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ITERATIVE METHODS FOR EFFICIENT GENERATION OF WAVE FIELDS IN HYDRODYNAMIC LABORATORY

H. Margaretha, F.P.H. van Beckum, Andonowati, E. van Groesen, A. Jamaludin, and A.M. Wijaya

Abstract. Motivated to be applied in the Indonesian Hydrodynamic Laboratory (LHI), we design an iteration scheme for the motion of a wavemaker on one side of a large wave tank in such a way that a desired energy spectrum for the wave field results. Due to imperfections of the mechanical operation of the wavemaker, the mapping from the wavemaker control to the generated wave field in the tank is not known. Taking some realistic qualitative assumptions for this mapping, we design and study possible steering strategies. From a class of iteration schemes, one preferred iteration scheme can be selected based on the global conditions for convergence; the iteration will enable to generate any realisable wave field. The use of the proposed iteration led to a satisfying result for an actual experiment, details of which will be described.

1. INTRODUCTION

The main aim of a hydrodynamic laboratory is to test ships and off-shore structures in wave fields that resemble as much as possible realistic wave conditions in the sea or ocean. Such wave fields are characterized by an energy (power) spectrum. A wave-maker, i.e. a controllable moving flap that pushes the waves on one side of a large wave tank, has to be moved in such a way to obtain the desired wave spectrum. Pre-designed wavemaker control software may not always lead to the desired result, in particular when mechanical problems that are not incorporated result into deviations. In this paper we will design an iteration method

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for the control of the wavemaker that is based on the wave field measured in the tank; the aim is to obtain a quickly converging iteration. We will describe some preliminary aspects first.

In a laboratory basin, the motion of the wavemaker has to be given to certain software that transfers the input data into related flap movements. The input is a power spectrum that is expected to be related directly to the power spectrum of the expected wave field generated in the basin.

There are two main reasons why the actual generated wave field may (and in practice, does) differ from the expected wave field. One reason is a consequence of the physical evolution of the wave field, namely the presence of nonlinear effects. These effects will deform a wave field generated by the wavemaker while running downstream in the tank in such a way that also the power spectrum changes with increasing distance. In principle this effect could be accounted for (in some approximation) by using advanced knowledge of the nonlinear effects (see e.g. [1], [2]). Therefore in this paper we will not deal with this aspect, and simply assume that the waves behave linearly (propagating without change of spectrum), which can be expected to be a valid approximation for sufficiently small wave heights. As a consequence of this assumption, the position where the power spectrum of the wave field is measured, is irrelevant. The second reason is of a technical origin and is directly attributed to mechanical imperfections of the wavemaker driving mechanism, resulting in the fact that the mapping from input (power spectrum of expected wavemaker motion) to actual motion is not known. Actually, the performance may even depend on the previous history (the time the wavemaker has been operating).

Irrespective of the origin of the deviations, we will try to achieve the goal to get the desired waves in the basin by only operating on the input in a sensible way. We will design an iteration scheme that is based on using the information from a measurement of the generated waves. That information is used to update the input, i.e. the power spectrum fed to the wavemaker.

This work is motivated by problems encountered by the Indonesian Hydrodynamic Laboratory (LHI), and executed in the framework of the EU-Jakarta Small Projects Facility, project "Building Academia - Industry Partnership in the Sectors of Marine and Telecommunication Technology". At present, the wave-maker of LHI is controlled by WAVESTIR 2.10, developed by Delft Hydraulics in the early nineties of the previous century. It is software that generates steering signals for the wave flaps. With increasing time, it is becoming more difficult to get a desired wave spectrum using the instruction given in the manual [3]. Based on the new iteration to be described below, a recent experiment at LHI was performed; we will report on this experiment in Section 4.

In section 2 we will specify the actual problem in mathematical language, and motivate and describe the simplifications of a class of iteration schemes to which we will restrict. Then in the main section 3 we will design an appropriate iteration scheme, having formulated explicitly some qualitative assumptions about the map from wavemaker spectrum to the spectrum of the actual generated wave field. In section 4, results from the model-designed laboratory experiment in LHI will be presented, and section 5 will be concluding remarks.

2. PROBLEM SPECIFICATION AND SIMPLIFICATIONS

We will use the following notation for the three different spectra that are relevant in the rest of the paper. For shortness, we will talk about spectrum when a power (quadratic-amplitude) spectrum is meant in the following.

