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The influence of learner-generated domain representations on learning combinatorics and probability theory

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ABSTRACT

The aim of the current study was to examine the effects of providing support in the form of tools for constructing representations, and in particular the differential effects of the representational format of these tools (conceptual, arithmetical, or textual) in terms of perceived affordances and learning outcomes. The domain involved was combinatorics and probability theory. A between-subjects pre-test–post-test design was applied with secondary education students randomly distributed over four conditions. Participants completed the same tasks in a simulation-based learning environment. Participants in three experimental conditions were provided with a representational tool that could be used to construct a domain representation. The experimental manipulation concerned the format of the tool (conceptual, arithmetical, or textual). Participants in a control condition did not have access to a representational tool. Data from 127 students were analyzed. It was found that the construction of a domain representation significantly improved learning outcomes. The format in which students constructed a representation did not directly affect learning outcomes or the quality of the created domain representations. The arithmetical format, however, was the least stimulating for students to engage in externalizing their knowledge.

1. Introduction

Learning to understand science and mathematics is hard for many students. The current study seeks to facilitate the learning process by offering students tools to create external representations while learning. It does so in a subdomain of mathematics which is known to be notoriously difficult: combinatorics and probability theory.

One of the more general reasons for students' difficulties with science and mathematics problems is that novices often have a tendency to focus on superficial details rather than on understanding the principles and rules underlying a science or mathematics domain (Chi, Feltovich, & Glaser, 1981; de Jong and Ferguson-Hessler, 1986; Reiser, 2004). Science and mathematics problems require students to go beyond the superficial details in order to recognize the concepts and structures that underlie the problem and to decide which operations are required to solve it (e.g., Fuchs et al., 2004). In the case of probability instruction, for example, identifying the approach that needs to be taken to solve a problem depends a great deal on correct classification of the problem (Lipson, Kokonis, & Francis, 2003).

A second reason for students' difficulties is that the abstract and formal nature of often used arithmetical representations does not illustrate the underlying principles or concepts as explicitly as pictorial and textual representations. Most students tend to view mathematical symbols (e.g., multiplication signs) purely as indicators of which operations need to be performed on adjacent numbers, rather than as reflections of principles and concepts underlying these procedures (Atkinson, Catrambone, & Merrill, 2003; Cheng, 1999; Greenes, 1995; Nathan, Kintsch, & Young, 1992; Niemi, 1996; Ohlsson & Rees, 1991). Therefore, they easily lose sight of the meaning of their actions. Correctly processing formal notations thus becomes an end in itself (Cheng, 1999), not for the purpose of understanding and communicating concepts but for getting high scores on tests (Greeno & Hall, 1997). Learning of arithmetical procedures without conceptual understanding tends to be error prone, easily forgotten, and not readily transferable (Ohlsson & Rees, 1991).

Third, the formal, abstract way in which subject matter is represented makes it hard for students to relate the subject matter to everyday life experiences. Fuson, Kalchman, and Bransford (2005) argue that the knowledge students bring into the classroom is often set aside in mathematics instruction and replaced by procedures that disconnect problem solving from meaning making. The integration of theory and everyday life experience is particularly important in probability and combinatorics, because the principles of probability often appear to conflict with students' experiences and how they view the world (Garfield & Ahlgren, 1988; Kapadia, 1985). The conflicts arise because probabilities do

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not always match students' conceptions and intuitions (e.g., Batanero & Sanchez, 2005; Fischbein, 1975; Greer, 2001). An example of a misconception is the gambler's fallacy, that is, the belief that the outcome of a random event can be affected by (and therefore predicted from) the outcomes of previous events.

These reasons are by no means exhaustive, but summarize some of the main problems encountered in the instruction of combinatorics and probability theory. What can be learned from these points to help improve instruction in this domain? One of the suggestions that follows from this list is that the (abstract and formal) way in which information is presented plays a critical role. The effects of format were tested in a previous study, where different formats (tree diagrams, mathematical equations, texts, or combinations of these) were compared in terms of their effects on learning outcomes and cognitive load (Kolloffel, Eysink, de Jong, & Wilhelm, in press). Learning outcomes improved when using a text describing solution steps on the basis of everyday life situations and simultaneously presenting an equation repeating the same information in an arithmetical format.

The aim of the current study is to find out whether more can be done to support and scaffold students to help them overcome the problems described above. A promising approach traditionally found to help students gain a better understanding and focus more on the underlying principles and concepts of the domain is having students themselves construct representations of the domain, for example by means of writing a summary, creating a drawing, building a runnable computer model, or constructing a concept map.

1.1. Constructing representations

Constructing representations can have different purposes. For example, for students with advanced levels of domain knowledge, constructing a representation may serve as an aid to accessing information stored in long term memory and as a summary of their processing, which decreases working memory load and thus helps them to concentrate on reasoning (Tabachneck-Schijf, Leonardo, & Simon, 1997). For students unfamiliar with the domain, constructing representations can support learning and understanding (e.g., Greeno & Hall, 1997; Lesh & Lamon, 1992). Gaining a full understanding of a domain requires students to recognize which information is relevant, to combine pieces of information into a coherent and internally connected structure (e.g., a mental representation), and to relate newly acquired knowledge to prior knowledge (Mayer, 2003, 2004; Shuell, 1986, 1988; Sternberg, 1984). Cox (1999) argues that the process of constructing a representation elicits self-explanation effects and consists of dynamic iterations and interactions between the constructed representations and mental representations and therefore helps students to refine and disambiguate their domain knowledge.

Evidence from studies in which students (collaboratively) constructed representations indicates that the format in which students construct representations plays a significant role in knowledge construction processes (e.g., Suthers & Hundhausen, 2003; van Drie, van Boxtel, Jaspers, & Kanselaar, 2005). Representational formats can differ with regard to the affordances students perceive.

