

Meissner effect in diffusive normal metal/*d*-wave superconductor junctions

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Abstract

The Meissner effect in diffusive normal metal/insulator/*d*-wave superconductor junctions is studied theoretically in the framework of the Usadel equation under the generalized boundary condition. The effect of midgap Andreev resonant states (MARS) formed at the interface of *d*-wave superconductor is taken into account. It is shown that the formation of MARS suppresses the susceptibility of the diffusive normal metal.

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1. Introduction

In diffusive normal metal/superconductor (DN/S) junctions, the DN acquires induced superconductivity, i.e. Cooper pairs penetrate into the DN. This proximity effect has been studied since the BCS theory was established. The proximity induced Meissner demagnetization in DN/S junctions was measured experimentally by Oda and Nagano [1] and Mota et al. [2]. It has $T^{-1/2}$ dependence in the dirty limit. The quasiclassical Green's function theory was used earlier to study the Meissner effect in proximity structures.

The quasiclassical Green's function theory was developed by Eilenberger [3] and was generalized by Eliashberg [4], Larkin and Ovchinnikov [5] in order to study the nonequilibrium state. This theory was applied by Zaikin [6] and Kieselmann [7] to studying the Meissner effect in DN/S junctions. Narikiyo and Fukuyama [8] calculated the Meissner screening length in a semi-infinite system containing an Anderson impurity. Higashitani and Nagai studied the Meissner effect in the clean limit [9]. Belzig et al. [10,11] have considered more realistic systems by assuming a perfectly transparent N/S interface. Up to now the boundary

conditions derived by Kupriyanov and Lukichev (KL) [12] were widely used to study proximity effect in DN/S structures.

A more general boundary conditions was derived by Nazarov [13] based on the Keldysh-Nambu Green's function formalism [14] within the framework of the Landauer-Büttiker scattering formalism. The merit of this boundary condition is that the BTK theory [15] is reproduced in the ballistic limit while in the diffusive limit with a low transmissivity of the interface, the KL boundary condition is reproduced. Although almost all previous papers on Meissner effect in mesoscopic NS junctions are either based on the KL boundary conditions or on the BTK model, in the actual junctions, the transparency of the junction is not always small and the impurity scattering in the DN cannot be neglected. Tanaka et al. [16] and Yokoyama et al. [17] calculated tunneling conductance by using the Nazarov's boundary condition.

It is well known in *d*-wave superconductors that the midgap Andreev resonant states (MARS) are formed at the interface of *d*-wave superconductor. The MARS crucially influence various physical quantities [18]. One of the authors (Y.T.) recently generalized the boundary condition of the Keldysh-Nambu Green's function formalism to unconventional superconductor junctions [19,20]. It is revealed that in DN/*d*-wave superconductor junctions the proximity effect and the MARS strongly compete with each other [19], while they coexist in DN/triplet superconductor junctions. The newly

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obtained boundary conditions expressed in the Keldysh-Nambu Green's function are useful for the calculation of various physical quantities. The timely problem is to study theoretically the Meissner effect in DN/*d*-wave S junctions using the new boundary conditions [19]. In the present paper, we calculate the susceptibility of the DN layer in DN/*d*-wave S junctions for various junction parameters such as the height of the insulating barrier at the interface and the angle between the normal to the interface and the crystal axis of a *d*-wave superconductor.

The organization of the paper is as follows. In Section 2, we will provide the derivation of the expression for the susceptibility of the DN. In Section 3, the results of calculation are presented for various types of junction. In Section 4, the summary of the obtained results is given. In the present paper we set $c = k_B = \hbar = 1$.

2. Formulation

In this section, we introduce the model and the formalism. We consider a junction consisting of vacuum (VAC) and superconducting reservoirs connected by a quasi-one-dimensional diffusive conductor (DN) with a length L much larger than the mean free path. We assume that the interface between the DN conductor and the S electrode at $x=L$ has a resistance R_b , the DN/VAC interface at $x=0$ is specular, and we apply the generalized boundary conditions by Tanaka [19] to treat the interface between DN and S. A weak external magnetic field H is applied in z -direction (see Fig. 1). The vector potential can be chosen to have only the y component which depends on x .

We describe the insulating barrier between DN and S by using the δ -function (i.e., $U(x) = H\delta(x-L)$), which provides the transparency of the junction $T_m = 4 \cos^2 \phi / (4 \cos^2 \phi + Z^2)$, where $Z = 2H/v_F$ is a dimensionless constant, ϕ is the injection angle of a quasiparticle measured from the interface normal to the junction and v_F is Fermi velocity.

