The effects of representational format on learning combinatorics from an interactive computer simulation

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Abstract   The current study investigated the effects of different external representational formats on learning combinatorics and probability theory in an inquiry based learning environment. Five conditions were compared in a pre-test post-test design: three conditions each using a single external representational format (Diagram, Arithmetic, or Text), and two conditions using multiple representations (Text + Arithmetic or Diagram + Arithmetic). The major finding of the study is that a format that combines text and arithmetics was most beneficial for learning, in particular with regard to procedural knowledge, that is the ability to execute action sequences to solve problems. Diagrams were found to negatively affect learning and to increase cognitive load. Combining diagrams with arithmetical representations reduced cognitive load, but did not improve learning outcomes.

Keywords   External representations · Multiple representations · Interactive representations · Simulation-based learning · Inquiry learning · Mathematics · Cognitive load

Introduction

The format of external representations (symbols, diagrams, etc.) is known to play a critical role in learning and understanding (Ainsworth and Loizou 2003; Cheng 1999; Zhang 1997). External representations are usually classified into two categories: nonverbal (e.g., diagrams) and verbal representations (e.g., natural and arithmetical languages) (Klein 2003; Paivio 1990). In general, diagrams are associated with superior performance compared to textual material (Goolkasian 2000; Marcus et al. 1996). Diagrams are considered to be most useful as

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an aid to understanding when materials are complex or difficult to understand (Levin 1981). In a meta-analysis Levin et al. (1987) found that diagrams are most effective in enhancing learning if they organize events into a coherent structure, clarify complex and abstract concepts, or assist learners in recalling important information. Diagrams are effective because they make relations among elements explicit and make information more concise by summarizing or highlighting what is essential (Levin and Mayer 1993).

Many researchers have observed that, despite a vast body of research on the facilitative effects of diagrams, not much is known about the cognitive processes underlying these effects (Cheng 1999; Glenberg and Langston 1992; Goolkasian 2000; Scaife and Rogers 1996; Zhang 1997). In addition, not much is known about the behavioral aspects of representations. These are important because modern instructional computer technology increasingly provides learners with interactive representations. In simulation-based instruction, for example, learners can manipulate representations in order to explore the concepts and principles underlying the domain (de Jong and van Joolingen 1998). Cheng (1999) argues that the active manipulation of representations makes underlying domain relations more accessible compared to static representations. Rogers (1999) adds that interactive representations can reduce the amount of "low-level" cognitive activities (e.g., drawing and redrawing) normally required when learning, and in turn allow learners to devote their cognitive resources to more "high-level" cognitive tasks, such as exploring more of the problem space.

Representational formats

The properties of a representation are assumed to influence which information is attended to and how people tend to organize, interpret, and remember the information presented. Larkin and Simon (1987) state that the value of representations depends on two factors: informational and computational efficiency. *Informational efficiency* refers to how representations organize information into data structures. *Computational efficiency* refers to the ease and rapidity with which inferences can be drawn from a representation. Even when a diagram and a text are informationally equivalent, meaning that all of the information in one representation can be inferred from the other and vice versa, the diagram is often more effective because inferences can be drawn more quickly and easily from diagrams (Koedinger and Anderson 1990; Larkin and Simon 1987).

We will use an example relevant to the understanding of combinatorics, the problem of a thief guessing the four-digit PIN-code (5526) of a credit card he has just stolen, to illustrate different ways of organizing information into data structures. Guessing the PIN-code can be conceived as a process of four interrelated steps: the first step is selecting the first digit of the code. The thief has 10 options (0 through 9) of which one is correct. The second step is selecting the second digit, and again there are 10 options of which one is correct. This is also true for the third step (selecting the third digit) and the fourth step. These four steps can be represented in different ways, such as by means of diagrams, texts, or arithmetic (see Fig. 1).

The different representations presented in Fig. 1 are informationally equivalent; that is, given each of the representations the other two can be constructed.