In the first place, the properties of representations influence which information is attended to and how people tend to organize, interpret, and remember the information (Ainsworth & Loizou, 2003; Cheng, 1999; Larkin & Simon, 1987; Zhang, 1997). This is called *constraining* (Ainsworth, 2006; Scaife & Rogers, 1996; Stenning & Oberlander, 1995), *representational bias* (Utgoff, 1986), or *representational guidance* (Suthers, 2003; Suthers & Hundhausen, 2003). For example, constructing concept maps directs attention to concepts and their mutual relationships (Nesbit & Adesope, 2006) and using formal arithmetical representations may focus attention on procedures rather than on principles and concepts (Atkinson et al., 2003; Cheng, 1999; Ohlsson & Rees, 1991).

In the second place, the perceived affordances of formats for expressing knowledge also depend on familiarity with the format and domain; some formats may seem easier or more appropriate for constructing a representation than others. For example, to students with advanced levels of mathematical knowledge, symbols and formulas may be the easiest and most appropriate way to express their knowledge. To them, these formats are a common and efficient way to express both procedures and underlying principles and concepts. For students relatively new to the domain, using symbols and formulas to construct a representation might seem too difficult or inappropriate, resulting in incorrect and/or incomplete representations, or even a failure to construct a representation at all. Such students may lack the knowledge to use this formal language or may be prone to the misconception that those formats reflect only procedures and not underlying concepts or principles. Tarr and Lannin (2005) found that in conditional probability instruction, students initially avoid using conventional ways of representing probabilities (i.e., using ratios or odds, or formal numerical probabilities). Instead they use alternative forms of representation, such as textual statements. When they reach more advanced levels of knowledge, some of them start using more conventional representations.

1.2. Research questions

The aim of the current study was to examine the effects of providing support in the form of tools for constructing representations, and in particular the differential effects of the representational format of these tools in terms of perceived affordances and learning outcomes. The following questions guided this study. Do representational formats have differential effects on the likelihood that students use the support and engage in constructing representations? Does format have differential effects on the quality of the representations students construct? Does the construction of a representation of a domain lead to better learning outcomes than not constructing a representation? And, if students construct a representation, does format have differential effects on domain understanding?

On the basis of the arguments outlined in the previous section, it is hypothesized that constructing a representation of a domain is beneficial for learning and understanding (e.g., Cox, 1999). The format in which such a representation is constructed is assumed to have differential effects on knowledge construction and domain understanding. Three of the most commonly used formats in the domain of combinatorics and probability theory were compared: (a) a conceptual format, (b) an arithmetical format, and (c) a textual format.

Constructing a conceptual representation like a concept map is thought to focus the student's attention on the identification of concepts and their mutual relationships (Nesbit & Adesope, 2006). A concept map is not a very complicated format, in particular when the number of concepts and relations is not too large (van Drie et al., 2005). It is hypothesized that the construction of a conceptual representation will make students focus in particular on the concepts underlying the domain. Therefore, it is expected that constructing a conceptual representation will result in enhanced levels of knowledge about the conceptual aspects of the domain, rather than procedural or situational aspects.

Constructing representations in an *arithmetical* format is assumed to draw the student's attention primarily to operational aspects. Therefore, it is hypothesized that constructing an arithmetical representation will enhance procedural knowledge, rather than conceptual and situational aspects. Regarding the likelihood that students will construct a representation using the arithmetical format, it is hypothesized that compared to other formats students may have difficulty constructing arithmetical representations (Tarr & Lannin, 2005).

The third format for constructing a domain representation that is considered here is a *textual* format. This format particularly allows students to express their knowledge in their own words. The current domain can easily be described in terms of everyday life contexts and situations. Constructing textual representations is assumed to direct the student's attention to situational and conceptual aspects, although the conceptual issues might not be as strongly stressed as for students who construct a conceptual representation. It is expected that students will not experience too many difficulties using this format. Overall, in educational settings this is the most commonly used format.

2. Method

2.1. Participants

In total, 133 secondary education students, 65 boys and 62 girls (six students did not indicate their gender), participated. The average age of the students was 14.63 years (SD = .62). The domain of combinatorics and probability theory is part of the regular curriculum and the experiment took place a few weeks before this subject would be covered. The students completed the experiment during regular school time; therefore, participation was obligatory. They received a grade based on their post-test performance.

2.2. Design

The experiment employed a between-subjects design with four conditions: three experimental conditions and one control condition. All students (including those in the control condition) had to complete the same tasks: completing a pre-test, working through a simulation-based learning environment, and completing a post-test. The only difference between the control condition and the experimental conditions was that students in the experimental conditions were asked to construct a representation of the domain. Their learning environments were equipped with an additional tool: the representational tool. The difference between the three experimental conditions concerned the format of the representational tool, which could be conceptual, arithmetical, or textual. The students in the experimental conditions were informed in advance about the (general) beneficial effects on learning of constructing representations. Students were assigned randomly to conditions. Afterwards, six students were excluded from the analyses because they missed one or more experimental sessions. Of the remaining 127 students, 33 were in the Conceptual condition, 30 in the Arithmetical condition, 32 in the Textual condition, and 32 in the Control condition.

2.3. Domain

The domain of instruction was combinatorics and probability theory. An example of a problem in this domain is: what is the probability that a thief will guess the 4-digit PIN-code of your credit card correctly the first try? The essence of combinatorics is determining how many different combinations can be made with a certain set or subset of elements. In order to determine the number of possible combinations, one also needs to know (1) whether elements may occur repeatedly in a combination (replacement) and (2) whether the order of elements in a combination is of interest (order). On basis of these two criteria, four so-called problem categories can be distinguished. The PIN-code example matches the category "replacement; order important". When the number of possible combinations is known, the probability that one or more combinations will occur in a random experiment can be determined.

2.4. Learning environment

The instructional approach used in this study is based on inquiry learning (de Jong, 2005, 2006). Computer-based simulation is a technology that is particularly suited for inquiry learning (e.g., de Jong and van Joolingen, 1998a,b). Computer-based simulations contain a model of a system or a process. By manipulating the input variables and observing the resulting changes in output values the student is enabled to induce the concepts and principles underlying the model (van Joolingen and de Jong, 1991, de Jong and van Joolingen, 1998a,b).