In the following, we solve the Usadel equations [21] with using the standard θ -parameterization. The parameter $\theta(x)$ is a measure of the proximity effect in DN and obey the

following equation

$$D \frac{\partial^2}{\partial x^2} \theta(x) - 2\omega_n \sin[\theta(x)] = 0, \quad (1)$$

where D and ω_n denote the diffusion constant and the Matsubara frequency, respectively. The boundary condition for $\theta(x)$ at the DN/S interface is given in Ref. [19]. The interface resistance R_b is given by

$$R_b = R_0 \frac{2}{\int_{-\pi/2}^{\pi/2} d\phi T(\phi) \cos \phi} \quad (2)$$

with $T(\phi) = 4 \cos^2 \phi / (4 \cos^2 \phi + Z^2)$. Here R_0 is Sharvin resistance $R_0^{-1} = e^2 k_F^2 S_c / (4\pi^2)$, where k_F is the Fermi wave number and S_c is the constriction area. The current distribution is given by

$$j(x) = -8\pi e^2 N(0) D T \sum_{\omega_n > 0} \sin^2 \theta(x) A(x), \quad (3)$$

where $A(x)$, $N(0)$ and T denote the vector potential, the density of states at the Fermi energy and the temperature of the system respectively. The Maxwell equation reads

$$\frac{d^2}{dx^2} A(x) = -4\pi j(x). \quad (4)$$

The boundary conditions for $A(x)$ are given by

$$\frac{d}{dx} A(0) = H, \quad A(L) = 0, \quad (5)$$

where we have neglected the penetration of magnetic fields into the superconductor by assuming a small penetration depth in S.

Finally we obtain the expression of the susceptibility,

$$-4\pi\chi = 1 + \frac{A(0)}{HL}. \quad (6)$$

The *d*-wave pair potentials in directional space are given by $\Delta_{\pm} = \Delta(T) \cos 2(\phi \mp \alpha)$, where $\Delta(T)$ is the magnitude of pair potential at a given temperature T and α denotes the angle between the normal to the interface and the crystal axis of a *d*-wave superconductor.

3. Results

In the following, we focus on the magnitude of the diamagnetic susceptibility χ induced by the proximity effect. Figs. 2 and 3 show the susceptibility for $Z=10$ and $Z=0$ respectively where $K = 16\pi e^2 N(0) D^2$. For $\alpha=0$, the temperature dependencies of $-4\pi\chi$ are not much different. For $\alpha=0.125\pi$, the magnitude of χ for $Z=10$ is much stronger suppressed than that for $Z=0$. At the same time, we find that the magnitude of χ decreases with increasing α . We note in the case of $\alpha=0.25\pi$ that the susceptibility completely vanishes, (i.e., $-4\pi\chi=0$). This is because the proximity effect is absent in diffusive metals due to angular averaging [19]. The absence of the proximity effect is

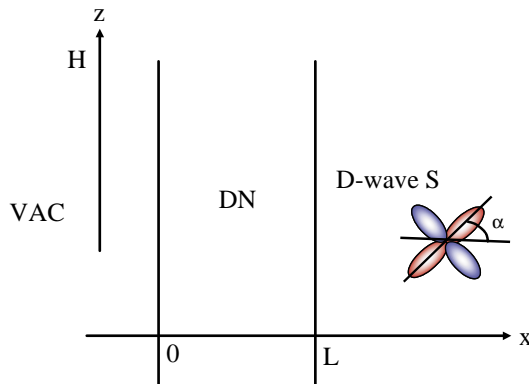


Fig. 1. Schematic illustration of the model.

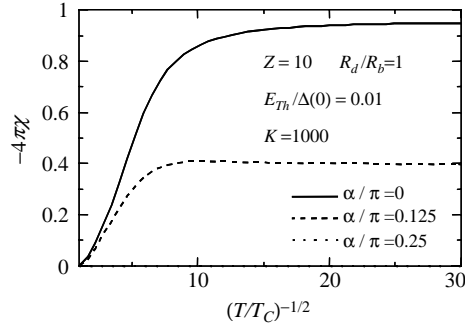


Fig. 2. Susceptibility for low transparent junctions with $Z=10$.

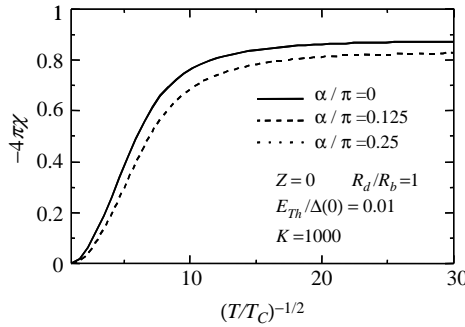


Fig. 3. Susceptibility for high transparent junctions with $Z=0$.

a significant feature specific for junctions containing unconventional superconductors.