The desired spectrum in the wave tank (e.g. Jonswap, PM, etc) will be denoted by $S(\omega)$, where ω is the frequency. For the iteration scheme that we will design, a subscript will denote the iteration step. The input spectrum fed as input into the wave maker software in the n-th iteration step is then denoted by $F_n(\omega)$, and $M_n(\omega)$ will denote the (power) spectrum calculated from the wave signal that is measured at a certain position in the tank as a result of the input $F_n(\omega)$.

The measured spectrum M_n is clearly a result of the input spectrum F_n, and we express this relation in a mapping E:

$$M_n = E(F_n) \tag{1}$$

As was argued above, the presence of deviations resulting from mechanical malfunctioning leads to the fact that this mapping is not, or badly, known. *Therefore we will treat the mapping E as being unknown.* (If it were known, and we could calculate its inverse, the input spectrum we are looking for would be given by $F = E^{-1}(S)$.) Although it was described above that the mapping E may change during (long periods of) operation, *we will assume that the mapping E is time independent*, indicated above already by the fact that E does not depend on the iteration number.

So, any information about E has to be obtained from (successive) measurements of wave fields resulting from the given input. Since each iteration is costly because it requires the operation of the basin and extensive data analysis, it is desirable to have the number of iterations as small as possible. Hence the problem we have to tackle is the following: *design an iteration scheme for the input such that the wave spectrum converges to a prescribed target S*, i.e. $F_1(\omega) \to ... \to F_n(\omega) ... \to F_\infty(\omega)$ such that $M_1(\omega) \to ... \to M_n(\omega) ... \to S(\omega)$.

2.1. Simplifications: Single-Step and Pointwise Iteration

For the actual design of the iteration we will make several simplifications which we will now make explicit. Relaxing each of these simplifications may result in better (faster converging) schemes, at the cost of increased complexity of the design.

The iteration scheme we will design will be a single-step scheme only: in the n-th iteration step, only the input F_n and the output M_n are used (without information from prior iterations) to construct in some way the new input F_{n+1} from the old one; symbolically denoted by

$$F_{n+1} = T_n(F_n), \tag{2}$$

where the mapping T_n depends on M_n.

Furthermore, we will restrict to look for pointwise iterations. To explain this, it should be noted that the result of the measurement for a certain frequency, $M_n(\omega_0)$ will in general depend on the information of the full input, not only on $F_n(\omega_0)$. This means that in general we have to take into account that the map T_n is really a mapping from one function into another function. Assuming pointwise relations means that we assume that $M_n(\omega_0)$ only depends on $F_n(\omega_0)$, and that we design the map also in this way:

$$F_{n+1}(\omega_0) = T_n(F_n(\omega_0)). \tag{3}$$

This means, amongst others, that the map T_n cannot depend on derivatives, or translations, of the input.

The restriction to pointwise iterations has huge consequences. One consequence is that, instead of with an infinite dimensional problem (function to function), we essentially deal with a one-dimensional problem (number to number). Since the target spectrum is not constant, a second consequence is that we have to make the iteration scheme in such a way that it will converge to any desired (realistic) value s, i.e. any value from $\omega \to S(\omega)$. Of course these values have to be restricted to the practically possible values, i.e. to the maximal possible value that can be measured for any flap input.

Summarising, and adapting notation to the one-dimensional setting, we get the following.

Problem: *Let $u_{\max}$ be the maximal value of the input, and let $s_{\max}$ be the maximal possible value in the wave field, $s_{\max} = E(u_{\max})$. Then the task is to design mappings Ψ_n such that for any $s < s_{\max}$ a convergent iteration scheme $u_{n+1} = \Psi_n(u_n)$ is obtained for which all u_n are practically realizable, i.e. $u_n \in [0, u_{\max}]$, and such that $E(u_n) \to s$. The mappings Ψ_n to be found will depend on $E(u_n)$ and on s. The convergence implies that the input will converge to some input $\hat{u}$ such that $E(\hat{u}) = s$.*

3. DESIGN OF A CONVERGENT ITERATION SCHEME

To motivate the following, consider the following reasoning. A given input u_1 produces the measurement $M_1 = E(u_1)$. To achieve the desired result s, the change of input must depend on the difference $M_1 - s$, or, equivalently, on the quotient $q_1 := s/E(u_1)$. We aim to a value $q_1 = 1$, and so if $q_1 < 1$ or $q_1 > 1$ (the measurement is too large or too small respectively) we want to change the input in a direction such that M will decrease or increase respectively. Hence we have to know the qualitative dependence of E on u. For the application of wave generation, it is natural to adopt the following assumption.

