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Anticipatory routing of police helicopters

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ABSTRACT

We have developed a decision support application for the Dutch Aviation Police and Air Support unit for routing their helicopters in anticipation of unknown future incidents. These incidents are not known in advance, yet do require a swift response. A response might include the dispatch of a police helicopter to support the police on the ground. If a helicopter takes too long to arrive at the crime scene, it might be too late to assist. Hence, helicopters have to be proximate when an incident happens to increase the likelihood of being able to support the police on the ground in apprehending suspects. We propose the use of a forecasting technique, followed by a routing heuristic to maximize the number of incidents where a helicopter provides a successful assist. We have implemented these techniques in a decision support application in collaboration with the Dutch Aviation Police and Air Support. Using numerical experiments, we show that our application has the potential to improve the success rate with a factor nine. The Dutch Air Support and Aviation Police are now using the application.

1. Introduction

Police and criminals are in an ongoing race to outsmart each other. In this race, the Dutch Aviation Police and Air support (LVP) has recently renewed it's fleet of helicopters with state-of-the-art police equipment. Having good equipment is one step forward, using it effectively and efficiently will make a real difference. Because police helicopters are expensive and subject to strict aviation rules for maintenance, only a limited number of helicopters is available. As this number is insufficient to have a helicopter ready at any time at any location, the positioning of the helicopters is key to efficient usage of the available flying hours.

Incidents happen over time, are unknown in advance, and require a swift response. This makes efficient usage of a limited number of police helicopters complicated. In this research, the bases of the helicopters are considered as input as the choice for a location is a long term decision. As the number of helicopters is not sufficient to cover all incidents, i.e., being sufficiently close to the location of the incident when they occur, a solution has to come from operational decisions to maximize the number of successful assists.

The total flight duration per helicopter is considered fixed, as this is given by tactical decisions. Therefore we reformulate the problem to a maximization problem, in which we maximize the weighted expected number of covered incidents, where the probability of covering an incident depends on the proximity of a helicopter, and where the weight represents the priority given to such an incident.

In this paper, we propose a forecasting technique, followed by a routing heuristic to maximize the weighted expected number of covered incidents. We believe this approach can be used for efficient positioning of any type of emergency vehicle, with minor modifications to account for the difference in freedom of movement. Furthermore, the approach could also be applied to for example taxis in New York City, where the 'incident' would be a passenger requiring a taxi and who takes the first one that becomes available at close proximity. As with police helicopters, covering a passenger's demand requires a swift response.

To the best of our knowledge, the combination of forecasting incidents and scheduling the routes of the helicopters on a few minutes basis has not yet been researched. The scientific contribution of our research is filling this gap, as well as proposing algorithms and models to support a planner of emergency vehicles to route the vehicles in anticipation of emergencies in such a way that the total expected success rate of the emergency vehicles is maximized. Furthermore, our research combines the Dynamic Vehicle Routing Problem with the Location Covering Problem, and combines vehicle positioning with incident forecasting.

The practical contribution of our research is a decision support application that enables the Dutch Aviation Police and Air Support to improve their success rate by proactively positioning their helicopters. The application can be used on a daily basis. Furthermore, it allows for rerouting a single helicopter within a minute. This allows adjusting the route of a police helicopter after it has assisted the ground police.

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The remainder of this paper is structured as follows. Section 2 describes the related literature. The formal problem definition is given in Section 3. Forecasting of incidents is discussed in Section 4, followed by the routing of helicopters in Section 5. Section 6 contains an overview of the instrument developed for the LVP. We end with conclusions in Section 8.

2. Literature

In this research, we focus on the positioning of vehicles in anticipation of unknown requests which require a quick presence. Among other vehicle types, this applies to emergency vehicles. These emergency vehicles can be ambulances, police cars, and helicopters. Although various vehicle types have different restrictions on their movement, the concept behind the models to solve these problems can be used for any vehicle.

The problem of the positioning of police helicopters to cover (unknown) incidents is related to the Vehicle Routing Problem (VRP) and the Location Covering Problem (LCP). The VRP is about routing one or more vehicles to fulfill demand, whereas the LCP is about locating a unit (e.g., a gas station) to cover the demand of an area. Our problem is on the edge of both problems, as we aim to route helicopters to maximize the coverage they give.

Among the first proposed models suitable for solving the problem of positioning emergency vehicles are the Location Set Covering Problem (LSCP) by Toregas et al. (1971), and the Maximal Covering Location Problem (MCLP) by Church and ReVelle (1974). In the LSCP, a set of locations is given where a facility might be opened. A facility is an object that gives coverage to a given area around it. Furthermore, a set of demand points is given as well as the distance from each possible facility location to the demand points. The objective of the LSCP is to minimize the number of required facilities such that each demand point is at most a predefined distance away from the closest facility. Like the LSCP, the MCLP also has a set of locations where facilities might be opened, and a set of demand points. However, in the MCLP, a fixed number of facilities is given. Therefore, the objective function is to maximize the number of demand points lying within a predefined distance from their closest facility.

As Gendreau et al. (2006) state, both the LSCP and the MCLP make sense in practice for use with emergency vehicles: the LSCP can be used to determine the required number of emergency vehicles to cover all demand, whereas the MCLP can be used to optimally position emergency vehicles when insufficient vehicles are available to cover every demand point. Schilling et al. (1979) propose an extension to take different types of facilities into account in the context of the Baltimore City Fire Protection System. However, they argue their findings are general and can be used for other emergency vehicles as well. Daskin and Stern (1981) added a second objective to the LSCP to measure the number of times a point is covered above its required coverage. Hogan and Revelle (1986) continued on this work by introducing the backup coverage problem. For an overview of extensions, we refer to Li et al. (2011).

The LSCP and the MCLP are static models. In order to account for a vehicle being dispatched to a call, probabilistic models have been developed. Larson (1974) was among the first to research the concept of emergency vehicles being a server in a region with demand. The demand arrives over time and enters the queue of uncovered demand of the emergency vehicle. As soon as the emergency vehicle has finished one request, it will start handling the next request. A request might also leave the queue when it cannot be handled in time. Daskin (1983) developed an integer programming formulation for the probabilistic covering problem, the Maximal Expected Covering Location Problem (MEXCLP). Batta et al. (1989) made an extension to the MEXCLP, the Adjusted Maximal Expected Covering Location Problem (AMEXCLP), which relaxes the assumptions that servers operate independently, servers have the same busy probabilities and are invariant with respect to their locations. Repede and Bernardo (1994) also made an extension to the MEXCLP by adding time variation, known as the TIMEXCLP. For situations with insufficient data on the travel times to make a statistical distribution, Davari et al. (2011) presented a Fuzzy Maximal Covering Location Problem (FMCLP) to take into account the variation estimated by experts. We refer to Owen and Daskin (1998) for a detailed review of probabilistic covering models.