The learning environment used in the current study, called Probe-XMT (see Fig. 1), was created with SimQuest authoring software (van Joolingen and de Jong, 2003).

In the box on the left-hand side of the simulation (see Fig. 1), students could manipulate input variables. On the right-hand side of the simulation the resulting effects of the manipulations on the output values could be observed. In this case the output consisted of a text and an equation that changed whenever the input variables were changed. Probe-XMT consisted of five sections. Four of these sections were devoted to each of the four problem categories within the domain of combinatorics. The fifth section aimed at integrating these four problem categories. Each section used a different cover story, that is, an everyday life example of a situation in which combinatorics and probability played a role. Each cover story exemplified the problem category treated in that section. In the fifth (integration) section, the cover story applied to all problem categories.

Each of the five sections in the learning environment contained a series of questions (both open-ended and multiple-choice items), all based on the cover story for that particular section. These questions involved determining which problem category matched the given cover story (situational knowledge), calculating the probability in a given situation (procedural knowledge), and selecting a description that matched the relation between variables most accurately (conceptual knowledge). In the case of the multiple-choice items, the students received feedback from the system about the correctness of their answer. If the answer was wrong, the system offered hints about what was wrong with the answer. Students then had the opportunity to select another answer. In the case of the open-ended questions, students received the correct answer after completing and closing the question.

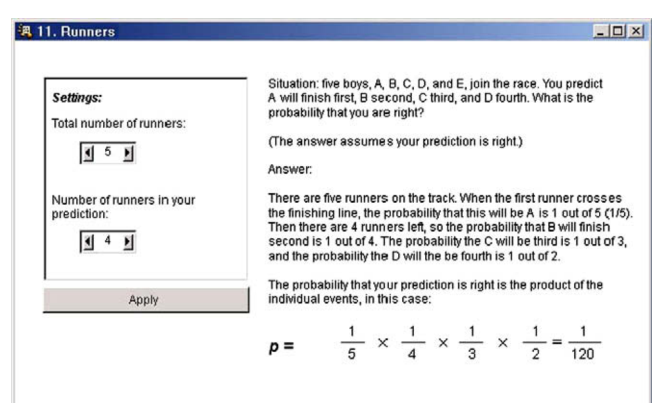

Fig. 1. Screendump Probe-XMT simulation.

Most of the questions were accompanied by simulations that could be used to explore the relations between variables within the problem category. In the simulations, students could manipulate variables and observe the effects of their manipulations on other variables. The simulations used a combination of textual and arithmetical representations. This combination of representations was found to have benefits in terms of learning outcomes and mental effort (Kolloffel et al., in press).

The learning environment automatically registered student actions. User actions that were logged included measures such as user path through the learning environment (which parts of the learning environment were opened, when, for how long, and in what sequence) and the number and nature of manipulations carried out in the simulations (how many experiments were carried out and the input values of each experiment).

2.5. Representational tools

Students in the experimental conditions were encouraged to construct a representation of the domain that would be meaningful to themselves and a fictitious fellow student. This representation could be used to summarize principles underlying the domain, the variables playing a role in the domain, and their mutual relationships. Students could create their representations by means of an electronic on-screen representational tool. There were three types of representational tools, one for each experimental condition: (a) a conceptual representational tool, (b) an arithmetical representational tool, and (c) a textual representational tool.

The *conceptual representational tool* (see Fig. 2) could be used to create a conceptual representation of the domain. Students could draw circles representing domain concepts and variables. Keywords could be entered in the circles. The circles could be connected to each other by arrows indicating relations between concepts and variables. The nature of these relations could be specified by attaching labels to the arrows.

In the *arithmetical representational tool* (see Fig. 3), students could use variable names (N, K, and P), numerical data, and mathematical operators (division signs, equals signs, multiplication signs, and so on) in order to express their knowledge.

Finally, the *textual representational tool* (see Fig. 4) resembled simple word processing software, allowing textual and numerical input.

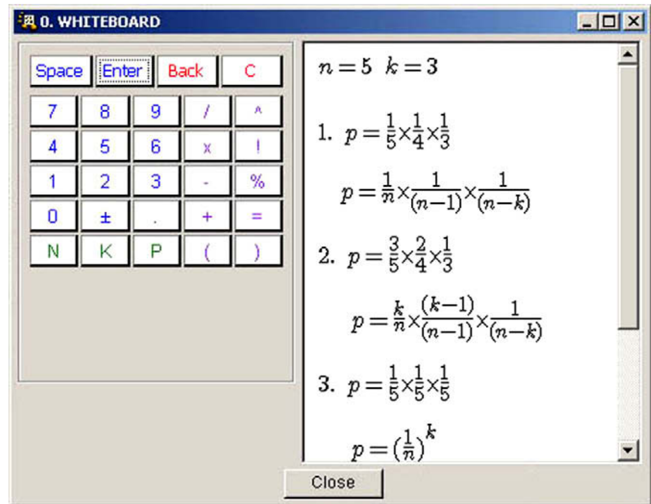

Fig. 3. Arithmetical representational tool.

The contents of the representational tools were stored automatically.

2.6. Knowledge measures

Two knowledge tests were used in this experiment: a pre-test and a post-test. The tests contained 12 and 26 items, respectively. The pre-test aimed at measuring the prior knowledge of the students. The post-test aimed at measuring the completeness of students' schemas related to this domain. Sweller (1989, p. 458) defined a schema as "a cognitive construct that permits problem solvers to recognize problems as belonging to a particular category requiring particular moves for solution". A complete schema therefore rests on three pillars: situational knowledge, conceptual knowledge, and procedural knowledge. Situational knowledge (de Jong and Ferguson-Hessler, 1996) enables students to analyze, identify, and classify a problem, to recognize the underlying concepts, and to decide which operations are required to solve the problem. There were four multiple-choice items measuring this type of knowledge on the post-test (see Fig. 5 for an example).