Model Assumption 1: *The mapping* $u \to E(u)$ *is monotonically increasing.*

Indeed, by increasing the input spectrum, the wave spectrum will increase. Hence, we want to increase or decrease the input u_1 if $q_1 > 1$ or $q_1 < 1$ respectively. This can be achieved by choosing:

$$u_{n+1} = \tau\left(s/E\left(u_n\right)\right)u_n \equiv \Phi\left(u_n\right)$$

for a suitable monotonically increasing function τ. The mapping Φ defined in this way must be such that

$$u_n \to \hat{u}, \text{ and } \hat{u} = \Phi\left(\hat{u}\right).$$

Hence, we have now formulated the problem as a fixed point problem for the mapping Φ. The relation at the fixed point $\hat{u} = \Phi(\hat{u})$ requires that the function τ satisfies $\tau(1) = 1$.

For instance, if we take the function τ to be the identity function,

$$\tau(q_n) = q_n,$$

we get the iteration

$$\Phi\left(u\right) = \frac{s}{E\left(u\right)}u. \tag{4}$$

If the measurement would be linearly depending on the input, $E(u) = au$ for some a, there results the map $\Phi(u) = s/a$ with the desired limit since $M = E(s/a) = s$. Hence in this special case, knowing the linearity of the map E, only one iteration is needed to find the value of $a := E(u_1)/u_1$ and then the second iteration $u_2 = s/a$ produces the desired result. The conclusion from this paper will be that this choice (4) is the best possible choice even when the map E is more realistic.

3.1. Considerations for Convergence

We now have to investigate the convergence of the iteration process $u_{n+1} = \Phi(u_n)$, that is to say to find suitable functions τ that will lead to a convergent iteration process.

There is only one general condition that can be written down to guarantee the convergence of an iteration scheme for a fixed point problem, and that is Banach Fixed Point theorem. The condition is that in a certain closed interval around the fixed point $\hat{u}$ the mapping Φ should be a strict contraction, meaning that there is a positive constant $\kappa < 1$, the so-called contraction constant, such that for any two points v, w in the interval

$$\left|\Phi\left(v\right) - \Phi\left(w\right)\right| \leq \kappa\left|v - w\right|.$$

For differentiable mappings Φ, with derivative $\Phi'(\hat{u})$ at $\hat{u}$, the value in nearby points is given by $\Phi(\hat{u}+\xi) = \Phi(\hat{u}) + \Phi'(\hat{u})\xi + O(\xi^2)$. This implies that a suitable contraction constant $\kappa < 1$ can be found provided $|\Phi'(\hat{u})| < 1$.The convergence rate of the iteration $u_{n+1} = \Phi(u_n)$ is given by the contraction constant, and the distance to the fixed point is given by $|\hat{u} - u_n| \leq [\kappa^n/(1-\kappa)]\,|u_1 - u_0|$.

With Φ given as above by

$$\Phi(u) = \tau\left(\frac{s}{E(u)}\right)u,$$

this contraction requirement will produce additional conditions on the choice of the function τ, which we will now investigate.

3.2. Conditions for Local Convergence

First we will look at the behaviour of the map near a fixed point, and see what conditions on the function τ this produces. For this local analysis we have to investigate $\Phi'(\hat{u})$. The derivative of Φ can be explicitly written like

$$\Phi'(u) = \tau\left(\frac{s}{E(u)}\right) - \tau'\left(\frac{s}{E(u)}\right)\frac{s}{E(u)}\frac{E'(u)\cdot u}{E(u)}$$

At any fixed point $\hat{u} \neq 0$ for which $s/E(\hat{u}) = 1$, and using the fact that $\tau(1) = 1$, there results

$$\Phi'(\hat{u}) = 1 - \beta\frac{E'(\hat{u})\cdot\hat{u}}{E(\hat{u})} \text{ with } \beta = \tau'(1).$$