Availability after a vehicle has been dispatched is ignored by static models (Brotcorne et al., 2003), and probabilistic models only take this into account to some extent. In order to really take dispatches into account, dynamic models have been developed in which vehicles are relocated after a dispatch or when new information arrives. Kolesar and Walker (1974) note that in case of a large fire or multiple smaller fires, a new positioning of fire trucks will yield a better coverage. They have developed a model for the New York City Fire Department; however, they state their algorithm should be applicable to other cities as well. Gendreau et al. (2001) propose a dynamic model that uses Tabu Search and is based on the model of Gendreau et al. (1997). Furthermore, they note that more challenging problems can be solved with the use of parallel processing. Rajagopalan et al. (2008) propose the use of a reactive Tabu Search algorithm for relocation of emergency vehicles. Boctor et al. (2011) define the Emergency Vehicle Relocation Problem in which the cost of relocation is taken into account. Furthermore, they propose two heuristics to solve this problem.

The aforementioned methods have in common that they require some forecast. A forecast is an estimate of what future observations will be if the underlying process continues as it has in the past (Brown, 2004). Gorr and Harries (2003) note that conventional forecasting methods are not or hardly effective for forecasting the moment an individual will commit a crime. They question whether crime forecasting is possible due to the uniqueness of crimes. Their answer on this question is that patterns can be recognized on a aggregated level. Sherman et al. (1989) discuss the phenomenon of hot spots, areas that have relatively much overall criminal activity. Block (1995) proposes a statistical tool for law enforcement decisions named Spatial and Temporal Analysis of Crime (STAC). STAC aims at discovering and describing hot spot areas. Felson and Poulsen (2003) discuss that crime varies by time of the day. Liu and Brown (2003) propose the use of a point-pattern-based density model, which uses criminal preferences obtained from past crimes. Deadman (2003) reviews forecasts made by Dhiri and Great Britain (1999). These forecasts were made in 1999 and were for the years 1998–2001. Deadman (2003) notes that time series models perform reasonably well. It can be concluded that from an aggregated level, it is possible to make probabilistic statements on the occurrence of crimes depending on time and place.

Corcoran et al. (2003) note that a continuous updating forecasting tool will help the real-time allocation of police resources. As discussed by Gorr et al. (2003), the forecasting errors become acceptable when the number of crimes used for a forecast is at least in the order of thirty or more. Field (1999) shows a correlation between the expenditure in the last four years by real consumers and the number of incidents. For every percent increase in the expenditure, incidents increase with two percent. Furthermore, Field (1999) shows there is a positive correlation between the number of incidents in an area and the number of young males in this area. An increase of one percent in the number of males in the age of fifteen to twenty, results in an increase of incidents of one percent in that area. It can be concluded that there are dependencies when looking at the characteristics on an aggregated level. These dependencies allow for use in forecasting. The challenge here is to generalize from specific events (e.g., an incident at a specific time and day, at a specific location by a specific group of criminals) to a criminal intensity factor for an area. For this, there are many generalization methods available. We refer to Sutton and Barto (1998) for an overview.

Anticipatory decision-making is the concept of making decisions in anticipation of future events that are unknown at the time of decision-making. In the field of vehicle routing, this is known as anticipatory routing. Anticipatory routing is often used to route trucks, as carriers know only part of their orders when the initial plan is made, or to avoid traffic jams (Claes et al., 2011). In this section, we focus on routing in anticipation of future demand, as traffic jams do not apply to the police helicopters.

Anticipatory routing is part of the larger domain of the Dynamic Vehicle Routing Problems (DVRP). DVRP is a generalization of the Vehicle Routing Problem (VRP), which is the problem where multiple vehicles are available to fulfill known requests (Laporte, 1992). For DVRP, these future request are unknown. Many extensions and special cases of this problem exist. Proposed methods include (i) rolling horizon approaches, (ii) replanning when new information becomes available, and (iii) including forecasted orders in the initial plan. We refer to Eksioglu et al. (2009) for an extensive overview of vehicle routing.

In anticipatory vehicle routing, routes are made for vehicles, in which the vehicles drive around and wait in anticipation of future demand. When and where to drive and wait is based on forecasts of future demand. Examples of anticipatory routing can be found in Branke et al. (2005), where additional customer requests are being scheduled in otherwise fixed tours; Ichoua et al. (2006), where dummy orders are included in the plan; Thomas (2007), where the number of customers served by a single incapacitated truck is maximized; and Mes et al. (2010), where look-ahead strategies are considered for full truckload pick up and deliveries. Once the anticipated demand becomes known, real-time rerouting is required, see e.g., Liao and Hu (2011).

Our problem of positioning police helicopters such that the number of successful assists is maximized can be described as a DVRP with soft time windows. In our problem, there are no hard time windows, as it is allowed not to send air support. There is a soft time window, which starts at the moment the incident happens. Although describing our problem as a DVRP is straightforward, proposed models to solve the DVRP are less suitable for our problem due to the short available time for decision making. Furthermore, it is sufficient to be approximate to incidents. Therefore, the problem is better described as a combination of the DVRP and the Location Covering Problem. This allows to route police helicopters in such a way that they cover as many future incidents as possible. We propose to name this special case the Anticipatory Emergency Vehicle Routing Problem (AEVRP).

The scientific contribution of our research is the combination of forecasting incidents and scheduling the routes of the helicopters on a few minutes basis. Specifically, we propose a model to support a planner of emergency vehicles to route the vehicles in anticipation of emergencies in such a way that the total expected success rate of the emergency vehicles is maximized. Furthermore, our research combines the Dynamic Vehicle Routing Problem with the Location Covering Problem, and combines vehicle positioning with incident forecasting.

3. Problem definition

The AEVRP has two aspects: where will incidents happen and where to position the helicopters. The first aspect is described in Section 4, the latter is described in detail in Section 5. As the incident forecasts made by the model in Section 4 will be used for positioning helicopters, the requirements implied by the positioning model have to be taken into account. Therefore we first give a formal definition of the problem.