In Fig. 1a the PIN-code problem is represented diagrammatically as a tree diagram. In pictures or diagrams, information is indexed by a two-dimensional location in a plane, explicitly preserving information about topological and structural relations (Larkin and Simon 1987). Comprehension of diagrams involves the establishment of some conventions specifying the meaning of the diagram (Cobb 1989; Fischbein 1987). Tree diagrams could
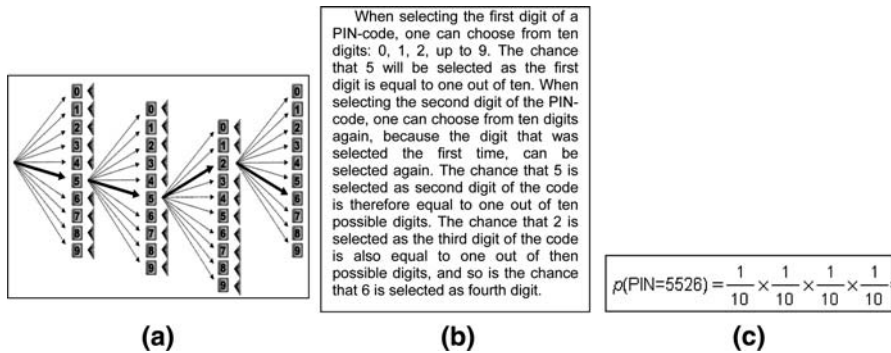
When selecting the first digit of a PIN-code, one can choose from ten digits: 0, 1, 2, up to 9. The chance that 5 will be selected as the first digit is equal to one out of ten. When selecting the second digit of the PIN-code, one can choose from ten digits again, because the digit that was selected the first time, can be selected again. The chance that 5 is selected as second digit of the code is therefore equal to one out of ten possible digits. The chance that 2 is selected as the third digit of the code is also equal to one out of then possible digits, and so is the chance that 6 is selected as fourth digit.

$$p(\text{PIN}=5526) = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$$

(a)   (b)   (c)

Fig. 1 Different but informationally equivalent ways of representing the PIN-code problem: (**a**) diagram; (**b**) text; (**c**) arithmetic

also preserve sequential relations, specifying the available options at each step. Tree diagrams are considered a powerful tool for teaching combinatorics and probability theory (e.g., Fischbein 1987; Greer 2001). They are especially effective in assessing the probability of various options (Fischbein 1987; Halpern 1989).

In Fig. 1b the PIN-code problem is represented textually. The use of natural language facilitates the relating of information in the text to everyday experiences and situations. On the other hand, problems with text comprehension may hamper problem solving performance (Koedinger and Nathan 2004; Lewis and Mayer 1987; Nathan et al. 1992). What distinguishes a text from a mere set of sentences is that a text is cohesive. Sentences in a text build upon and refer to one another. Textual representations emphasize other relational features than do diagrams. In textual representations information is organized sequentially, preserving temporal and logical relations (cf. Larkin and Simon 1987), rather than topological and structural relations. This has consequences for the way representations permit accessing and processing of information: diagrams allow simultaneous access (Koedinger and Anderson 1990; Larkin and Simon 1987), whereas in order to access and process a text, the reader has to keep certain elements of the text highly activated in working memory, while comparing newly encountered elements with those held in working memory (Glenberg et al. 1987). Keeping elements activated is thought to burden working memory considerably (Leung et al. 1997; Sweller et al. 1998).

Figure 1c displays an arithmetical representation of the PIN-code problem. Again, the representation is informationally equivalent with the previous representations, although recognizing the parallels may strongly depend on the learner's knowledge of the meaning of arithmetical representations. One needs to know, for example, the *conceptual* meaning of the multiplication sign. In arithmetical representations the underlying principle or concept is not as explicit as in diagrams and texts, and as a result most learners tend to view mathematical symbols (e.g., multiplication signs) purely as indicators of which operations to perform on adjacent numbers (Atkinson et al. 2003; Cheng 1999; Greenes 1995; Nathan et al. 1992; Niemi 1996; Ohlsson and Rees 1991).