(The point $\hat{u} = 0$ can be a fixed point and has then to be treated separately.) The assumption that E is monotonically increasing, and the above condition of monotonicity for τ, implies that $\beta E'(\hat{u}) > 0$. Since we aim for convergence for all practical values s of the desired spectra that we want to generate, say $s \in [0, s_{\max})$, the condition $|\Phi'(\hat{u})| < 1$ leads to a condition for the value of β that guarantees convergence:

$$0 < \beta < \frac{2E(u)}{E'(u)\cdot u}, \text{ for all } u \in (0, u_{\max}]$$

where $E(u_{\max}) = s_{\max}$. Hence

$$\beta < \beta_{\max} := \min\left\{ \frac{2E(u)}{E'(u)\cdot u} \;\middle|\; u \in (0, u_{\max}] \right\}.$$

For monotone convergence, $0 < \Phi'(\hat{u}) < 1$ the value of β has to be restricted further, $\beta < \beta_{mon}$ with

$$\beta_{mon} = \beta_{\max}/2.$$

As is to be expected, the property of the iteration depends strongly on the map E, and the expression above makes this dependence explicit. To proceed further, we have to know more about this map, which is why we make the previous assumption stronger.

Model Assumption 2: *We assume that the map* E *can be described on* $[0, u_{\max}]$ *as*

$$E(u) = cu(2u_{\max} - u)$$

for some positive constant c.

This assumption is motivated by the idea that for small values of u, the behaviour will be linear: $E(u) \approx au$, where we write $a = 2cu_{\max}$. That $E(0) = 0$ is immediate since any flap motion will produce some waves. That the relation is linear for small u, i.e. for small flap motions and small waves, will hold true if the major part of the power inserted by the flap will be transferred to the waves, as can be expected. The linear relation cannot hold for all inputs. In fact the inputs are bounded above by $u_{\max}$, and will give rise to a highest possible value of the output $s_{\max}$ for the maximal power at the wavemaker $u_{\max}$. This saturation to a highest value in the output, in a monotone way, is described by the proposed quadratic relation. Note that $s_{\max} = cu_{\max}^2$.

For this model it holds that the quotient $2E(u) / (E'(u) \cdot u)$ is always larger than 2, monotonically increasing from the value 2 at $u = 0$ to unbounded values for $u \to u_{\max}$. As a consequence we find the following.

Condition for local convergence: $0 < \beta < \beta_{\max}$ *with* $\beta_{\max} = 2$*, and for monotone convergence* $\beta < \beta_{mon} = 1$.

3.3. Conditions for Global Convergence

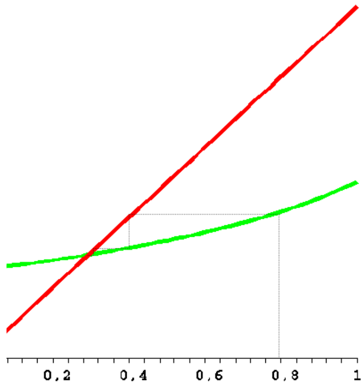
The above local considerations are not complete yet to obtain a statement about the global convergence of the iteration scheme. For that we need to show that Φ maps $[0, u_{\max}]$ into itself, and that the graph of the function Φ intersects the graph of the linear map u in precisely one point in the interval $(0, u_{\max}]$ (the point $u = 0$ may be somewhat special as we shall see). Then the existence of a unique fixed point, with convergence for the iteration scheme from any initial point, will be guaranteed. This will lead to additional conditions on the function τ, which we will now investigate.

The behaviour of Φ near $u = 0$ strongly depends on the behaviour of $\tau(s/E(u))$ for small u. If we assume that the linear approximation for E applies, $E(u) \approx au$ for u small, the argument of τ will become unbounded. In order to restrict the many possibilities, we now select a functional form for τ:

For the following we will take the function τ *defined for* $\xi > 0$ *to be the homogeneous function*

$$\tau(\xi) = \xi^{\beta} \text{ for } \beta > 0.$$

Observe that this function is monotonically increasing and satisfies $\tau'(1) = \beta$, as