In order to give air support, the LVP has a homogeneous fleet of helicopters $\mathcal{H}$. Each helicopter $h \in \mathcal{H}$ has a location $w \in \mathcal{W}$ at any time interval $t \in \mathcal{T}$. A location $w \in \mathcal{W}$ is defined as a single connected area of any shape or size, and a time interval is a duration specified in time. The movement of a helicopter $h \in \mathcal{H}$ is restricted by its speed and other air traffic. Helicopters are either flying, standby on the ground, or out of order due to, for example, maintenance. When a helicopter is standby on the ground, it takes p_w time intervals to get airborne depending on its location w. Here, there is a difference due to presence of other air traffic, which differs per location due to, for example, the proximity of an airport. Each helicopter $h \in \mathcal{H}$ has a set of attributes. These attributes are (i) cruising speed, (ii) remaining fuel level, (iii) attached equipment, (iv) maximum flight duration before the next scheduled maintenance to comply with aviation rules, and (v) time windows for which crew is available.

In this research, we chose to not explicitly take into account the preventive effect coming from the presence of a police helicopter, which has not been studied yet. As incidents are not known in advance, we define the objective to maximize the total weighted expected coverage. To compute this, we need (i) the fraction of coverage G_{wt} for location w during time interval t, and (ii) the criminal intensity factor i_{wt}. The value of G_{wt} can be seen as the probability a successful assist will be made if an incidents happen at location w during time interval t. Intuitively, i_{wt} depends on incidents that happened in the past at or approximate to the location w during or approximate to time interval t. In Section 4, we discuss i_{wt} in more detail. We define weighted expected coverage of a location $w \in \mathcal{W}$ during time interval $t \in \mathcal{T}$ as $G_{wt} \cdot i_{wt}$.

We give a formal definition of the AEVRP as a Mixed Integer Linear Program (MILP). We assume a forecast is available for use in this mathematical model, and is made for the set of locations $\mathcal{W}$ and the set of time intervals $\mathcal{T}$. This leads to the MILP given by Eqs. (1)–(11). Eq. (1) is the objective function, which maximizes the total weighted expected coverage.

$$\max \sum_{w \in \mathcal{W}, t \in \mathcal{T}} (G_{wt} \cdot i_{wt}) \tag{1}$$

Eqs. (2) and (3) restrict the fraction of coverage per area and per time interval to at most the minimum of 1 and the sum of all coverage G_{wt}. As i_{wt} are the nonnegative criminal intensity factors, the objective function is bounded by the minimum of Eqs. (2) and (3). The availability of a helicopter in time interval t is represented by the parameter d_{ht}, and the coverage a helicopter at location a gives to a location w is c_{wa}. This coverage is based on a decreasing probability function $g(w,a)$, proposed by experts from the LVP, and depends on the distance between two locations w and a. In this function, support within ten minutes is considered to be always successful, the probability of success decreases between ten and fifteen minutes, and support after fifteen minutes is considered unsuccessful. The binary variable L_{wht} indicates whether helicopter h is at location w in time interval t.

$$G_{wt} \leqslant \sum_{a \in \mathcal{W}, h \in \mathcal{H}} (L_{aht} \cdot c_{wa} \cdot d_{ht}) \quad w \in \mathcal{W}, t \in \mathcal{T} \tag{2}$$

$$G_{wt} \leqslant 1 \quad w \in \mathcal{W}, t \in \mathcal{T} \tag{3}$$

Eqs. (4) and (5) restrict the movement of a helicopter to exactly one area in the subset W'_{wht}, which contains the locations that can be reached within one time interval from the location w of helicopter h in the time interval $t-1$. The subset W'_{wht} can be computed upfront and used as input for the algorithm.

$$L_{wht} \leqslant L_{ah,t-1} \quad w \in \mathcal{W},\ h \in \mathcal{H},\ t \in \mathcal{T},\ a \in \mathcal{W}'_{wht} \tag{4}$$

$$\sum_{w \in \mathcal{W}} L_{wht} = 1 \quad h \in \mathcal{H},\ t \in \mathcal{T} \tag{5}$$

The duration of flights is limited by the fuel capacity. We define f_h to be the maximum flight duration in time intervals. The remaining flight duration is represented by F_{ht}. When being refueled, a helicopter h gains remaining duration worth g_h time intervals per time interval it is being fueled. The remaining flight duration e_h represents the starting fuel level in time intervals. Eqs. (6)–(9) take the fuel consumption into account. As helicopters can only be refueled when on the ground, we introduce binary variables A_{ht} that indicate whether helicopter h is airborne at time interval t.

$$F_{h,t+1} \leqslant F_{ht} - A_{ht} + g_h \cdot (1 - A_{ht}) \quad h \in \mathcal{H},\ t \in \mathcal{T} \tag{6}$$

$$F_{ht} \leqslant f_h \quad h \in \mathcal{H},\ t \in \mathcal{T} \tag{7}$$

$$F_{ht} = e_h \quad h \in \mathcal{H},\ t = 0 \tag{8}$$

$$A_{ht} \leqslant F_{ht} \quad h \in \mathcal{H},\ t \in \mathcal{T} \tag{9}$$

Eq. (10) restricts the movement of a helicopter such that it can only move when airborne. Eq. (11), restrict the total time a helicopter is allowed to fly during the planning horizon, due to for example regulations on time between maintenance.

$$L_{wh,t+1} \geqslant L_{wht} - A_{ht} \quad w \in \mathcal{W},\ h \in \mathcal{H},\ t \in \mathcal{T} \tag{10}$$

$$\sum_{t \in \mathcal{T}} A_{ht} \leqslant n_h \quad h \in \mathcal{H} \tag{11}$$

The police sometimes has intelligence about potential future incidents, which is obtained, for example, via an undercover operation. This intelligence can be taken into account, by setting l_{wht} equal to 1 for a helicopter h during time interval t such that location w is covered during this time interval. We may set multiple adjacent l_{wht} to 1 to allow for longer coverage. This is done using Eq. (12).

$$L_{wht} \geqslant l_{wht} \quad w \in \mathcal{W},\ h \in \mathcal{H},\ t \in \mathcal{T} \tag{12}$$

Often, it is undesired to hover over a single location for a longer period, as this leads to too much noise for citizens. To take this into account, we added Eq. (13). This constraint restricts the total duration above a location to be at most m time intervals during a single flight.

$$\sum_{h \in \mathcal{H}, t \in \mathcal{T}} L_{wht} \leqslant m \quad w \in \mathcal{W} \tag{13}$$

In this section we have defined the problem of routing the police helicopters as a MILP. Solving this MILP will yield the optimal positioning of all helicopters. However, solving this MILP is not possible for problems of practical size. We propose a solution to this problem in Section 5, but first we propose a method to come up with the incident forecast i_{wt} in Section 4.