Representational format and learning

How information is organized in an external representation is assumed to influence learning and understanding. Gaining a full understanding of a domain requires learners to

acquire meaningful schemata. Sweller (1989, p. 458) defined a schema as "(...a cognitive construct that permits problem solvers to recognize problems as belonging to a particular category requiring particular moves for solution". A complete schema therefore rests on three pillars: conceptual knowledge, procedural knowledge, and situational knowledge. *Conceptual knowledge* is "implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain" (Rittle-Johnson et al. 2001, p. 346). Conceptual knowledge develops by establishing relationships between pieces of information or between existing knowledge and new information. *Procedural knowledge* is "the ability to execute action sequences to solve problems" (Rittle-Johnson et al. 2001, p. 346). *Situational knowledge* (de Jong and Ferguson-Hessler 1996) enables learners to analyze, identify, and classify a problem, to recognize the concepts that underlie the problem, and to decide which operations need to be performed to solve the problem.

Schemata can be acquired by performing cognitive activities such as selecting, organizing, and integrating information (Mayer 2003, 2004; Shuell 1986, 1988; Sternberg 1984). *Selecting* involves recognizing which information is relevant and which is not. *Organizing* involves combining pieces of information into a coherent and internally connected structure (e.g., a mental representation). *Integrating* refers to relating newly acquired knowledge to already existing knowledge structures (prior knowledge).

The representational format in which instructional material is presented might influence the way learners select, organize, and integrate this information. If representational formats have differential effects on these cognitive activities, this might result in different emphases on conceptual, procedural, and situational aspects of the learners schemata. For example, diagrams summarize or highlight essential information, make relations among elements explicit, and organize information into coherent structures (Levin 1981; Levin et al. 1987; Levin and Mayer 1993). Therefore, the features of diagrams seem particularly suited for the acquisition of conceptual knowledge. In the case of arithmetical representations, the emphasis is on operational aspects rather than on conceptual or situational aspects. This is assumed to result in schemata that rely more on procedural knowledge. With respect to textual representations the use of natural language facilitates of the relation of information in the text to everyday experiences and situations. Textual representations allow learners to analyze and understand problem statements, in particular stressing situational domain aspects.

Combining two or more representational formats into what is called a multiple representation (e.g., van Someren et al. 1998) is assumed to have some additional effects on schema construction processes (Ainsworth 1999, 2006; Seufert 2003). First, different formats can complement each other; for example, combining an equation and a diagram might be helpful in focusing the learners' attention on not only operational aspects but also conceptual aspects of the domain. Second, one representation might constrain the interpretation of the other. For example, when an arithmetical representation such as an equation is accompanied by a textual representation, the latter might help learners to better understand the equation. Third, learners' integration of information from different representations is thought to support the construction of deeper understanding (Ainsworth 1999, 2006; van der Meij and de Jong 2006). However, combining different formats of equivalent information is not always beneficial for learning. It may interfere with cognitive processing (e.g., split-attention effects), and a multiple representation may contain redundant information, which is assumed to increase cognitive load (e.g., Leung et al. 1997).

Representational format and cognitive load

Cognitive load theory (CLT) is based on the idea that working memory capacity is limited (Miller 1956). CLT distinguishes between three types of working memory load: intrinsic, extraneous, and germane load. *Intrinsic load* is generated by the complexity (element interactivity) of the learning material. *Extraneous load* is determined by the way in which the material is organized and presented (e.g., diagrams or text formats). *Germane load* refers to load caused by mental activities relevant to schema acquisition, such as organizing the material and relating it to prior knowledge (DeLeeuw and Mayer 2008; Paas et al. 2004; Sweller et al. 1998). Cognitive load principles may support or even determine decisions as to which representational format to use (Leung et al. 1997). Carlson et al. (2003) found that where intrinsic load was low (low element interactivity), the extraneous load caused by instructional format was of little consequence. However, in the case of high intrinsic load (high element interactivity), the extraneous load caused by instructional format turned out to be critical. Here, learners who were presented with diagrams performed better and reported significantly lower levels of cognitive load compared to learners presented with textual representations. Yet, empirical findings about the relation between representational format, cognitive load, and learning outcomes are far from straightforward: the advantage of a particular format in learning is not universal, but depends on a complex interaction among the nature of the task and the material, the student's ability, prior knowledge, and practice time. For example, Dee-Lucas and Larkin (1991) found that learning with verbal material was superior to learning with equivalent equations. Leung et al. (1997) replicated these findings but also found that this effect only applies to less able students. Moreover, as material required more complex verbal statements but simpler equations, equations turned out to be more effective, suggesting lower levels of cognitive load. Finally, Leung et al. also found that additional practice with equations reduced cognitive load and increased performance.