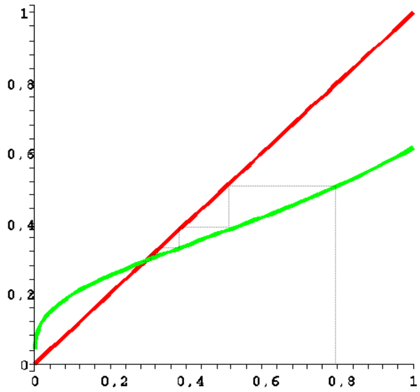


Figure 1: Illustration of the convergence of the map Φ to the non-zero fixed point, for $c = 1, u_{\max} = 1, s = 1/2$ for $\beta = 1$ (left), and $\beta = 0.7$ (right).

assumed above. Then the map is given by

$$\Phi(u) = \left(\frac{s}{E(u)}\right)^{\beta} u.$$

Since $\Phi(u) \approx (s/a)^{\beta} u^{1-\beta}$ for u small, we observe that Φ becomes unbounded for $\beta > 1$ which forces us to choose $\beta \leq 1$.

Proposition 3.3.1. *Assume that Model Assumption 2 is satisfied. Then for any β with $0 < \beta \leq 1$ and $s < s_{\max}$ the map Φ is monotically increasing and has one unique fixed point in the interval $(0, u_{\max}]$.*

This can be shown with elementary mathematics. The result can be read of immediately from simple plots in Figure 1, which show that the graph of Φ (dashed line) intersects the graph of the identity function u in exactly one interior point. In Figure 1 we present the graphs for $c = 1, u_{\max} = 1, s = 1/2$ for $\beta = 1$ at the left, and $\beta = 0.7$ at the right. Note that the convergence for $\beta = 1$ is faster.

Briefly, the monotonicity of Φ is seen from the derivative that is given by

$$\Phi'(u) = \left(\frac{s}{E(u)}\right)^{\beta} \left[1 - \beta \frac{E'(u) \cdot u}{E(u)}\right].$$

Since $E'(u)\, u < E(u)$, it follows that $\Phi'(u) > 0$ for $\beta \leq 1$, and hence that Φ is monotonically increasing. For $\beta < 1$ it holds that $\Phi(0) = 0$ (and hence that $u = 0$ is a fixed point). Since $\Phi'(0) = \infty$ (hence $u = 0$ is unstable), the graph of the function Φ lies above the graph of the linear function u near $u = 0$. For $\beta = 1$ it holds that $\Phi(0) = s/a > 0$. (Note that since $s < s_{\max} < a u_{\max}$ as a consequence

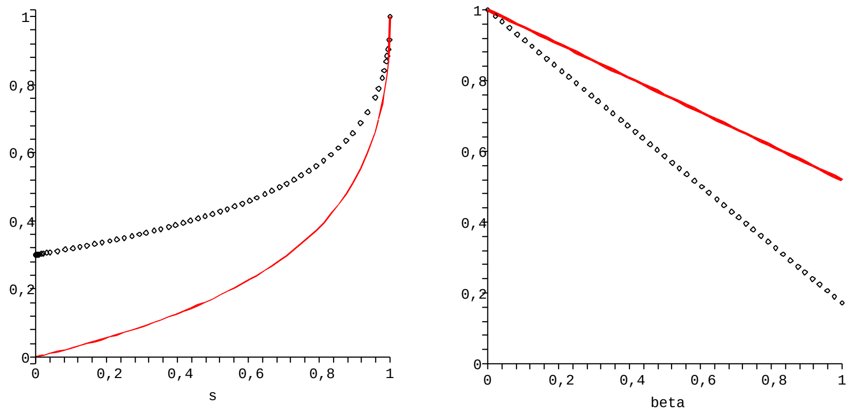


Figure 2: The rate of convergence $\Phi'(\hat{u})$ at a fixed point $E(\hat{u}) = s$ as function of s (left) for $\beta = 1$ and $\beta = 0.7$ (dotted line); and as a function of β (right) for $s = 1/2$ (dotted) and $s = 0.9$.

of Model Assumption 2, it holds that $\Phi(0) < u_{\max}$). The behaviour near $u_{\max}$ is found based on Model Assumption 2 to be

$$\Phi(u) = \left(\frac{s}{E(u)}\right)^{\beta} u \approx \left(\frac{s}{s_{\max}}\right)^{\beta} u_{\max} \text{ for } u \text{ near } u_{\max}.$$

If we restrict s to satisfy $s < s_{\max}$, this shows that $\Phi(u) < u_{\max}$ for u near $u_{\max}$. These properties near the endpoints of the interval and the monotonicity imply that the graph of Φ intersects the graph of the linear function u exactly once in $(0.u_{\max}]$.