4. Incident forecasting

We propose to split the problem as defined in Section 3 into a forecast problem, and a routing problem. The latter is discussed in Section 5. As the forecasting problem and routing problem are related, we first define the relation between these problems.

The input we have is the historic data about incidents. These incidents have the attributes (i) location, (ii) date, (iii) time, and (iv) weight. The weight given to an incident is relative to other weights, where a higher value means a higher weight. The input we require for the routing problem is a criminal intensity factor i_{wt} for each location $w \in W$ and time interval $t \in \mathcal{T}$. To reduce the forecast error, sufficient data has to be available to create a reliable forecast i_{wt} for each $w \in \mathcal{W}$ and $t \in \mathcal{T}$. In Section 4.2, we propose doing this by generalization for the time and location, and by conversion as explained for the seasonality observed in months and weekdays. However, we first discuss the forecast area classification in Section 4.1.

4.1. Forecast area classification

As the forecast for the Netherlands as a whole does not give any insight on where to fly, we have to divide the Netherlands in smaller areas $w \in \mathcal{W}$. We define a grid of areas to use for forecasting. The use of circles is a natural first choice, as it represents the area a helicopter can reach in during a given period. However, equally sized circles will either not cover the entire area or have overlap. Uncovered areas are not favorable, as incidents in such an area are not covered. Overlap is not favorable either, as it leads to the possibility of being in multiple areas at the same time. Therefore, we use an alternative shape. For each point in an area, we want the closest center to be the center of that area. Therefore, only convex shapes are considered. Grunbaum and Shephard (1977) note there are only three regular convex polygons that give an edge-to-edge tiling. These shapes are the equilateral triangle, the square, and the regular hexagon.

During routing, we allow helicopters to move from one area to another area only if the two areas share an edge. It can be verified that the hexagonal tiling has the best worst-case distance error, which is defined as the maximum difference between the shortest travel time and the travel time when moving over the edges of adjacent areas. Besides the smaller distance error, hexagons also allow for more flight directions each step. Therefore, we propose the use of a hexagonal grid. In this research, we assume all helicopters have an identical cruising speed of two nautical miles per minute. This assumption corresponds with the use of helicopters of the type Eurocopter 135. In this research, we set the inner radius of the hexagon to 1 min or 2 nautical miles. This implies the distance between the centers of two neighboring hexagons is 2 min or 4 nautical miles.

4.2. Forecasting algorithm

Although more incidents happen than desired, the number of incidents that took place within an hexagon of size 4 nautical miles will be relatively low. As helicopters are considered to be the fastest emergency vehicle, this also applies to other emergency vehicles. Therefore, we use generalization techniques, and use all historic data to reduce the forecast error. To use all historic data, we have to convert each incident, such that it can be used for the date under consideration. The generalization techniques extrapolates an incident at one specific location and time to neighboring areas and some time periods around the incidents.

As crime rates differ between moments of the day, days of the week, and months of the year, we have to take this into account. In order to do so, we have defined two conversion factors: a *FactorWeekday*(*weekday*, *hour*) and a *FactorMonth*(*month*, *hour*). The *FactorMonth* converts the month of an incident into the month of the day to be forecasted. This conversion is done for each hour of the day, such that the percentage of weighted incidents in an hour is equal for both months. This is done by Algorithm 1, with *TargetMonth* the month for which we create a forecast. For this algorithm, we introduce the set $\mathcal{I}$ as the set of historic incidents. Based on the attributes date and time, we can derive for each incident its year, month, day of the week, and hour.

Algorithm 1: Generate *FactorMonth* for a given *TargetMonth*.

for each *incident* in $\mathcal{I}$ **do**
  $Hourly[incident_{month}, incident_{hour}] += incident_{weight}$
  $Total[incident_{month}] += incident_{weight}$
end for
for *month* = January **to** December **do**
  for *hour* = 0 **to** 23 **do**
    $Fraction(month, hour) = Hourly[month, hour]/Total[month]$
  end for
end for
for *month* = January **to** December **do**
  for *hour* = 0 **to** 23 **do**
    $FactorMonth(month, hour) = \frac{Fraction[TargetMonth, hour]}{Fraction[month, hour]}$
  end for
end for

The algorithm gives the *FactorMonth* for each combination of month and hour with the *TargetMonth*. By replacing all references to month with weekday, Algorithm 1 gives the *FactorWeekday* for each combination of weekday and hour with the *TargetWeekday*. This enables us to use all historic incidents from all months and weekdays to generate a forecast.

In order to generate $i_{w,t}$ for all w and t, we use Algorithm 2. As we move forward in time, older incidents in $\mathcal{I}$ are likely to become a less accurate predictor of future incidents. Therefore we propose the use of a forget factor α that represents the percentage of weight an incident loses every month. The calculation can be found on lines 2–3 of Algorithm 2. Lines 4–7 of this algorithm apply the FactorMonth and FactorWeekday. Line 8 creates the forecast without any generalization. Lines 10–11 generalize in space and time and are discussed after Algorithm 2. In line 11, ϕ_{z,σ^2} represents the probability density function of the normal distribution with average z and variance σ^2.

Algorithm 2: Generate i_{wt}.

1: **for** each incident in $\mathcal{I}$ **do**
2:   $MonthsOld = 12 * (TargetYear - incident_{year}) + TargetMonth - incident_{month}$
3:   $incident_{weight} = (1 - \alpha)^{MonthsOld}$
4: **end for**
5: Execute Algorithm 1
6: Execute Algorithm 1 for Weekdays
7: **for** each incident in $\mathcal{I}$ **do**
8:   $incident_{weight} *= FactorMonth[incident_{month}, incident_{hour}]$
9:   $incident_{weight} *= FactorWeekday[incident_{weekday}, incident_{hour}]$
10:   $i''_{incident_{location}, incident_{timeInterval}} += incident_{weight}$
11: **end for**
12: $i'_{wt} = \sum_a i''_{at} / distance(w, a)^2$
13: $i_{wt} = \sum_z i'_{wz} * \phi_{z,\sigma^2}(t - z)$

We assume generalization in the space dimension is justified as data about one location gives information about its surrounding area, similar to a thermometer in one room, which gives an indication about the temperature in the room next to it. We assume generalization in time is justified in a similar way as space. The current temperature in a room gives information about what it could have been fifteen minutes ago or in half an hour. This also applies to the second room mentioned in the space dimension example, although this indication might become less reliable.