Assessing the effects of representational format

From the previous sections it should be clear that decisions as to which representational format to use in instruction are particularly critical in the case of learners with little or no prior knowledge who are dealing with complex instructional material (high intrinsic load). The aim of the current study was to investigate the effects of different representational formats on learning combinatorics and probability theory. The PIN-code example presented earlier is a typical problem for this domain. Understanding and solving problems like these requires learners to process many interrelated elements concurrently (high interactivity and therefore high intrinsic load). It has been demonstrated that instructional material in this domain can be represented in several informationally equivalent ways (see e.g., Fig. 1). In this study, five conditions were compared: three conditions each using a single external representational format (Diagram, Arithmetic, or Text), and two conditions using multiple external representations, that is combinations of single representational formats (Text + Arithmetic or Diagram + Arithmetic). In theory, more combinations of external representations would be possible (e.g., diagram plus text), but due to screen size limitations, these combinations were not feasible without severely hampering the readability of information presented to subjects in the study. The effects of format could be isolated by varying only the representational format and by keeping the different representations informationally equivalent. The main focus was on the effects of formats on schema construction, cognitive load, and interactiveness (learners manipulating interactive representations).

Method

Participants

A total of 123 pupils participated in the study: 61 boys and 62 girls. The average age of the participants was 15.61 years ($SD = 0.59$). Three participants were excluded from the analyses because their post-test scores deviated more than 2 SDs from the mean scores within their condition. The participants participated in the experiment during regular school time, so that participation was obligatory.

Design

The experiment employed a between-subjects pre-test post-test design, with the representational format in which the domain was presented (diagram, arithmetic, text, a combination of text and arithmetic, or a combination of diagram and arithmetic) as the independent variable. The distribution of the participants across conditions is displayed in Table 1.

Domain

The domain of instruction was combinatorics and probability theory. Combinatorics can be used to determine the number of combinations that can be made with a certain set or subset of elements. Probability theory can be used to calculate the chance that a certain combination will be observed empirically. The PIN-code problem presented in the introductory section is a typical problem for this domain. In order to determine the number of possible combinations, one also needs to know (1) whether elements may occur repeatedly in a combination (replacement) and (2) whether the order of elements in a combination is of interest (order). On the basis of these two criteria, four problem categories can be distinguished (for an overview, see Fig. 2).

The PIN-code example matches category 3 (replacement; order important).

Table 1 Number of participants per condition

	Representational format				
	Diagram	Arithmetic	Text	Text + Arithmetic	Diagram + Arithmetic
Participants	24	25	24	24	23

Fig. 2 Problem categories within the domain of combinatorics

		ORDER IMPORTANT?	
		Yes	**No**
REPLACEMENT?	**No**	Category 1: No replacement; Order important	Category 2: No replacement; Order not important
	Yes	Category 3: Replacement; Order important	Category 4: Replacement; Order not important

Learning environment

The instructional approach used in this study is based on inquiry learning (de Jong 2005, 2006). Computer-based simulation is a technology that is particularly suited for inquiry learning. Computer-based simulations contain a model of a system or a process. The learner is enabled to induce the concepts and principles underlying the model by manipulating the input variables and observing the resulting changes in output values (de Jong and van Joolingen 1998).