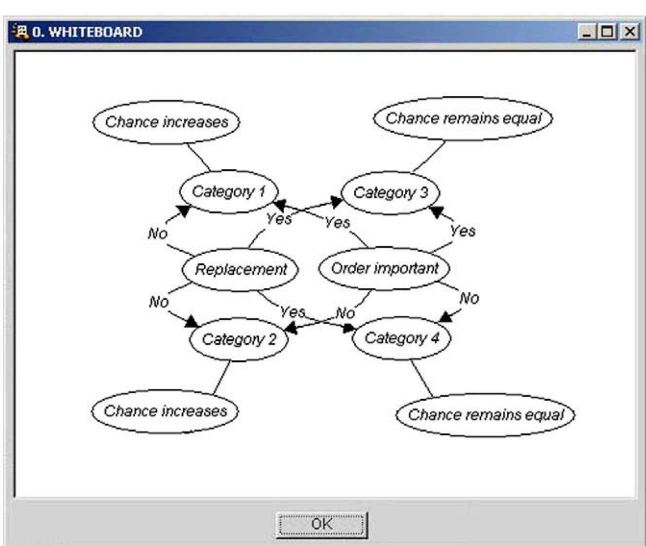

Fig. 2. Conceptual representational tool.

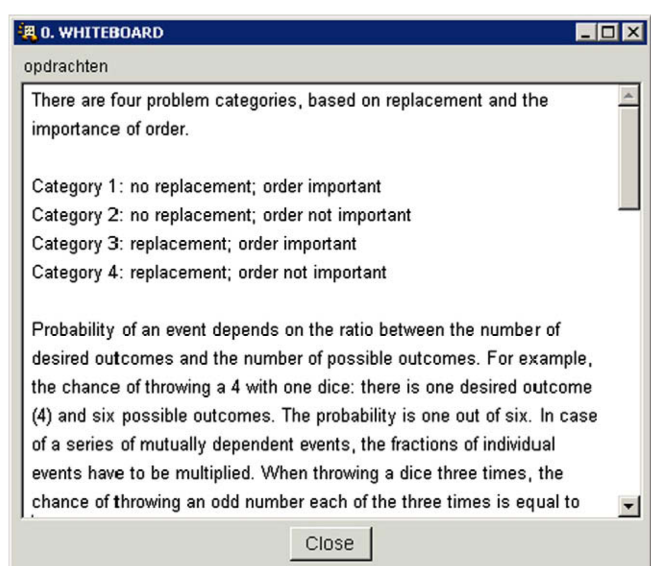

Fig. 4. Textual representational tool.

You throw a die 3 times and you predict that you will throw two sixes and a one in random order. What is the characterization of this problem?

a. order important; replacement
b. order important; no replacement
c. order not important; replacement
d. order not important; no replacement

Fig. 5. Post-test item measuring situational knowledge.

There is a man at a fair who says he will predict the 2 months in which you and your companion were born. The man does not have to specify who was born in which month. When he correctly predicts both months, he wins the stake; when his prediction is not correct, you win the stake and can choose a cuddly toy. You and your friend decide to take the chance. You were born in July and your friend was born in May. What is the chance that the man correctly predicts these months and wins?

Fig. 7. Post-test item measuring procedural knowledge.

Conceptual knowledge is "implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain" (Rittle-Johnson, Siegler, & Alibali, 2001, p. 346). Conceptual knowledge develops by establishing relationships between pieces of information or between existing knowledge and new information. The post-test contained 13 multiple-choice items aiming at measuring conceptual knowledge (see Fig. 6 for an example).

Procedural knowledge is "the ability to execute action sequences to solve problems" (Rittle-Johnson et al., 2001, p. 346). The post-test contained 9 open-ended items aiming at measuring procedural knowledge (see Fig. 7 for an example).

The correct answers to the items presented in Figs. 5, 6 and 7, are, respectively: answer C; answer B; and $(2/12) \times (1/12) = 1/72$.

2.7. Procedure

The experiment was carried out in a real school setting in three sessions, each separated by a 1-week interval. Students worked individually and they were told that they could work at their own pace.

The first session started with some background information with regard to the experiment (general purpose of the research, the domain of interest, learning goals, etc.). This was followed by the pre-test. It was announced that the post-test would contain more items of greater difficulty than the pre-test, but that the pre-test items nonetheless would give an indication of what kind of items to expect on the post-test. At the end of the pre-test the students received a short, printed introductory text introducing the domain. The first session was limited to 50 min in duration. During the last 15 min of the session, the use of the learning environment was demonstrated. Use of the representational tools was demonstrated for students in the experimental conditions. They were informed of the beneficial effects on learning of constructing representations and they were told that they could use the tool any time they wanted while working in the learning environment. During the second session, students worked with the learning environment; students in the experimental conditions were encouraged to use the representational tool to construct a domain representation while working with the learning environment. Again they were informed of the beneficial effects of constructing representations and they were told that they could use the tool any time they wanted while working in the learning environment. The duration of this session was set at 70 min. Although it was possible to take a non-linear path through the learning environment, students were advised to go through the sections in order because they build upon each other. While working with the representational tool, some students asked the experimenter if the quality of the representation they constructed would count as well for their final grade. They were told that it was very important to use the tool, that the constructed representation could possibly play some role in determining the grade, but that it in any case would be very helpful for preparing oneself for the post-test.

You play a game in which you have to throw a die twice. You win when you throw a 3 and a 4. Does it matter if these two numbers must thrown in this specific order?

a. Yes, if you have to throw the numbers in a specific order your chance is greater than when the order doesn't matter
b. Yes, if you have to throw the numbers in a specific order your chance is smaller than when the order doesn't matter
c. No, both events are equally likely to occur
d. This depends on what the other players in the game throw

Fig. 6. Post-test item measuring conceptual knowledge.

The third session was set at 50 min. First, students were allowed to use the learning environment for 10 min in order to refresh their memories with regard to the domain. Then all students had to close their domain representations and learning environments, and had to complete the post-test. When students finished the test they were allowed to leave the classroom.