In order to generalize in the time and space dimensions, we propose to first generalize in space for each time interval. Next, the result of the forecast in the space dimension can be used as input for the generalization in the time dimension. This yields a forecast that is suitable for helicopter routing.

The assumptions for the space generalization distribution and time generalization distribution are based on expert opinions. As can be seen in the algorithm, all data can be used and scaled based on various attributes, such that we learn about the entire history.

Although we believe the hexagonal grid is the best solution for police helicopters, we are aware this is not the case for land or water vehicles as roads and waterways have to be followed. In order to overcome this problem, the functions used to calculate whether a hexagon can be reached within one time interval can be substituted by a parameter δ_{ij} indicating whether area j is at most one time interval away from area i. This also gives the opportunity to take different travel speeds for different streets into account.

After the incidents have been converted and generalized in space and time, we have obtained the criminal intensity factor i_{wt} for each location $w \in W$ and time interval $t \in T$, which is more reliable than the criminal intensity factor i''_{wt}, given the relative small number of data points.

5. Helicopter routing

In this section, we consider the forecast obtained in Section 4 as a fact. This allows us to use the earlier defined MILP to solve the problem. However, as the problem becomes too complicated for solving practical sized problems, we have to resort to a heuristic. In this section, we propose two heuristics to tackle this problem. First, we propose a heuristic that uses Random Search for finding a good departure time, and runs for the maximum available duration. Second, we propose a heuristic that makes use of a algorithm to find the most promising start time for each flight.

In order to derive the neighbors of a location without an additional parameter, we define a hexagonal coordinate system. With this system we define a location $w \in \mathcal{W}$ from now on as a location $(x, y) \in (\mathcal{X}, \mathcal{Y})$. Fig. 1 shows the hexagonal coordinate system we use for this research. As can be seen, a hexagon at location $(x, y) \in (\mathcal{X}, \mathcal{Y})$ is surrounded by the hexagons with coordinates $\{(x, y+1), (x+1, y+1), (x+1, y), (x, y-1), (x-1, y-1), (x-1, y)\}$. This enables us to refer to the six neighbors of any location with a single function.

5.1. RDDT heuristic

The first heuristic is the Randomly Determined Departure Time (RDDT) heuristic. As shown in Fig. 2, it starts with the forecast obtained in Section 4 as input, as well as a list of available helicopters, the number of flights, and the duration, start, and end location of

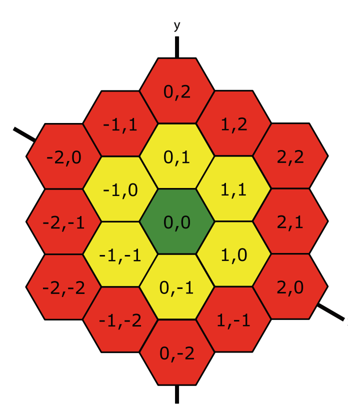

Fig. 1. Hexagonal coordinate system developed for this research.

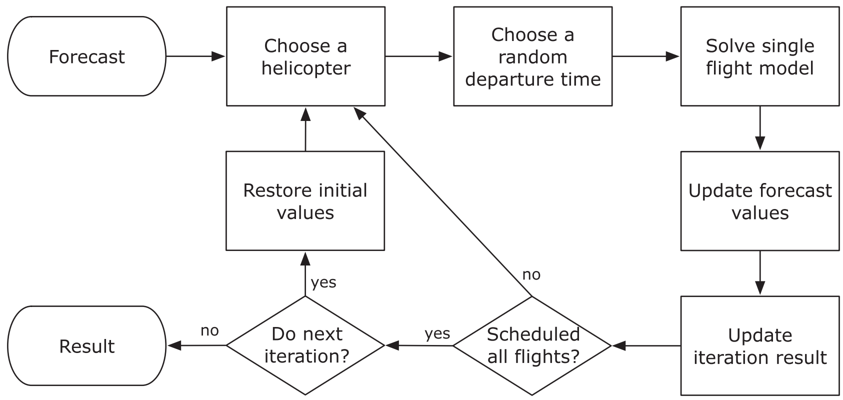

Fig. 2. Graphical representation of the RDDT heuristic.

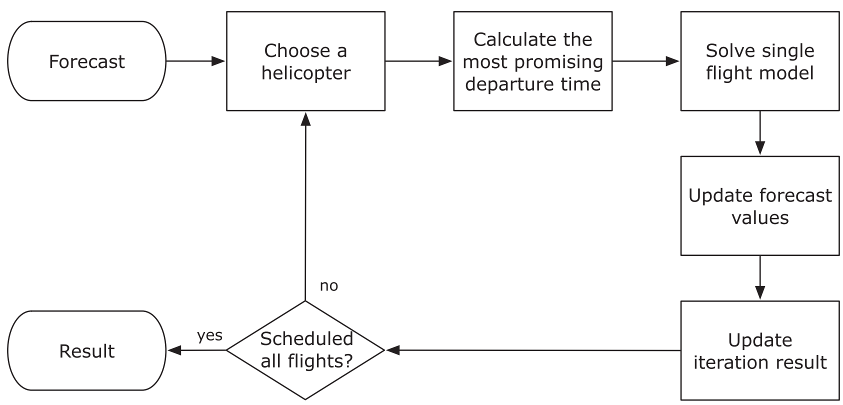

Fig. 3. Graphical representation of the MPDT heuristic.

each flight. Flights are scheduled one at a time and the departure time is chosen randomly, where the probability a time interval is chosen is given by the criminal intensity of the time interval divided by the criminal intensity of all time intervals. The chosen distribution corresponds with the relative density of the incidents.

After a departure time is chosen, a MILP is solved to optimize the single flight. This MILP is based on the MILP defined in Section 3. We defined the locations $w \in \mathcal{W}$ to be of hexagonal shape, and redefined the set of locations to $(x,y) \in (\mathcal{X},\mathcal{Y})$. All variables and parameters formerly indexed with w are now indexed with (x,y). Furthermore, we simplify some constraints and consider some decisions to be made in advance, such as the departure time.