The learning environments used in the current study were created with SIMQUEST authoring software (van Joolingen and de Jong 2003). A learning environment was developed for each experimental condition. The learning environments contained simulations in which learners could manipulate the number of possible options (generally indicated by N) and the number of element selections (usually indicated by K). The output values displayed the corresponding situation and probability of an event, given the input variables. All five learning environments were identical to each other, except for the representational format of the simulation output. The representational format was diagram, arithmetic, text, a combination of text and arithmetic, or a combination of diagram and arithmetic. The learning environments consisted of five sections. Four of these sections were devoted to each of the four problem categories within the domain of combinatorics. The fifth section aimed at integrating these four problem categories. Each section used a different cover story, that is, an everyday life example of a situation in which combinatorics and probability played a role. Each cover story exemplified the problem category treated in that section. In the fifth (integration) section, the cover story applied to all problem categories.

Each of the five sections contained a series of task assignments (both open-ended and multiple-choice), all based on the cover story for that particular section. Learners could use the simulations to complete the assignments. The assignments involved determining which problem category matched the given cover story (situational knowledge), calculating the probability in a given situation (procedural knowledge), and selecting the description that matched the relation between variables most accurately (conceptual knowledge). In the case of the multiple-choice items, the learners received feedback from the system about the correctness of their answer. If the answer was wrong, the system offered hints about what was wrong with the answer. Learners then had the opportunity to select another answer. For the open-ended questions, learners received the correct answer after completing and closing the question.

In this study we measured cognitive load. The literature mentions several ways to measure cognitive load (Ayres 2006; Brünken et al. 2003, 2004; DeLeeuw and Mayer 2008; Paas et al. 2003). The current study employed self-reports; after each section a questionnaire regarding perceived cognitive load appeared on the screen. Self-reports have been found to be valid, reliable, unintrusive, and sensitive to relatively small differences in cognitive load (e.g., Ayres 2006; Paas et al. 2003). The questionnaire used in the current study consisted of six items (see Table 2). One item intended to measure overall load. This item was adopted from a study by Paas (1992). The set of remaining items was an adapted and extended version of the SOS-scale (Swaak and de Jong 2001). These five items were intended to be indicative of intrinsic (1 item), extraneous (3 items), and germane load (1 item). The learners indicated their amount of mental effort on 9-point Likert scales. Each time the cognitive load questions were presented, they appeared in a different order.

Table 2 Cognitive load items

Type of cognitive load	Item
Intrinsic load	How easy or difficult do you consider probability theory at this moment?
Extraneous load 1	How easy or difficult is it for you to work with the learning environment?
Extraneous load 2	How easy or difficult is it for you to distinguish important and unimportant information in the learning environment?
Extraneous load 3	How easy or difficult is it for you to collect all the information that you need in the learning environment?
Germane load	How easy or difficult was it to understand the simulation?
Overall load	Indicate on the scale the amount of effort you had to invest to follow the last simulation

Table 3 Overview of test items

Knowledge type	Description	Number and type of items
Pretest		
Conceptual	Items about relations between variables	4 mc[a]
Procedural	Solving combinatorial problems	8 Open
Total		12 items
Post-test		
Conceptual	Items about relations between variables	4 Open and 21 mc[a]
Procedural	Solving combinatorial problems	14 Open
Situational	Analyzing, identifying, and classifying problems	1 Open and 4 mc[a]
Total		44 Items

[a] mc = Multiple choice

Knowledge measures

Two knowledge tests were used in this experiment: a pre-test and a post-test. These tests contained 12 and 44 items respectively. An overview of the test items and underlying knowledge types is presented in Table 3

In cases where learners had to calculate the outcome of an item, a calculator was provided on-screen. Both tests were administered via the Internet. The questions were presented screen-by-screen without allowing participants to skip questions or turn back to previous questions. Learner responses on both multiple-choice and open-ended items were collected and recorded electronically.

Procedure

The experiment was carried out in a real school setting in one three hour session including a 15 min break. Learners worked individually. They were told that they could work at their own pace, and that the three hours would be more than enough to complete all assignments. The participants started the session by logging on to the electronic pre-test. It was

announced that the post-test would contain more items of greater difficulty than the pre-test, but that the pre-test items nonetheless would give an indication of what kind of items to expect on the post-test. After completing the pre-test the participants received a printed introductory text introducing the domain. Along with the introductory text the participants received information about how to enter the learning environment. After finishing the last section of the learning environment, the participants received log-on instructions for the post-test environment.