To get an idea about the rate of convergence, we plot in the left picture of Figure 2 the rate of convergence $\Phi'(\hat{u})$ at a fixed point $E(\hat{u}) = s$ as function of s for two values of β, namely $\beta = 1$ and $\beta = 0.7$ (dotted line). In the right picture, for fixed values of s, the (linear) convergence rate as function of β is given for $s = 1/2$ (dotted) and $s = 0.9$.

This shows clearly the expected convergence: at fixed β, the convergence is slowest for near the largest possible values of the target value s, while for fixed s the convergence improves with increasing value of β to $\beta = 1$.

Summarising the findings, we get the following result.

Conclusion 3.3.2. *Suppose that the mapping E satisfies the Model Assumption 2, and consider the iteration scheme*

$$u_{n+1} = \Phi(u_n) \text{ with } \Phi(u) = \left(\frac{s}{E(u)}\right)^{\beta} u.$$

Then for the choice $\beta = 1$ *it holds that for any* $s < s_{\max} = E(u_{\max})$ *the iteration converges from any initial point* $u_1 \in [0, u_{\max}]$*, and that* $E(u_n) \to s$.
For $\beta < 1$*, the origin is also a fixed point, which is unstable. Then for any* s *with* $0 < s < s_{\max}$ *the iteration converges from any nonzero initial point* $u_1 \in (0, u_{\max}]$ *and* $E(u_n) \to s$*. The fastest convergence for each* s *is obtained for* $\beta = 1$.

Translating this result in terms of the wavemaker problem, we have the following result.

Corollary 3.3.3. *Suppose that the Model Assumption 2 is satisfied. Let* $S(\omega)$ *be a given target spectrum that is realizable in the sense that* $S(\omega) < M_{\max}(\omega)$ *where* $M_{\max}(\omega)$ *is the maximal power that can be generated by the flap for that frequency; if* $F_{\max}(\omega)$ *is the maximal power of the flap motion,* $M_{\max}$ *is given by* $M_{\max}(\omega) = E(F_{\max}(\omega))$*. Then, starting with any initial spectrum* $F_1(\omega)$ *the iteration process*

$$F_{n+1}(\omega) = \left(\frac{S(\omega)}{E(F_n(\omega))}\right)^{\beta} F_n(\omega)$$

converges, and the wave field spectrum converges to the desired spectrum $E(F_n(\omega)) \to S(\omega)$ *for any* β *satisfying* $\beta \leq 1$*, with fastest convergence for* $\beta = 1$*. The slowest convergence is then determined by the difference* $\max\{\, M_{\max}(\omega) - S(\omega) \;|\, \omega\}$ *which happens for frequencies where this difference applies.*

3.4. Comparison with Laboratory Practice

In the laboratory practice, usually the recommendations as stated in the WAVESTIR manual [3] are followed. In this manual it is described that the procedure implemented in the software is indeed an iterative procedure, where the mapping T_n is a pointwise multiplication with a certain function. This means, writing this function with the same symbol T_n, that for each ω:

$$F_{n+1}(\omega) = T_n(\omega) F_n(\omega). \tag{5}$$

Specifically, the function $T_n(\omega)$ is calculated from the previous iteration by considering the (local) ratio of the desired spectrum S and the measured spectrum M_n:

$$T_n(\omega) = \tau(q_n)$$

with

$$q_n = \frac{S(\omega)}{M_n(\omega)}. \tag{6}$$

The function τ is the same function as considered above, $\tau(\xi) = \xi^{\beta}$. Somewhat different from the conclusion above that one can choose any value $\beta \leq 1$, in WAVESTIR the value of β is adviced to take differently, without explanation or further specification than what follows in the following quote taken from page 12 of the Wavestir - CCS manual [3].

> "Practical experience has shown the following rules of thumb: ...In the formula used to calculate the new transfer factor (from energy comparison), a power value of 0.25 is used. This usually leads to a too low correction. A value of 0.5 gives a too high correction. Depending upon the wave at hand, an intermediate value such as 0.35 may be useful...The number of iterations to reach a satisfactory spectrum usually ranges from 5 to 15 and depends mainly upon the skills of the operator."

