

Proc. Eurosensors XXVI, September 9-12, 2012, Kraków, Poland

## Extensive Modeling of a Coaxial Stub Resonator for Online Fingerprinting of Fluids

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### Abstract

A straightforward method of extensive modeling of a lossy stub resonator system for online fingerprinting of fluids is presented in this paper. The proposed model solves the telegrapher's equations including the skin effect and dielectric losses and describes the amplitude versus frequency response of lossy coaxial stub resonators with a fluid under investigation as dielectric. The adequacy of the method is demonstrated by comparing simulations with experimentally obtained data. Even though we applied the model to a coaxial stub resonator for the online fingerprinting of fluids (e.g., for water quality monitoring), the potential applicability of the method reaches further. Indeed, the method introduced here may be useful for different types of sensors based on lossy transmission line theory.

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Keywords: Lossy Transmission Line, Coaxial Stub Resonator, Water Quality Monitoring.

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### 1. Introduction

Coaxial stub resonator systems can be simulated by the electrical equivalent of open-ended or closed circuits. Previously, we applied the lumped element model for the description of a quarter wave length

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open-ended coaxial stub resonator [1, 2]. This first model was shown to adequately predict the amplitude versus frequency plot near the base resonant frequency of the stub resonator but was limited to the first (basic) resonance frequency. The extensive model introduced here allows simulation of all resonance frequencies within a defined frequency range. Our method, based on transmission line theory, is a general solution of the telegrapher’s equations and takes into account both the skin effect and dielectric losses [3 pp.49-64, 4]

**2. Model of the lossy stub resonator system**

Fig. 1 gives a schematic overview of the coaxial stub resonator sensing system applied in this study for analyzing the dielectric properties of a fluid.

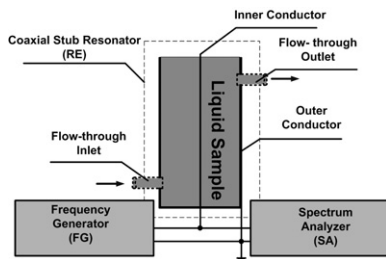


Fig. 1. Basic principle of the coaxial stub resonator sensing system consisting of a function generator (FG), a spectrum analyzer (SA) and the coaxial stub resonator (RE). The dotted structures indicate that the flow-through resonator can be optionally used as batch resonator by plugging inlet and outlet. The liquid sample under investigation is applied as dielectric between inner and outer conductor.

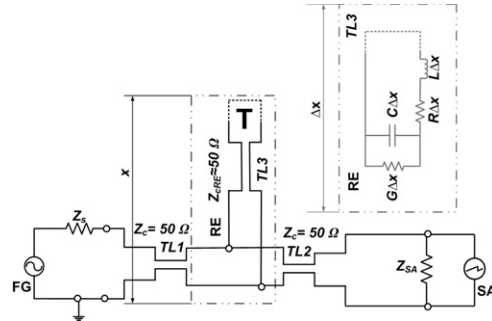


Fig. 2. The electrical equivalent circuit of the sensor system. Parameters  $L\Delta x$ ,  $C\Delta x$ ,  $G\Delta x$  and  $R\Delta x$  represent the resonator’s distributed element inductance, capacitance conductivity losses due to the skin effect in the inner and outer conductor, respectively, all with length  $x$ . Note, the dotted line on T stands for either an open-circuit or short-circuit, representing a  $\lambda/4$  or  $\lambda/2$  resonator, respectively.

Fig. 2 shows an electrical equivalent circuit of the experimental set-up in fig. 1 comprising the frequency generator (FG) with internal resistance  $Z_s$ , transmission line TL1, connecting the function generator with the coaxial stub resonator, and transmission line TL2, connecting the coaxial stub resonator to spectrum analyzer (SA) with internal resistance  $Z_{SA}$ . The coaxial stub resonator is described by a distributed elements model.

Based on the equivalent electric circuit of the sensor system in fig. 2, we will now derive a straightforward model for predicting the amplitude versus frequency plot (A-f plot) of a coaxial sensing system. Close inspection of fig. 2 reveals that the input impedance  $Z_{in}$  of the stub resonator is in parallel with the internal resistance of the spectrum analyzer  $Z_{SA}$  and in series with the internal resistance of the function generator  $Z_s$ .

From the electrical equivalent circuit in fig. 2, the following relation between the voltage supplied by the function generator  $V_{in}$  and the voltage recorded by the spectrum analyzer  $V_{out}$  can be derived (1):

$$V_{out} = V_{in} (Z_s \cdot Z_{SA} + Z_s \cdot Z_{in} + Z_{in} \cdot Z_{SA}) / (Z_{in} \cdot Z_{SA}) \tag{1}$$