Results

Pre-test

The overall pre-test scores are summarized in Table 4.

A one-way ANOVA performed on the pre-test scores established that there were no differences between conditions, $F(4,119)= 0.63$, *ns*.

Post-test

The post-test results are displayed in Table 5. All post-test measures were analyzed by one-way ANOVAs with representational format as factor.

Analysis of *post-test total scores* showed a significant effect of representational format, $F(4,119) = 2.57$, $P < .05$. A post-hoc least significant difference (LSD) analysis revealed that participants in the Text + Arithmetic condition outperformed participants in the Diagram condition ($P < .01$), and participants in the Diagram + Arithmetic condition ($P < .05$).

Table 4 Pre-test scores

	Representational format									
	Diagram		Arithmetic		Text		Text + Arithmetic		Diagram + Arithmetic	
	M	SD	M	SD	M	SD	M	SD	M	SD
Pre-test score	7.67	1.90	8.20	1.80	8.29	2.03	8.38	1.58	8.00	1.24

Table 5 Post-test scores

Knowledge type	Representational format									
	Diagram		Arithmetic		Text		Text + Arithmetic		Diagram + Arithmetic	
	M	SD	M	SD	M	SD	M	SD	M	SD
Conceptual	17.17	3.10	18.04	2.91	18.17	2.75	19.08	2.21	18.04	2.21
Procedural	5.83	2.73	6.80	3.06	6.75	2.45	8.08	2.84	6.13	2.12
Situational	3.54	0.93	3.40	1.12	3.38	1.10	3.75	0.85	3.70	1.02
Total	26.54	4.91	28.24	5.75	28.29	4.97	30.92	4.32	27.87	4.07

With regard to *conceptual knowledge* no main effect of format, $F(4,119) = 1.56$, *ns*, was found.

Analysis of *procedural knowledge* revealed a significant main effect of representational format, $F(4,119) = 2.52$, $P < .05$. A post-hoc LSD analysis showed that participants in the Text + Arithmetic condition outperformed participants in the Diagram condition ($P < .01$), and participants in the Diagram + Arithmetic condition ($P < .05$).

With regard to *situational knowledge* items no main effect of representational format, $F(4,119) = 0.66$, *ns*, was found.

Cognitive load

In the learning environment, the participants had to indicate on 9-point Likert scales the amount of perceived difficulty (cognitive load) of: the domain in general (intrinsic load); working with the learning environment (extraneous load 1); distinguishing between important and unimportant information (extraneous load 2); collecting needed information (extraneous load 3); understanding the simulation (germane load); and the amount of invested mental effort (overall load). The results are displayed in Table 6.

All cognitive load measures were analyzed by one-way ANOVAs with condition as factor.

Regarding the question of how easy or difficult the participant considered probability theory (*intrinsic load*), a main effect of representational format was found, $F(4,118) = 2.70$, $P < .05$. Post-hoc LSD analyses showed that participants in the Diagram condition considered probability theory more difficult than participants in the Text + Arithmetic condition ($P < .01$) and participants in the Diagram + Arithmetic condition ($P < .05$). Furthermore, participants in the Textual condition considered probability theory more difficult than participants in the Text + Arithmetic condition ($P < .05$).

Differences between conditions with regard to extraneous load were observed as well, $F(4,118) = 2.89$, $P < .05$. Participants in the Diagram condition experienced higher levels of extraneous load compared to participants in the Arithmetic condition ($P < .01$), the Text + Arithmetic condition ($P < .01$), and the Diagram + Arithmetic condition ($P < .01$). Analysis of the extraneous load sub measures (EL 1, EL 2, and EL 3) showed that the differences between conditions with regard to extraneous load, in general, could be