2.8. Data preparation

The domain representations constructed by the students were scored using a scoring rubric (see Appendix A). This rubric revolved around the principle that scoring of the domain representation should not be biased by the representational format of the representational tool, that is, all types of representations should be scored on the basis of exactly the same criteria. The maximum possible score was 8 points. The rubric was used to assess whether domain representations reflected the concepts of replacement and order, presented calculations, referred to the concept of probability, and indicated the effects of size of (sub)sets, replacement, and order on probability.

3. Results

3.1. Use of representational tools

The first aspect of our research question was whether representational formats have differential effects on the likelihood that students engage in constructing representations. Of the 33 students provided with a conceptual representational tool, 17 students (52 percent) created a domain representation. This was about the same for students provided with a textual representational tool: 15 of the 32 students (47 percent) constructed a representation. The arithmetical tool turned out to be used the least: 6 out of 30 students (20 percent) used the tool. A Chi-square analysis showed these differences between conditions were significant, $X^2(2, N = 95) = 7.45$, $p < .05$. The data show that the arithmetical format is clearly less ready-to-hand for creating external representations than the other

Table 1
Quality scores of constructed representations.

	Representational format											
	Conceptual ($n = 17$)				Arithmetical ($n = 6$)				Textual ($n = 15$)			
	M	*SD*	*Min*	*Max*	*M*	*SD*	*Min*	*Max*	*M*	*SD*	*Min*	*Max*
Score	2.38	1.03	1	4	2.67	1.97	1	6	2.67	0.98	1	4

two formats. The next question concerns the quality of the representations created by the students.

In Table 1 the average quality scores of the constructed representations as these were assessed with the scoring protocol are displayed. All representations were scored by two raters who worked independently. The inter-rater agreement was .89 (Cohen's Kappa).

A one-way ANOVA showed that the format in which a representation was constructed did not influence its quality, $F(2, 37) = 0.42$, MSE = 0.62, $p = .66$.

3.2. Time-on-task

The log files provided data about how much time students spent on the learning task (see Table 2).

The data presented in Table 2 were analyzed by means of 3 × 2 ANOVA with experimental conditions (Conceptual, Arithmetical, and Textual) and tool-use as factors. Subsequently, these data are compared to the control condition. With regard to time-on-task, no differences were observed between conditions ($F(2, 89) = 0.22$, MSE = 60.89, $p = .81$), tool-use ($F(1, 89) = 1.60$, MSE = 449.92, $p = .21$), and no interaction was observed ($F(2, 89) = 0.17$, MSE = 46.99, $p = .85$). A one-way ANOVA in which the tool-users from each experimental condition and the students from the control condition were included, showed that tool-users and students in the control condition spent the same amount of time on the learning task, $F(3, 66) = 0.65$, MSE = 156.48, $p = .59$. The same was true for no-tool-users and students in the control condition, they also spent the same amount of time on the task, $F(3, 85) = 0.26$, MSE = 74.19, $p = .85$.

3.3. Knowledge measures

Two measures of knowledge were obtained: prior knowledge (pre-test score), and post-test score. The reliability, Cronbach's α, was $\alpha = .40$ for the pre-test and $\alpha = .80$ for the post-test. The pre-test reliability was rather low, but sufficient for the purpose of verifying if students did not have too much prior knowledge and that there were no differences between conditions. The scores on the knowledge measures are displayed in Table 3. In this table and the subsequent analyses a distinction is made for the three experimental conditions between students who used the representational tool and those who did not.

The data presented in Table 3 were analyzed by means of 3 × 2 ANOVAs with experimental conditions (Conceptual, Arithmetical, and Textual) and tool-use as factors. After that, a separate analysis that compares the data of the experimental groups with the control group is presented.

Students were asked for their latest school report grade in mathematics. This grade, which can range from 1 (very, very poor) to 10 (outstanding) was interpreted as an indication of the student's general mathematics achievement level. An ANOVA showed that there were no differences between conditions with regard to math grade, $F(2, 89) = 0.22$, MSE = 0.49, $p = .80$. With regard to tool-use a difference was found: students who constructed representations (tool-use) in general had somewhat higher math grades than students who did not use their representational tool to construct a representation, $F(1, 89) = 10.01$, MSE = 22.38, $p < .01$. No interaction between condition and tool-use was found, $F(2, 89) = 0.24$, MSE = 0.54, $p = .79$.

With regard to prior knowledge (pre-test score) no differences were observed between conditions, $F(2, 89) = 0.58$, MSE = 1.52, $p = .56$, tool-use, $F(1, 89) = 0.97$, MSE = 2.55, $p = .33$, and no interaction was observed, $F(2, 89) = 0.05$, MSE = 0.12, $p = .96$.

Math grade was entered as a covariate in the analysis of learning outcomes. With respect to overall post-test scores, no differences were observed between conditions, $F(2, 88) = 1.39$, MSE = 20.23, $p = .26$. A main effect of tool-use was found: students who constructed a domain representation showed significantly higher overall post-test scores, $F(1, 88) = 5.65$, MSE = 82.41, $p < .05$, $\eta_p^2 = .06$. No interaction effects were observed between condition and tool-use, $F(2, 88) = 0.24$, MSE = 3.53, $p = .79$.

With regard to conceptual knowledge, no main effect of condition ($F(2, 88) = 1.58$, MSE = 4.46, $p = .21$), tool-use ($F(1, 88) = 3.54$, MSE = 9.99, $p = .06$), or interaction between the two was found ($F(2, 88) = 0.07$, MSE = 0.19, $p = .94$).

In the case of procedural knowledge no main effects were observed for condition ($F(2, 88) = 0.59$, MSE = 2.58, $p = .56$) and tool-use ($F(1, 88) = 0.96$, MSE = 4.20, $p = .33$), and no interaction effect was found ($F(2, 88) = 0.76$, MSE = 3.31, $p = .47$).

Regarding situational knowledge, condition did not play a significant role ($F(2, 88) = 0.93$, MSE = 1.45, $p = .10$), but tool-use did, $F(1, 88) = 9.60$, MSE = 14.97, $p < .01$, $\eta_p^2 = 0.10$. No interaction effect was observed, $F(2, 88) = 0.07$, MSE = 0.11, $p = .93$.