The objective (Eq. (14)) is to maximize the total weighted coverage of all areas. G_{xyt} is the total coverage obtained by the area $(x,y) \in (\mathcal{X},\mathcal{Y})$ at time interval t$\in \mathcal{T}$.

$$\max \sum_{(x,y)\in(\mathcal{X},\mathcal{Y}),t\in\mathcal{T}} (G_{xyt} \cdot i_{xyt}) \tag{14}$$

The coverage each area receives depends on the location of the helicopter and the coverage an area receives from another location (Eq. (15)). The variable L_{abt} is binary and represents whether the helicopter is at location (a,b)$\in (\mathcal{X},\mathcal{Y})$ at time interval t$\in \mathcal{T}$. The parameter c_{xyab} represents the fraction of coverage a helicopter delivers to location $(x,y) \in (\mathcal{X},\mathcal{Y})$ when flying at location $(a,b) \in (\mathcal{X},\mathcal{Y})$.

$$G_{xyt} = \sum_{(a,b)\in(\mathcal{X},\mathcal{Y})} L_{abt} \cdot c_{xyab} \quad (x,y) \in (\mathcal{X},\mathcal{Y}), t \in \mathcal{T} \tag{15}$$

Eq. (16) restricts the movement of the helicopter. and replaces Eq. (4). A helicopter can only be at a location at which it was in the previous time interval or in one of the surrounding locations. The seven terms on the right hand side of the equation correspond with the same location and its six neighbors.

$$\begin{aligned} L_{xyt} \leqslant\ & L_{xy,t-1} + L_{x-1,y-1,t-1} + L_{x-1,y,t-1} + L_{x,y+1,t-1} + L_{x+1,y+1,t-1} \\ & + L_{x+1,y,t-1} + L_{x,y-1,t-1} \quad (x,y) \in (\mathcal{X},\mathcal{Y}),\ t \in \mathcal{T} \end{aligned} \tag{16}$$

Eq. (17) forces a helicopter to be at exactly one location at every time interval t$\in \mathcal{T}$. Together, Eqs. (16) and (17) ensure the helicopter either stays at a location or moves to one of the surrounding locations.

$$\sum_{(x,y)\in(\mathcal{X},\mathcal{Y})} L_{xyt} = 1 \quad t \in \mathcal{T} \tag{17}$$

We added Eq. (18) to restricts the number of visits to each location to at most m.

$$\sum_{t\in\mathcal{T}} L_{xyt} \leqslant m \quad (x,y) \in (\mathcal{X},\mathcal{Y}) \tag{18}$$

After the single flight is solved, the forecast is updated to take into account locations that are already covered. A location is updated by removing a fraction of the criminal intensity equal to the fraction of the coverage gained in the current flight. Furthermore, the covered amount and the scheduled flight are added to the current overall solution. The RDDT heuristic runs until the time limit is reached, such that the result of the best iteration can be used.

Currently, the decision to do the next iteration is only based on the time available. However, any stopping condition can be implemented in this step such as, for example, after a number of iterations without an improved result.

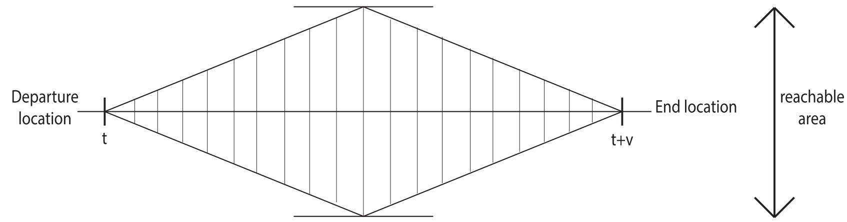

Fig. 4. Graphical representation of the areas that are reachable in a flight.

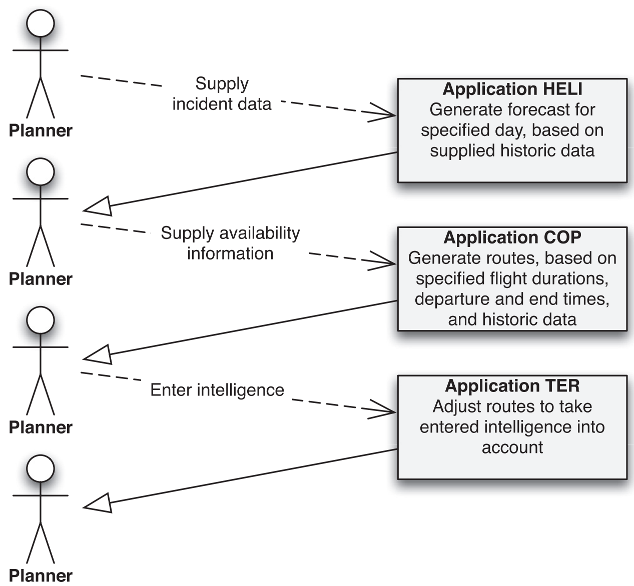

Fig. 5. Graphical representation of the interaction between the user and the application.

5.2. MPDT heuristic

The second heuristic, which we denote by the Most Promising Departure Time (MPDT) heuristic, is shown in Fig. 3. The MPDT heuristic is similar to the RDDT heuristic but differs on the method to define a departure time. Instead of randomly defining a starting time, the most promising starting time is calculated. Due to this calculation, which requires no random numbers, only a single iteration is required. Therefore, the MPDT heuristic is quicker than the RDDT heuristic.

The calculation of the most promising departure time is based on the concept that given a certain starting time, the reachable area of an helicopter is limited. When a helicopter leaves from a heliport at time t, it can be at most u minutes flying away from the heliport at time $t + u$. Similarly, when considering the end location, the helicopter should be at most w minutes flying away at time $t + v - w$, where v is the flight duration. Let us define the set $\mathcal{R}(z)$ as the set of locations and times that can be reached from the start location and end location, represented by combination z.

The total cover value for a departure time can be calculated by taking the sum over all locations at all time intervals that can be reached from the start location, and reach the end location. Taking the maximum over the set $\mathcal{T}^*$ of all possible departure times yields the most promising departure time. This leads to Eq. (19), which can be seen as the double sum over i'_{xyt} in the space dimension and time dimension. A graphical representation is given in Fig. 4.

$$\text{Most promising departure time} = \arg\max_{t\in\mathcal{T}^*} \sum_{z=t}^{t+v} \sum_{(x,y)\in\mathcal{R}(z)} i'_{xyz} \qquad (19)$$

The input in for the calculation of the most promising departure time is the criminal activity indicator i'_{xyt}, which is the criminal activity indicator i_{xyt} adjusted for the area already covered by previous scheduled flights (See Fig. 5).

5.3. Comparison

Obviously, the MPDT heuristic is faster, because we consider only one possible starting time for each helicopter. The RDDT heuristic is designed to run as long as possible. In practice this means a running time of around 23 h, as it is likely to be used today for tomorrows schedule. The computation time of the MPDT heuristic is linear dependent on the number of flights scheduled. When scheduling for one day, which typically consists of ten flights for five different helicopters, we observed the calculation time of the MPDT heuristic is a fraction of those 23 h.