Table 6 Cognitive load measures

Type	Representational format									
	Diagram		Arithmetic		Text		Text + Arithmetic		Diagram + Arithmetic	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Intrinsic (IL)	4.37	1.55	3.65	1.51	3.96	1.31	3.11	1.14	3.52	1.14
Extraneous (EL)	4.34	1.49	3.40	1.14	3.68	0.90	3.38	1.22	3.45	0.95
EL 1	4.36	1.62	3.14	1.21	3.68	0.95	3.35	1.51	3.22	1.05
EL 2	4.28	1.45	3.54	1.26	3.62	1.00	3.29	1.03	3.51	1.23
EL 3	4.40	1.67	3.51	1.25	3.75	1.05	3.52	1.30	3.61	1.06
Germane (GL)	4.36	1.56	3.26	1.23	3.69	1.38	3.29	1.27	3.43	1.03
Overall	4.22	1.62	3.25	1.32	3.75	1.05	3.03	1.14	3.54	1.35

Table 7 Time-on-task (minutes) and number of manipulations performed in the simulations

	Representational format									
	Diagram		Arithmetic		Text		Text + Arithmetic		Diagram + Arithmetic	
	M	SD	M	SD	M	SD	M	SD	M	SD
Time-on-task	29.96	8.64	32.52	7.61	34.21	11.05	36.63	8.84	33.00	9.48
Manipulations	12.52	9.15	23.63	15.33	17.00	12.33	21.56	11.08	17.88	7.88

entirely attributed to EL 1, ($F(4,118) = 3.55$, $P < .01$), that is: participants in the Diagram condition perceived their learning environment as more difficult compared to participants in the Arithmetic condition ($P < .01$), the Text + Arithmetic condition ($P < .01$), and the Diagram + Arithmetic condition ($P < .01$).

With regard to understanding the simulations (*germane load*), differences were found between conditions, $F(4,118) = 2.89$, $P < .05$. Participants in the Diagram condition reported having more difficulty understanding their simulations as compared to participants in the Arithmetic condition ($P < .01$), the Text + Arithmetic condition ($P < .01$), and the Diagram + Arithmetic condition ($P < .05$).

Furthermore, a difference between conditions was found concerning the amount of effort participants had to invest to complete the learning task (*overall load*), $F(4,117) = 2.85$, $P < .05$. Participants in the Diagram condition had to invest more effort than participants in the Arithmetic condition ($P < .05$), and the Text + Arithmetic condition ($P < .01$).

Interactiveness

A summary of the time spent in the learning environment (time-on-task), and the total number of manipulations performed in the simulations is displayed in Table 7. Three outliers have been excluded from the analysis of the number of manipulations. Furthermore, due to technical reasons the manipulation data of 25 learners were not available. This loss of data was equally distributed over experimental conditions. The data for the remaining 95 participants are displayed in Table 7.

A one-way ANOVA showed that there were no differences between conditions with regard to time-on-task, $F(4,119) = 1.62$, *ns*. With regard to the number of manipulations performed in the simulations, a significant difference between conditions was found, $F(4,90) = 2.74$, $P < .05$. Post-hoc LSD analyses showed that participants in the Diagram condition performed fewer manipulations than participants in the Arithmetic condition ($P < .01$) and participants in the Text + Arithmetic condition ($P < .05$).

Discussion and conclusion

Decisions on which representational format to use in instruction are particularly critical in the case of learners with little or no prior knowledge who must deal with complex instructional material. The aim of the current study was to investigate the effects of different representational formats (Diagram, Arithmetic, Text, or the combinations, Text + Arithmetic or Diagram + Arithmetic) on learning combinatorics and probability theory. The main focus was on the effects of formats on schema construction, cognitive load, and interactiveness.

It was expected that different formats would have different effects on the nature of the constructed schemata. Diagrams were expected to result in schemata emphasizing conceptual knowledge. Arithmetical representations were expected to lead to an emphasis on procedural knowledge, whereas textual representations were assumed to stress the situational aspects of the domain. Multiple representations were thought to serve complementing and constraining functions. In general, research findings have suggested that diagrams help to reduce extraneous cognitive load in complex domains.