Finally, Pearson product-moment correlation coefficients between the quality of the constructed representations (in general, but also for each format) and knowledge type scores were calculated. No correlations were observed except that between the quality of representations in general (regardless of format) and procedural knowledge ($r = .33$, $p < .05$).

It could be argued that students who use the representational tools are more compliant, interested, and motivated and that stu-

Table 2
Time-on-task.

No. of experiments	Condition							
	Conceptual ($n = 33$)		Arithmetical ($n = 30$)		Textual ($n = 32$)		Control ($n = 32$)	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Total time-on-task (min.)	69.64	13.95	66.95	17.61	66.64	18.32	65.48	15.57
Tool-use	70.84	14.17	70.62	15.98	70.50	16.85		
No-tool-use	68.38	14.05	66.04	18.19	63.23	19.38		

Table 3
Knowledge measures.

Knowledge measures	Condition							
	Graphical (n = 33)		Arithmetical (n = 30)		Textual (n = 32)		Control (n = 32)	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Math grade (max. 10)	6.46	1.61	5.89	1.55	6.25	1.54	6.27	1.64
Tool-use	7.12	1.52	6.62	1.63	6.73	1.37		
No-tool-use	5.75	1.41	5.71	1.51	5.82	1.58		
Pre-test (max. 12)	5.70	1.36	5.43	1.85	5.25	1.59	5.47	1.95
Tool-use	5.94	1.39	5.67	1.75	5.40	1.60		
No-tool-use	5.44	1.32	5.38	1.91	5.12	1.62		
Post-test								
Conceptual knowledge (max. 13)	9.21	1.97	9.77	1.50	9.28	1.67	9.06	1.90
Tool-use	9.76	1.64	10.33	0.82	9.80	1.47		
No-tool-use	8.63	2.16	9.63	1.61	8.82	1.74		
Procedural knowledge (max. 9)	4.15	2.32	3.67	2.70	3.66	2.04	3.75	2.57
Tool-use	4.82	2.33	5.17	3.19	3.87	1.96		
No-tool-use	3.44	2.16	3.29	2.49	3.47	2.15		
Situational knowledge (max. 4)	2.94	1.25	2.87	1.33	2.66	1.31	2.63	1.36
Tool-use	3.41	1.00	3.67	0.82	3.07	1.22		
No-tool-use	2.44	1.32	2.67	1.37	2.29	1.31		
Overall score (max. 26)	16.30	4.51	16.30	4.11	15.59	3.98	15.44	4.68
Tool-use	18.00	4.27	19.17	3.82	16.73	3.49		
No-tool-use	14.50	4.15	15.58	3.93	14.59	4.21		

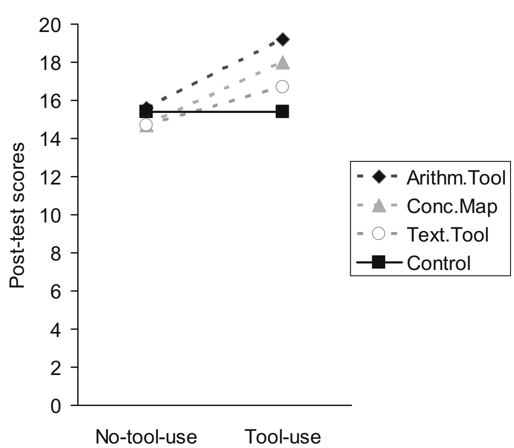

Fig. 8. Post-test scores of experimental conditions and control condition.

dents who do not use the tools are the less compliant, interested and motivated students. Differences between these groups as they were found could then be attributed to these characteristics and not to the factor tool-use. A comparison of the data from the experimental groups with the data from the control group makes this assumption highly unlikely. Students in the control condition came from the same population as the students in the experimental conditions. They formed a cross-section of both the group of students who chose not to use the tool and the group who did use the tool. The performance of the control group can therefore be considered average. The supposedly less motivated, compliant, and/or self-regulated no-tool-users are expected to perform below average, the tool-users to perform above average. However, the data do not confirm this expectation as can be observed in Fig. 8.

Fig. 8 indicates that the learning outcomes of students who did not use a representational tool are equal to learning outcomes in the control condition. A statistical comparison of the overall post-test scores of the no-tool-use group and the control condition confirmed this picture, $t(87) = -0.48$, $p = .63$. Furthermore, it was established that the no-tool-use group and the control condition obtained equal scores with regard to conceptual knowledge ($t(87) = 0.10$, $p = .92$), procedural knowledge ($t(87) = -0.69$, $p = .49$), and situational knowledge ($t(87) = -0.45$, $p = .65$). Therefore, in terms of learning results the no-tool-use group and the control condition perform equal.

When the tool-use group and the control condition are compared with each other, it is found that with regard to overall post-test scores the tool-use group outperformed the control condition, $t(68) = 2.19$, $p < .05$, Cohen's $d = .52$. Both obtained comparable scores on conceptual knowledge ($t(57.45) = 1.96$, $p = .05$) and procedural knowledge ($t(68) = 1.28$, $p = .21$). With regard to situational knowledge, the tool-use group outperformed the control condition, $t(58.32) = 2.33$, $p < .05$, Cohen's $d = .57$.

4. Conclusion and discussion

Constructing a representation of a domain is thought to be beneficial for learning and understanding (e.g., Cox, 1999). The format in which such a representation is constructed is assumed to have differential effects on knowledge construction and domain understanding. In the current study three formats for constructing representations in the domain of combinatorics and probability theory have been compared in a between-subjects pre-test–post-test experiment with three experimental conditions and one control condition. The three experimental conditions were provided with a simulation-based inquiry learning environment that was equipped with a representational tool, that is, a support tool that could be used by students to construct domain representations. The experimental conditions differed with respect to the format of the tool, that could be conceptual (i.e., a concept map), arithmetical, or textual. The following questions guided this study. Do representational formats have differential effects on the likelihood that students use the support and engage in constructing representations? Does format have differential effects on the quality of the representations students construct? Does the construction of a representation of a domain lead to better learning outcomes than not constructing a representation? And, if students construct a representation, does format have differential effects on domain understanding? In order to gain a more full understanding of the effects of support in the form of representational tools, the data of the experimental groups were also compared with students in a control condition using the same learning environment as used in the experimental conditions, but without a representational tool.

