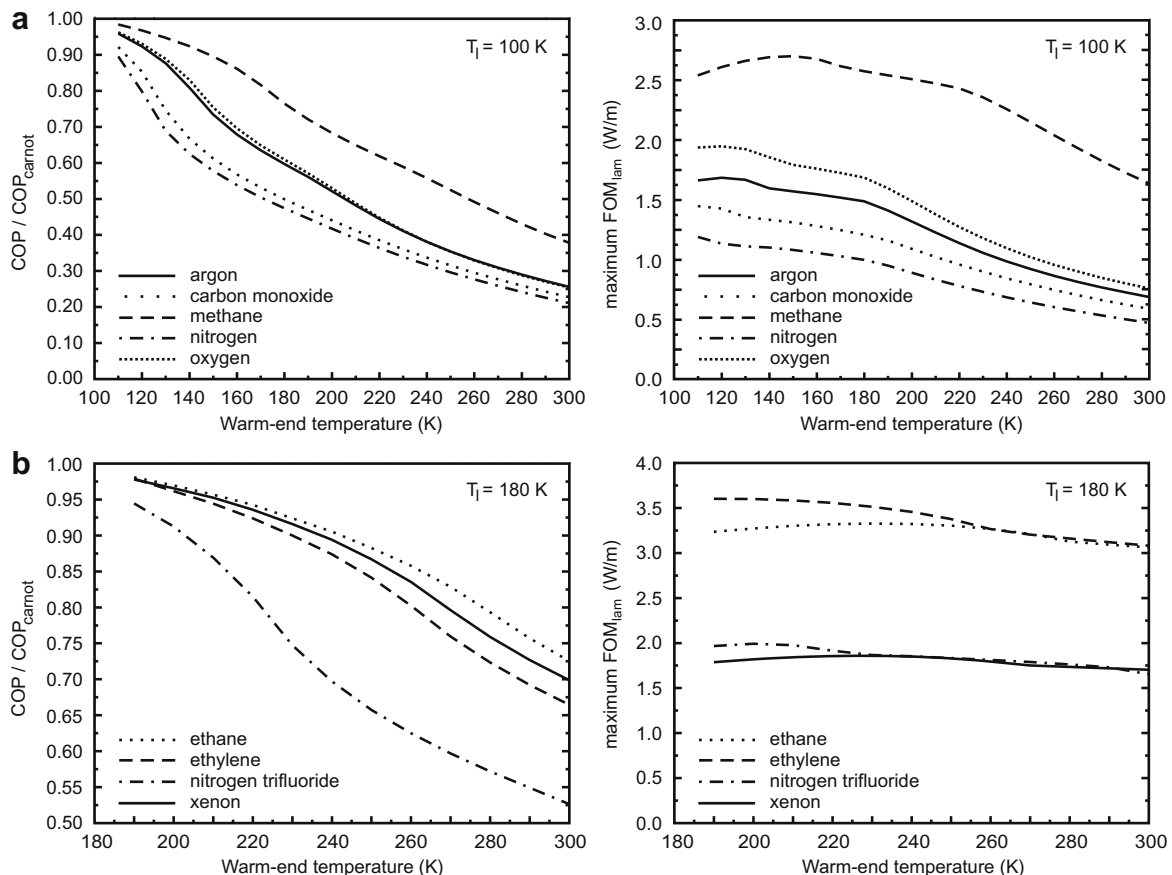


Fig. 7. Maximum *COP* and laminar-case *FOM* as a function of the warm-end temperature for a cold-tip temperature of (a) 100 K and (b) 180 K. The *COP* is normalized to the Carnot efficiency.

affects the lowest temperature that can be achieved using a specific fluid. Changing that boundary, therefore, affects the transition temperature from one optimum fluid to the next. For instance, if the low pressure is reduced from 0.2 bar to 0.1 bar, the transition of carbon monoxide to oxygen reduces from 77.1 K to 72.7 K.

## 6. Conclusions

The working fluid in a JT cold stage operating at different temperatures within a wide range of 2–250 K is optimized. The most suitable working fluid is selected on the basis of the *COP*, defined as the ratio of the gross cooling power to the change in Gibbs free energy of the fluid during compression ('available work'). In addition, a *FOM* for laminar and turbulent flow is evaluated. Both are a measure for the efficiency of the heat exchange in the CFHX. The larger the *FOM*, the shorter the CFHX can be. The most suitable working fluids in a small temperature range are helium-3 (2.0–2.9 K), helium-4 (2.9–5.2 K), hydrogen (16–25 K), neon (25–38 K), nitrogen (65–70 K), carbon monoxide (70–77 K), oxygen (77–95 K), methane (95–124 K), nitrogen trifluoride (124–146 K), ethylene (146–159 K), ethane (159–188 K), hydrogen sulfide (188–212 K) and ammonia (212–250 K). Non-toxic alternatives are suggested for the toxic working fluids. It is also shown that, in general only the laminar-case *FOM* has to be taken into account in the selection of the most suitable working fluid. Furthermore, in the temperature range 65–250 K, the warm-end temperature and the maximum attainable high pressure do not play a role in the selection of the most suitable working fluid. In the range 2–65 K, however, the selection of the working fluid is affected by these two parameters.

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