For the open-ended coaxial stub resonator i.e., a transmission line of length  $x$ , the input impedance  $Z_{in}$  is given by equation (2) [3]:

$$Z_{in}=Z_c \cdot \tanh(\gamma x) \quad (2)$$

The complex characteristic impedance  $Z_c$  of the lossy transmission line and the complex propagation constant  $\gamma$  can be calculated using general expressions (3) and (4) [3]:

$$Z_c=((R+j\omega L)/(G j\omega C))^{1/2} \quad (3)$$

$$\gamma=((R+j\omega L) \cdot (G j\omega C))^{1/2}=\alpha+j\beta \quad (4)$$

In these equations  $R$  is the resistance per unit length, [ $\Omega/m$ ];  $L$  is the inductance per unit length, [ $H/m$ ];  $G$  is the conductance of the dielectric per unit length, [ $S/m$ ];  $C$  is the capacitance per unit length, [ $F/m$ ];  $\omega$  is the angular frequency,  $\omega=2\pi f$ , [ $rad/s$ ];  $\beta$  is the phase of the propagation constant  $\gamma$ , [ $rad/m$ ] and  $\alpha$  is the attenuation of propagation constant  $\gamma$ , [ $Np/m$ ].

The attenuation of the propagation constant  $\gamma$  i.e.,  $Re(\gamma)=\alpha$  represents all losses in the stub resonator comprising metal resistive losses (in inner and outer conductors), dielectric losses and radiation losses. In coaxial transmission lines, radiation losses are negligible.

### 3. Experimental

All experiments were performed with a HAMEG HMS3010 3 GHz Spectrum Analyzer with Tracking Generator, both with an internal resistance  $Z_s$  of 50  $\Omega$ .

Table 1 gives an overview of the dimensions of the flow-through coaxial stub resonator and the batch resonator applied in this study respectively; see also Fig.1.

Table 1. Geometric parameters of the coaxial batch and flow-through resonators. The outer and the inner conductors were made from copper for both resonators.

Parameters	Flow-through Resonator	Batch resonator
Length, $x$	101.0 [cm]	34.0 [cm]
Inner conductor diameter, $d$	0.5 [mm]	0.5 [mm]
Inner diameter of the outer conductor, $D$	22.0 [mm]	22.0 [mm]
Diameters of the fluid inlet and outlet	7.0 [mm]	
Conductivity of copper, $\sigma$	5.7·10 <sup>7</sup> [ $S \cdot m^{-1}$ ]	

The main algorithm for the model simulations was written in MATLAB code (MATLAB 2007b).

### 4. Results and Discussion

Fig. 3 shows A-f plots for ethanol as determined with the experimental set-up shown in fig.1 while using either the batch resonators or the flow-through resonators of table 1 (red curves). The blue curves show model simulations performed with literature values of  $\epsilon_r$ , the resistance per unit length  $R$  and values of  $L$  and  $C$  calculated from the stub resonator geometry respectively. Subsequently, equations (1) to (4) were solved with the loss tangent ( $\tan \delta$ ), which is related to  $G$ , as only unknown parameter. By minimizing the difference between the measured A-f plot and the model simulation, the value of  $\tan \delta$ , which is frequency dependent, was determined. Table 2 summarizes some major modeling results.

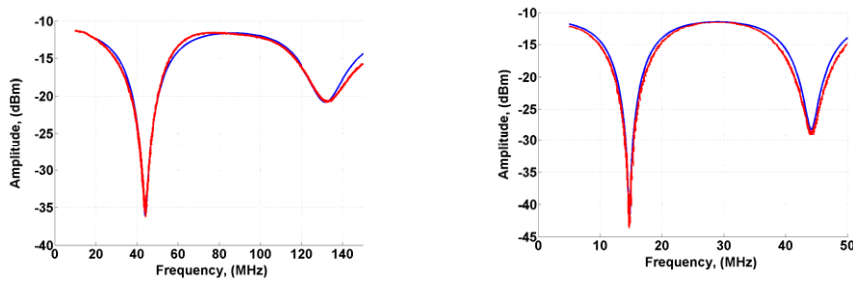


Fig. 3. Experimentally obtained amplitude versus frequency plot of ethanol (red) using a quarter wave length open-ended batch (left) or flow-through (right) resonator, respectively. Simulation data are shown for comparison (blue).

Table 2. Modeling results for the coaxial resonator stubs in Table 1, filled with ethanol as dielectric respectively.

Resonator	Resonance frequency [MHz]	$\epsilon$ [-], experimental	$R$ , [ $\Omega/m$ ]	$\tan \delta$ [-]
Batch	1st 44.1	$25.0 \pm 0.3$	1.0	0.040
	2nd 130	$25.9 \pm 0.6$	1.8	0.110
Flow-through	1st 14.8	$25.2 \pm 0.5$	0.6	0.010
	2nd 44.1	$25.5 \pm 0.8$	1.0	0.035

Literature  $\epsilon$  values of ethanol are 25.13 and 24.35 [5].

## 5. Conclusions

The extensive model proposed here can be used to assess the dielectric characteristics of fluids, using a coaxial stub resonator in batch or flow-through mode. The result of this characterization can be used, e.g., to monitor water quality.

## Acknowledgements

This work was performed in the TTIW-cooperation framework of Wetsus, Centre of Excellence for Sustainable Water Technology ([www.wetsus.nl](http://www.wetsus.nl)). Wetsus is funded by the Dutch Ministry of Economic Affairs. The authors thank the participants of the research theme Sensoring for the fruitful discussions and their financial support.

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