The findings show that representational tools with a conceptual or a textual format are more readily used than a tool with an arithmetical format. With regard to tool-use by students, literature about tool appropriation suggests a distinction between knowing how to use a tool in the sense of knowing how to operate it and knowing how to apply or utilize the tool in such a way that its contribution to the learning process is maximized (Overdijk, Bernard, van Diggelen, & Baker, 2008; Overdijk & van Diggelen, 2008). If students understand how to operate the tool, but do not utilize it as intended by its designers, the extent to which the tool contributes to the learning process can be reduced or take a form that is different from what was intended. In the current study, operating the representational tools, including the arithmetical tool, was learned easily and quickly by students. The observation that the arithmetical tool was used less frequently during the learning process therefore could to have to do with problems regarding utilizing the tool rather than operating it. Apparently, the arithmetical format prevented many students from utilizing the tool, possibly because the format is too difficult for students to be used to express their understanding and construct a domain representation. This would to some extent corroborate the observation by Tarr and Lannin (2005) that students initially avoid using conventional, formal ways of representing probabilities, using instead alternative representational forms. When they reach more advanced levels of knowledge, some of them start using more formal representations. This could indicate that the affordances of different formats in this domain can change as a function of time and expertise. Such an expertise reversal effect (Kalyuga, Ayres, Chandler, & Sweller, 2003) was also observed by Leung, Low, and Sweller (1997) in the case of learning from arithmetical representations. They found that for arithmetically less able students, learning from words was more effective than learning from equations. The formal notations were found to interfere with learning. However, with practice and increasing expertise, the formal notations were found to be more effective for learning than words. In the case of the current study the participants were novices. Therefore, the arithmetical format might not have fitted their level of expertise, but it might be suited for students with advanced levels of expertise. In further research, post-treatment interviews with students could possibly reveal more about the affordances of the different formats as they perceive them.

It was found that the construction of a domain representation in general is related to higher post-test scores. Furthermore, it was found that constructing representations, regardless of the format, is associated with significantly higher levels of situational knowledge. This type of knowledge is a prerequisite for going beyond the superficial details of problems in order to recognize the concepts and structures that underlie the problem and to decide which operations are required to solve it (e.g., Fuchs et al., 2004). In the case of probability instruction, the approach that needs to be taken to solve a problem is very dependent on the correct classification of the problem (Lipson et al., 2003). The differences could not be attributed to time-on-task. Students in all conditions and regardless whether or not they constructed a domain representation, all spent the same amount of time on their learning task. It was also observed that formats do not have a differential effect on the quality of the constructed representations. The post-test scores show that there is no direct relation between the format of the domain representation and learning outcomes in terms of conceptual, procedural, or situational knowledge.

In Section 1 it was discussed that students often have a hard time understanding the domain of combinatorics and probability theory. The question was raised as to what could be done to improve instruction about this domain. In a previous study it was shown that the representational format in which the domain is presented to the learners affects learning outcomes (Kolloffel et al., in press). What the current study adds to the understanding of how instruction in this domain can be improved is that creating a representation of the domain can also be beneficial for learning outcomes. The format used to create this representation is found to play a critical but indirect role. Although format affects neither the quality of the representation nor the learning outcomes, it does influence the likelihood that students engage in constructing a representation. The activity of constructing a domain representation is primarily associated with becoming more knowledgeable about the problem categories in this domain so as to identify these categories in problem statements. Choosing a format that stimulates students to externalize their knowledge is a useful first step towards having them utilize and take advantage of this type of learning support. The findings also suggest that it is worthwhile to search for ways in which more students will engage in constructing representations, for example by exploring the use and the effects of the representational tools in a collaborative learning setting. In other studies (e.g., Gijlers & de Jong, 2005; Gijlers, Saab, van Joolingen, de Jong, & van Hout-Wolters, 2009; Suthers & Hundhausen, 2003; van Drie et al., 2005), in which the effects of representational tools were studied in other domains, they were found to mediate communication and knowledge construction processes between students. Studying the effects of the collaborative use of representational tools could be a promising option in the search for new ways to improve the instruction of combinatorics and probability theory as well.