Since a value β will correspond to a value of the power $\beta/2$ in a formula to calculate a new transfer factor (which is linear in the wave amplitude spectra, while the wave power spectrum is quadratic), these suggestions are not in line with the findings above which showed that a value $\beta = 1$ will produce the fastest convergence.

4. EXPERIMENT AT LHI

In December 2004 an experiment was conducted at the Indonesian Hydrodynamic Laboratory (LHI) as part of the execution of the EU-project mentioned before. The experiment was done in the towing tank of LHI. The wave maker in the towing tank has a main flap and a upper flap, with the hinges located at 2.95 and 4.67 meter above the bottom of the tank. When generating an irregular wave field, low frequency wave components are generated by the main flap, while the upper flap will generate the higher frequencies. In the experiment we only used the upper flap. The operational water depth is 5.50 m. The towing tank has a length of 225 m and a width of 11 m.

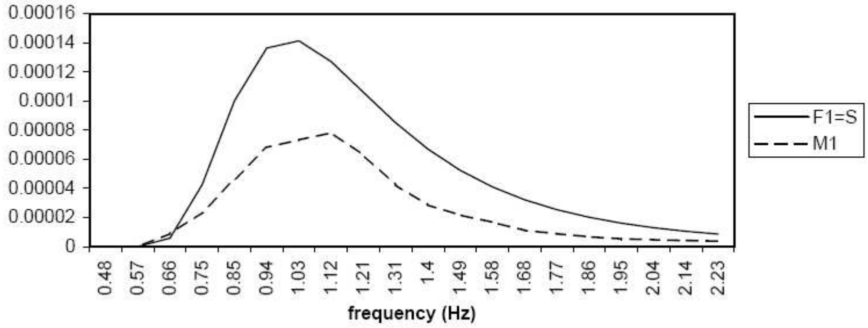


Figure 3: The solid line is the target spectrum, which also served as the first input for the wavemaker. The resulting wave field that is actually generated in the tank has spectrum shown by the dashed line, roughly a factor 2 too small.

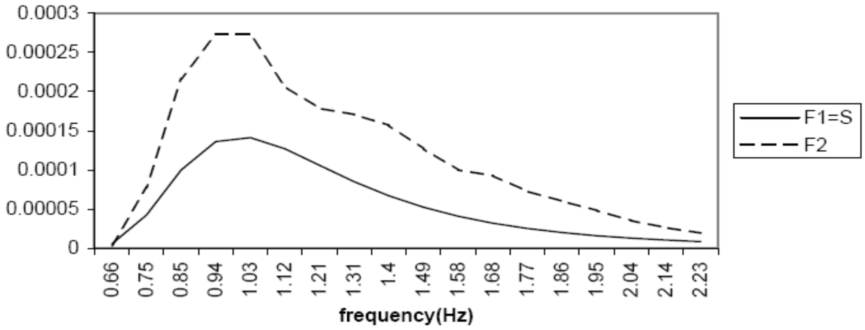


Figure 4: Based on the measured spectrum in the previous graph, the proposed iteration scheme leads to a second input for the wave flap F_2 (the dashed line) which is now roughly twice al large as the previous input.

In the experiment, we aimed to generate a random wave field with a Pierson - Moskovitch spectrum, a power spectrum that is often used to simulate wind driven waves on a sea. The choice of specific parameters allows the spectrum to resemble specific sea and wind conditions at the location of interest. For the experiment the target PM-spectrum was chosen with parameters to obtain a significant wave-height of 10 cm and a peak period of 1 s. The range of frequencies for this PM-spectrum is $[0.48, 2.23]\,Hz$ and allows the use of only the upper flap. The wave was measured at a position $100m$ from the wavemaker with sampling rate $50Hz$.

We used the iteration method described in Section 3 with $\beta = 1$. In Figures 3, 4, 5, results from the measurement are presented. Figure 3 shows the first input spectrum F_1 and the spectrum of the resulting wave field measured in the basin, M_1. Roughly speaking it is seen that the measured wave spectrum is a factor of two too small $M_1 \approx S/2$, and the maximum is shifted somewhat to the higher frequencies. Then the new input should be roughly a factor of two larger than the previous input: $F_2 = (S/M_1)\,S \approx 2S$; this is shown more precisely in Figure 4. Figure 5 shows the spectrum of the new wave field M_2, which is shown to be close to the target spectrum S, close enough to be acceptable for the laboratory practice.