Although at first glance, one might expect the RDDT heuristic to give a better result, this turned out to be incorrect. In a preliminary comparison run we did for one week, we obtained the results of Table 1. These results show with 99% confidence that the MPDT heuristic outperforms the RDDT heuristic when scheduling a total of 10 flights for five different helicopters with the running time for the RDDT heuristic capped at 23 h. Therefore, we propose the use of the MPDT heuristic, or start with the MPDT heuristic, and then use the RDDT heuristic to search for improvements.

6. Application

In order to enable the LVP to use the techniques proposed in this paper, we developed an application in the AIMMS software package. The intended use of the application is to route the helicopters on a daily basis. As the LVP is among the units in the Dutch police that see a clear benefit of using new techniques to make better use of available data, we developed the application in such a way that it can be used relatively easy to study multiple scenarios. We developed this application in AIMMS as the LVP was already familiar with this software. Furthermore, AIMMS allowed us to easily combine programming, graphical user interface building, and the power of MILP solvers such as CPLEX.

Our application consists of three major parts:

HELI *H*euristic *E*xpected *L*ocation of *I*ncidents
COP *C*overage *O*ptimization *P*rocess
TER *T*ool for *E*xpress *R*erouting

Table 1
Comparison results for the two heuristics.

Date	RDDT heuristic	MPDT heuristic	Difference (%)
1 Jan 2012	103,195	107,435	+4.1
2 Jan 2012	434,658	456,029	+4.9
3 Jan 2012	307,957	337,612	+9.6
4 Jan 2012	415,742	423,615	+1.9
5 Jan 2012	1,648,512	1,722,670	+4.5
6 Jan 2012	751,551	764,920	+1.8
7 Jan 2012	851,376	869,265	+2.1

The HELI part is where we implemented the forecasting techniques. The user can give the required input for forecasting and the resulting forecast will be shown here. The COP part is where the routes are generated. When the HELI part is already done, the user gives the input related to the routing and the resulting routes will be shown here. The TER part allows to take intelligence into account after the COP part is finished. The user enters the intelligence and is shown the updated route.

The first part of the application we discuss here is the HELI part, for which a screen capture is shown in Fig. 6. A conversion file from zipcodes to the hexagonal coordinate system is required first. This conversion file is not built into the application to enable the LVP to update the list of zipcodes, without requiring modification of the application. Next, the file with historic incidents has to be loaded. This file contains the date, time, zipcode, and weight of each incident. After the target day is set, the forecast is generated. This yields the map in the center of Fig. 6. The graphical representation of the forecast of the time interval shown on the map can be changed with the slider on the right.

After the forecast is generated, the user continues to the COP part as shown in Fig. 7. The user enters the crew shifts, and the available flights with the required details. after the maximum number of visits per location is entered, the routing process can be started. This process works as shown in Fig. 3. All routes are scheduled within approximately 60–90 s per flight. When all routes are scheduled, the map in the middle shows one of the flights. The slider on the right can be used to switch between planned flights. The flight information is shown below the slider. For a more detailed view, the routes can be exported to the KML format, which can be loaded in GIS software.

When required, the user can add points of interest in the TER part as shown in Fig. 8. The main difference between the two graphical user interfaces is the ability to enter intelligence instead of entering crew information. The latter can be done by entering the zipcode and time interval, or by looking up the hexagonal coordinates at the correct time interval. Rescheduling a flight takes approximately 60–90 s. This enables the LVP to dispatch a helicopter to an incident, and schedule the remainder of the flight when the helicopter is about to move away from that incident.

7. Validation

We have verified the outcome of both the forecasting technique and the MPDT heuristic with experts of the LVP. These experts confirmed that the forecast was in line with their expectations, and that the observed routes (covering criminal hotspots) were also in line with their expectations. Furthermore, an expert from Paragon Decision Technology, the company behind the AIMMS software, has verified both the code and the modeling techniques in our application.

To validate our heuristic and application quantitatively, we simulated the working of our algorithm and compared the results with the realization of the LVP in the same period. For this experiment, we used two years of data from the period October 2010 until and including September 2012. We used the first year of data only as initial input for our forecasting method. We used the second year of data (running from October 1, 2011) to simulate what would have happened when our application was used by the LVP during that year. For each day in the second year, we used all historic data available at the start of that day. This way, we used all available data, without using the data of the evaluated day to generate the flights. We used a crew planning similar to the one used by the LVP in this period, resulting in ten flights per day.

After generating the routes for a given day, we compared the routes with the incidents that happened on that day. When an incident happened, no helicopter is dispatched when the travel time was more than fifteen minutes. In other cases, we dispatched the helicopter with the highest expected success rate, or lowest helicopter identifier number in case of a tie. We limited the number of assists of a helicopter to at most one assist per flight, which implies a helicopter returns to base after it has been dispatched to an incident. When a helicopter was not flying, we assume it was standby to take off and took the appropriate time into account to become airborne.

The resulting computation times were in the order of five to six minutes for every day, apart from a single day, where it took 10 min. This was due to an exceptional situation where it turned out it was hard to find a near optimal route for a single flight. The generation process of this route was automatically ended after five minutes to ensure an upper bound on the running time. The results of our experiment are shown in Table 2. To maintain confidentiality, all values are normalized, where the realized number of successful assists of the LVP in the second year is normalized to 1. The indicators are based on the different parts of the success rate formula $g(w,a)$ discussed in Section 3.

When we compare the results of the LVP with our results, we see a clear difference in the number of successful assists of around a factor nine. We are aware of the fact that our results are based on a success rate formula based on expert opinions, which might have

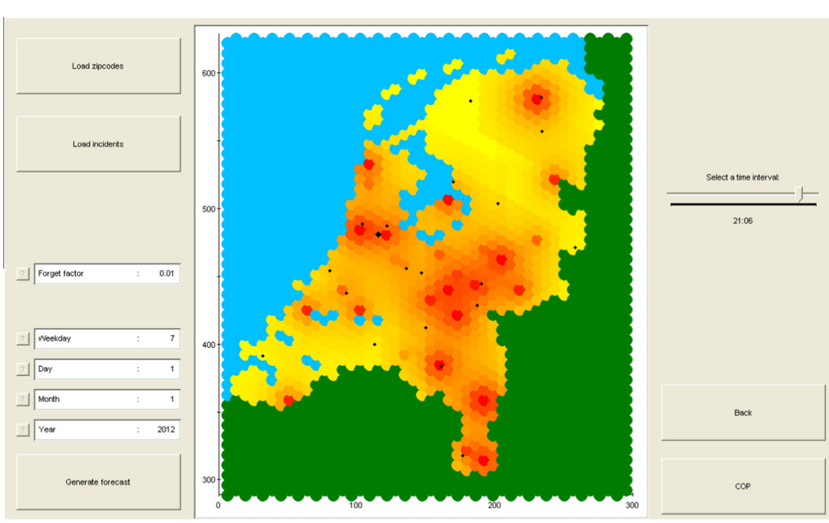

Fig. 6. Screen capture of the HELI part of the application.