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Appendix A

	Represented?	Conceptual tool	Arithmetical tool	Textual tool	PNT
A	The concept of "replacement"	Literally, or descriptive Examples: – "Replacement" – "Category 1: without replacement; order important" – "...[Runners, BK]... then you have to do $1/7 \times 1/6 \times 1/5$ because each time there is one runner fewer"	Two formulas or calculations in which "replacement" varies Examples: – "$(1/n) \times (1/n) \times (1/n) = p(1/n) \times (1/(n-1)) \times (1/(n-2)) = P$" – "$1/5 \times 1/4 \times 1/3 \times 1/5 \times 1/5 \times 1/5$" – "$p = 1/10 \times 1/10 \times 1/10$ $p = 1/5 \times 1/4 \times 1/3$"	Literally, or descriptive Examples: – "Replacement" – "Category 1: without replacement; order important" – "...If there are 7 runners, then the chance is 1 out of 7 (1/7), if that runner passes the finish, then there are 6 runners left, then there is a chance of 1 out of 6 (1/6), and so on"	1
B	The concept of "Order"	Literally, or descriptive Examples: – "Order" – "Category 1: without replacement; order important" – "...If there are 7 runners and you predict the top 3 without specifying the positions of specific runners in the top 3..."	Two formulas or calculations in which "order" varies Examples: – "$(1/n) \times (1/n) \times (1/n)$ $(k/n) \times ((k-1)/n) \times ((k-2)/n)$" – "$1/5 \times 1/4 \times 1/3$ $3/5 \times 2/4 \times 1/3$"	Literally, or descriptive Examples: – "Order" – "Category 1: without replacement; order important" – "...At a game of Bingo, order is not important"	1
C	Calculation	Formal, literally, descriptive, or a concrete calculation Examples: – p = acceptable outcomes/possible outcomes$1/5 \times 1/4 \times 1/3$ – ... when you also bet ont he order in which the marbles will be selected, your chance is: 1/5 and 1/4 is 1/20..."	Formal (formula) or a concrete calculation Examples: – "$(1/n) \times (1/n) \times (1/n)$" – "$1/5 \times 1/4 \times 1/3$"	Formal, literally, descriptive, or a concrete calculation Examples: – p = acceptable outcomes/possible outcomes – $1/5 \times 1/4 \times 1/3$ – ... when you also bet ont he order in which the marbles will be selected, your chance is: 1/5 and 1/4 is 1/20..."	1
D	Probability	Literal reference to the term "probability"/p, or a description of the concept Expression of a concrete probability (e.g., a fraction), but then it need to be made clear in the context (e.g., by a calculation) where the probability comes from Examples: – "In order to calculate 'p' the chances need to be multiplied" – $p = 1/5 \times 1/4 \times 1/3$ – "...In that case [student refers to a situation outlined earlier], the probability is 1/10"	Literal reference to the term "p" Expression of the outcome of a calculation Examples: – "$p = (1/n) \times (1/n) \times (1/n)$" – "$p = 1/5 \times ¼ \times 1/3$" – "$1/5 \times 1/4 \times 1/3 = 1/60$"	Literal reference to the term "probability"/p, or a description of the concept Expression of a concrete probability (e.g., a fraction), but then it need to be made clear in the context (e.g., by a calculation) where the probability comes from Examples: – "In order to calculate 'p' the chances need to be multiplied" – $p = 1/5 \times 1/4 \times 1/3$ – "...In that case [student refers to a situation outlined earlier], the probability is 1/10"	1
E	Effect of n on probability	Descriptive or on basis of calculations showing the effect (in the latter case, k needs to be constant) Examples: – "fewer options = higher chance" – "If fewer runners attend the race, the chance your prediction is correct will increase"	A formula or a series of calculations showing the effect (in the latter case, k needs to be constant) Examples: – "$(1/n) \times (1/n) \times (1/n) = 1/n^3$" – "$1/5 \times 1/4 \times 1/3 = 1/60$ $1/6 \times 1/5 \times 1/4 = 1/120$"	Descriptive or on basis of calculations showing the effect (in the latter case, k needs to be constant) Examples: – "If the number of elements you can choose from increases, the chance will be smaller that you will select a specific element" – "If fewer runners attend the race, the chance your prediction is correct will increase"	1

(*continued on next page*)

Appendix A (*continued*)

	Represented?	Conceptual tool	Arithmetical tool	Textual tool	PNT
F	Effect of k on probability	Descriptive or on basis of calculations showing the effect (in the latter case, n needs to be constant) Examples: Ù "with 1 choice → 1/possible; with more choices → number of choices/possible outcomes" – "If you only predict who will win the race and not the top 3, then the chance is greater that your prediction will be correct"	A formula or a series of calculations showing the effect (in the latter case, k needs to be constant) Examples: – "$(1/n) \times (1/n) = 1/n^2$ – $(1/n) \times (1/n) \times (1/n) = 1/n^3$" – "$1/5 \times 1/4 = 1/20$ $1/5 \times 1/4 \times 1/3 = 1/60$"	Descriptive or on basis of calculations showing the effect (in the latter case, n needs to be constant) Examples:"When your prediction is less elaborate, the probability that your prediction will be correct increases""If you only predict who will win the race and not the top 3, then the chance is greater that your prediction will be correct"	1
G	Effect of replacement on probability	Descriptive or on basis of calculations showing the effect (in the latter case, n and k need to be constant) Examples: – "If it is a matter of replacement, your chances will decrease" – "...if you have 10 different cell phones and you need to select one, your chance will be 1 out of 10, if you put the phone back your chance will be 1 out of 10 again, but if you leave it out your chance will increase that you will select the next phone as predicted"	A series of formulas or calculations showing the effect, but the outcome (p) needs to be represented as well and n and k need to be constant Examples: – "$(1/n) \times (1/n) = 1/n^2 (1/n) \times (1/(n-1)) = 1/(n^2 - n)$" – "$1/5 \times 1/4 \times 1/3 = 1/60$ $1/5 \times 1/5 \times 1/5 = 1/125$"	Descriptive or on basis of calculations showing the effect (in the latter case, n and k need to be constant) Examples: – "If it is a matter of replacement, your chances will decrease" – "...if you have 10 different cell phones and you need to select one, your chance will be 1 out of 10, if you put the phone back your chance will be 1 out of 10 again, but if you leave it out your chance will increase that you will select the next phone as predicted"	1
H	Effect of order on probability	Descriptive or on basis of calculations showing the effect (in the latter case, n and k need to be constant) Examples: – " If order is important, the chance your prediction will be right will decrease" – "...If there are 7 runners and you predict the top 3, then the probability is $1/7 \times 1/6 \times 1/5 = 1/210$, but without specifying the positions of specific runners in the top 3 the probability is $3/7 \times 2/6 \times 1/5 = 6/210$..."	A series of formulas or calculations showing the effect, but the outcome (p) needs to be represented as well and n and k need to be constant Examples: – "$(1/n) \times (1/n) = 1/n^2 (k/n) \times ((k-1)/n) = (k^2 - k)/n^2$" – "$1/5 \times 1/4 \times 1/3 = 1/60$ $3/5 \times 2/4 \times 1/3 = 6/60$"	Descriptive or on basis of calculations showing the effect (in the latter case, n and k need to be constant) Examples: – " If order is important, the chance your prediction will be right will decrease" – "...If there are 7 runners and you predict the top 3, then the probability is $1/7 \times 1/6 \times 1/5 = 1/210$, but without specifying the positions of specific runners in the top 3 the probability is $3/7 \times 2/6 \times 1/5 = 6/210$..."	1
Maximum number of points					8

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