The idea of diagrams being most beneficial for learning has found its way into everyday practice in classrooms. Tree diagrams are considered a powerful tool for teaching combinatorics and probability theory (e.g., Fischbein 1987; Greer 2001). However, the major finding of this study is that the use of tree diagrams in this domain is not as beneficial as generally thought. Although the learners had prior experience with tree diagrams, the results show that learning with tree diagrams leads to poorer performance at solving problems mathematically, and to higher levels of cognitive load. The measures intended to be indicative of intrinsic, extraneous, and germane load showed that participants presented with tree diagrams considered the domain more difficult (intrinsic load), found the learning environment more difficult (extraneous load), had more difficulty understanding the simulations (germane load), and had to invest more mental effort to complete their learning task. It was also found that the addition of equations to the tree diagrams (Diagram + Arithmetic condition) led to a significant reduction of intrinsic, extraneous, and germane cognitive load, but not to better learning outcomes.

The best learning outcomes were obtained by learners who were presented with a combination of text and equations (Text + Arithmetic condition). Their outcomes were significantly better than those of learners presented with tree diagrams or with a combination of tree diagrams and equations. Learners in all experimental conditions demonstrated equal levels of understanding of domain concepts, variables, and their mutual relations (conceptual knowledge) and were equally able to analyze, identify, and classify problems, to recognize the concepts underlying these problems, and to decide which operations needed to be performed to solve the problems (situational knowledge). However, learners who had been presented with a combination of text and equations during the learning phase turned out to be better at finding mathematically correct solutions to these problems (procedural knowledge).

The results indicate that the differential effects of formats on the nature of schema construction are not as straightforward as expected (e.g., diagrams resulting in schemata emphasizing conceptual knowledge, arithmetical representations stressing procedural knowledge, and so on). The effects of combining different representational formats into multiple representations are not straightforward either. The observation that learners presented with a combination of text and equations tended to outperform learners presented with text only or equations only suggests a positive effect of the combination of these formats. However, the data do not allow interpretation in terms of complementing, constraining, and arriving at deeper understanding by integrating. In the condition where equations were added to tree diagrams, the learning outcomes remained unaffected, but a significant reduction of cognitive load was observed. It remains unclear whether this reduction of mental load was caused by complementing and constraining functions. It is possible that learners in this multiple representation condition ignored the tree diagrams and focused on the equations.

Rogers (1999) suggested that interactive representations can reduce the amount of "low-level" cognitive activities (e.g., drawing and redrawing) normally required when learning, and in turn allow learners to focus their cognitive resources on more "high-level"

cognitive tasks, such as exploring more of the problem space. It was found that learners presented with tree diagrams did not perform as many manipulations on the representations as did learners in the other conditions. This might be an indication that tree diagrams inhibit exploration of the problem space.

In short, the major finding of this study is that the use of tree diagrams in this domain is not as beneficial as generally thought: the learning outcomes are relatively poor, the experienced levels of cognitive load are high, and the format seems to inhibit problem space explorations. The question remains of accounting for these findings. A possible explanation is provided by Tabachneck-Schijf et al. (1997). They argue that experts in particular benefit from diagrams because for them diagrams serve as an aid to access information stored in long term memory. Moreover, diagrams help to decrease the expert's working memory load, because elements of information that are displayed in the diagram do not have to be kept activated in working memory all the time. This frees up cognitive resources the expert can devote to reasoning and problem solving instead. This implies that diagrams are more suited for people who already possess the relevant schemata (e.g., math teachers), and just need the diagrams as mnemonic devices. This explanation also suggests that tree diagrams are less suited for learners to infer reasoning steps from, because the reasoning steps depicted in tree diagrams are quite implicit and require advanced knowledge about the diagram in order to identify them. This might be a disadvantage in domains like combinatorics and probability theory, where problem solving requires a set of reasoning steps that are taken in a specific order. This would explain the advantage of the Text + Arithmetic format. In the textual part of the representation the learners are taken by the hand as it were, and led step-by-step through an explicit and sequential line of reasoning described in everyday language, followed by an equation concisely repeating these steps in an arithmetical (and also sequential) way. The Text + Arithmetic format preserves the strictly sequential nature of problem solving in this domain, whereas the often-claimed advantage of diagrams, allowing for simultaneous processing, turns into a disadvantage.

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