5. CONCLUDING REMARKS

The research is motivated by the need of hydrodynamic laboratories to have an efficient iteration scheme to generate wave fields with a specified spectrum in the wave tank. The iteration of the input to the wavemaker uses the measured spectrum of the actually generated wave field. In this paper we started from scratch the design of a convergent iteration scheme. To that aim we restricted ourselves

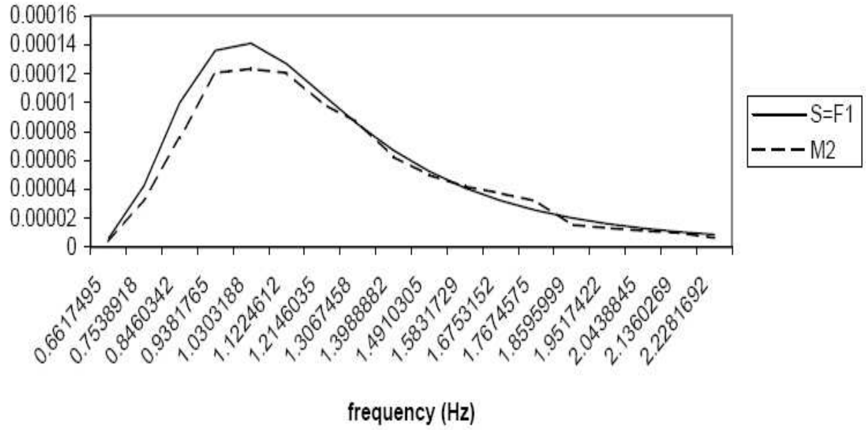


Figure 5: The result in the wave tank of the improved input leads to a spectrum of the wave field (M_2 dashed) that is close to the target spectrum S. For laboratory practice, this approximation is acceptable, and the quick convergence satisfying.

to the simplest possible schemes, namely to those that are one-step iterations, and pointwise. The convergence of such scheme depend very much on the unknown map from wavemaker to wave field, for which the measurements give only very limited information. We formulated a reasonable model assumptions about the qualitative behaviour of this map, with the aid of which it became possible to analyse in detail the convergence of the iteration depending on the choice of a power exponent in the correction coefficient. Although a local condition for convergence led to a range of acceptable exponents, the requirement for global convergence led to a further restriction $\beta \leq 1$, with $\beta = 1$ giving a unique iteration procedure which is fastest, simple and intuitively well understandable. This unique value of the exponent is out of the range of values suggested by the software manual that is in use at the laboratory. A real life experiment executed in the wave tank of LHI showed that the proposed iteration worked correctly in the least possible number of iterations.

In addition to the research presented here, some additional results have been obtained that may make the iteration scheme more efficient to operate. These results concern methods that do not adjust the wavemaker spectrum at all points, as in the method treated here, but only at a limited number of frequencies. These methods will be further developed, and more experiments will be needed in order to justify all the proposed methods.

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H. MARGARETHA: Applied Analysis and Mathematical Physics Group, Department of Applied Mathematics, University of Twente, 7500AE Enschede, The Netherlands. (*Present address*: Department of Mathematics, Universitas Pelita Harapan, UPH Tower, Lippo Karawaci, Tangerang 15811, Indonesia.)
E-mail: h_margaretha@yahoo.com; h.margaretha@bdg.centrin.net.id

F.P.H. VAN BECKUM: Applied Analysis and Mathematical Physics Group, Department of Applied Mathematics, University of Twente, 7500AE Enschede, The Netherlands.
E-mail: f.p.h.vanbeckum@math.utwente.nl

ANDONOWATI: Department of Mathematics, Institut Teknologi Bandung, Bandung 40132, Indonesia.
E-mail: aantrav@attglobal.net

E. VAN GROESEN: Applied Analysis and Mathematical Physics, Department of Applied Mathematics, University of Twente, 7500AE Enschede, The Netherlands.
E-mail: groesen@math.utwente.nl

A. JAMALUDDIN: Indonesian Hydrodynamic Laboratory (LHI), Surabaya, Indonesia
E-mail: andi99@engineer.com

A.M. WIJAYA: Modeling and Simulation Co-Laboratory, P2MS, Institut Teknologi Bandung, Bandung 40132, Indonesia