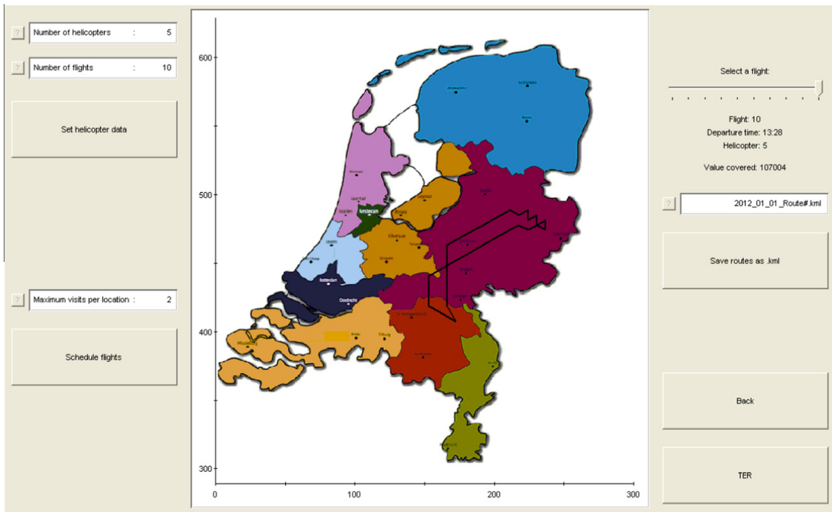

Fig. 7. Screen capture of the COP part of the application.

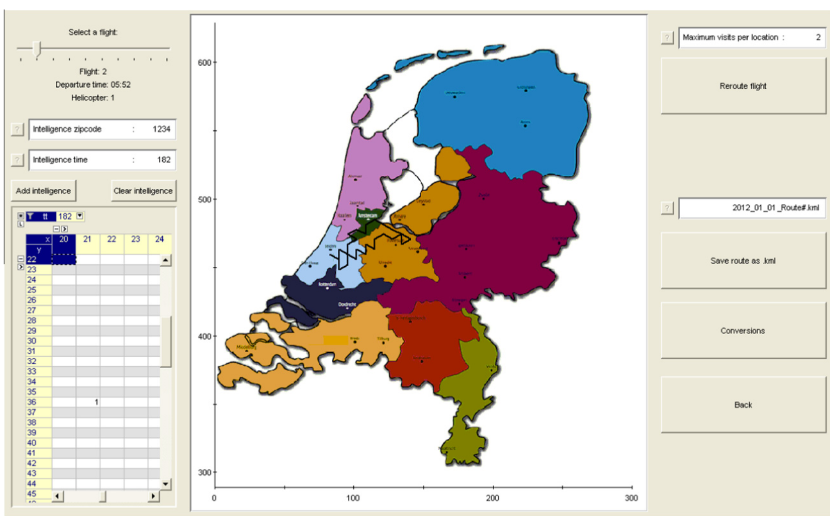

Fig. 8. Screen capture of the TER part of the application.

influenced the results. However, our results are on the pessimistic side, as (i) we cannot take intelligence into account in this simulation, which is possible in practice, and (ii) we cannot simulate the assist and therefore we assumed the helicopter will abort it's route and head back to base, whereas in practice it can fly a new route, based on remaining fuel and location. Considering the size of the gap between practice and our results, it is safe to state that our application is a valuable addition to the tool set of the Dutch Aviation Police and Air Support.

Besides the technical validation, we also made sure our application is comfortable to work with, using an appropriate Graphical User Interface (GUI). In order to give a good experience to the user, the application was built iteratively. This allows us to take user feedback into account in the next iteration. The final application is now used in practice by the LVP.

An extensive validation of the proposed instrument in practice is currently carried out by the Dutch Aviation Police and Air Support. Besides the extensive validation, we also propose to research which events and conditions have impact on the quality of the forecast as well as on the coverage given. For example, weather conditions and type of incidents might have an impact on the success rate or the effect on neighboring areas.

Table 2
Validation results. The numbers are normalized such that 1.0 equals the number of successful assists of the LVP in the same period.

Indicator	Result
Uncovered incidents	24.8
Incidents with a helicopter within 10 min	6.3
Incidents with a helicopter 10–15 min away	3.8
Expected number of successful assist	9.0
Average covered incidents	25.8%

8. Conclusion

In this paper, we defined a special case of both the Location Covering Problem and the Vehicle routing problem, which we called the Anticipatory Emergency Vehicle Routing Problem. We proposed a combination of two techniques to tackle this problem: a forecasting technique to cope with the uncertainty about future incidents, and a routing technique to maximize the weighted total expected coverage of incidents.

The proposed forecasting technique allows to make a forecast for a large number of locations with a relative low number of incidents with a relative small forecast error. This is achieved using generalization techniques in both the time and space dimensions. Furthermore, we proposed conversion techniques to convert incidents in any month or on any weekday, to the target month and weekday. We propose to take a forget factor into account such that recent data has a higher weight. Finally, incidents can be given different priorities to take the nature of the incident into account.

In the proposed routing technique, instead of solving the entire routing problem at once, it is solved sequentially. In each step, only a single helicopter has to be routed, with a known departure location, end location, and flight duration. This significantly reduced the complexity of the problem. For the departure time, we proposed a formula to calculate the most promising departure time for a helicopter. The solution of the MPDT heuristic is good and is directly usable. If sufficient calculation time is available, iterations of the RDDT heuristic can be used to search for an improved schedule.

We have performed validation of our methods and application qualitatively by the use of experts, and quantitatively by an simulation study. In this simulation study, we used one year of data solely as input, and a second year of data to simulate the use of our application during one year. Based on this simulation study, we conclude that number of successful assist can be increased by a factor nine.

We have implemented both the forecasting technique and the routing technique in an application. This application is considered to be a valuable addition to the tools of the Dutch Aviation Police and Air Support (LVP). The application is currently used and tested in practice by the LVP in the Netherlands.

Acknowledgments

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