We also plot the α dependencies of the susceptibility at $T/T_C=0.01$ and $T/T_C=0.1$ in Fig. 4. For all cases, χ is a decreasing function of α . At $T/T_C=0.01$, the magnitude of χ for $Z=10$ rapidly decreases with the increase of α . The results imply that the MARS suppresses the proximity effect in low transparent junctions and low temperatures.

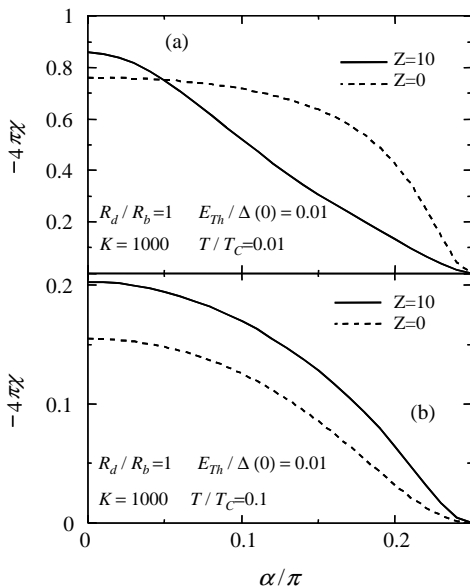


Fig. 4. α dependences of the susceptibility at $T/T_C=0.01$ (upper panel) and $T/T_C=0.1$ (lower panel).

4. Conclusions

In the present paper, we have calculated the induced Meissner effect by the proximity effect in DN region of DN/ d -wave superconductor junctions. We have solved the Usadel equation under a general boundary condition [19] in which the formation of the MARS is fully taken into account [18]. The magnitude of χ decreases with the increase of α up to 0.25π . At $\alpha=0.25\pi$, where all quasiparticles feel MARS, the χ becomes zero. It might be interesting to check experimentally such an anomalous proximity effect in DN. Another future problem is a similar calculation of the induced Meissner effect with a p -wave triplet superconductor instead of a d -wave one, since dramatic new phenomena are recently predicted in DN/triplet junctions [20].

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References

- [1] Y. Oda, H. Nagano, Solid State Commun. 35 (1980) 631.
- [2] A.C. Mota, D. Marek, J.C. Weber, Helv. Phys. Acta 55 (1982) 647.
- [3] G. Eilenberger, Z. Phys. 214 (1968) 195.
- [4] G.M. Eliashberg, Sov. Phys. JETP 34 (1971) 668.
- [5] A.I. Larkin, Yu.V. Ovchinnikov, Sov. Phys. JETP 41 (1975) 960.
- [6] A.D. Zaikin, Solid State Commun. 41 (1982) 533.
- [7] G. Kieselmann, Phys. Rev. B 35 (1987) 6762.
- [8] O. Narikiyo, H. Fukuyama, J. Phys. Soc. Jpn. 58 (1989) 4557.
- [9] S. Higashitani, K. Nagai, J. Phys. Soc. Jpn. 64 (1995) 549.
- [10] W. Belzig, C. Bruder, G. Schön, Phys. Rev. B 53 (1996) 5727.
- [11] W. Belzig, C. Bruder, A.L. Fauchère, Phys. Rev. B 58 (1998) 14531.
- [12] M.Yu. Kupriyanov, V.F. Lukichev, Zh. Exp. Teor. Fiz. 94 (1988) 139 [Sov. Phys. JETP 67 (1988) 1163].
- [13] Yu.V. Nazarov, Superlattices Microstruct. 25 (1999) 1221 [condmat/9811155].
- [14] A.V. Zaitsev, Sov. Phys. JETP 59 (1984) 1163.
- [15] G.E. Blonder, M. Tinkham, T.M. Klapwijk, Phys. Rev. B 25 (1982) 4515.
- [16] Y. Tanaka, A.A. Golubov, S. Kashiwaya, Phys. Rev. B 68 (2003) 054513.
- [17] T. Yokoyama, Y. Tanaka, A.A. Golubov, J. Inoue, Y. Asano, Phys. Rev. B 71 (2005) 094506.
- [18] Y. Tanaka, S. Kashiwaya, Phys. Rev. Lett. 74 (1995) 3451; Y. Tanaka, S. Kashiwaya, Phys. Rev. B 56 (1997) 892; S. Kashiwaya, Y. Tanaka, Rep. Prog. Phys. 63 (2000) 1641; Yu.S. Barash, M.S. Kalenkov, J. Kurkijärvi, Phys. Rev. B 62 (2000) 6665.
- [19] Y. Tanaka, Yu. V. Nazarov, S. Kashiwaya, Phys. Rev. Lett. 90 (2003) 167003; Y. Tanaka, Yu. V. Nazarov, A.A. Golubov, S. Kashiwaya, Phys. Rev. B 69 (2004) 144519.
- [20] Y. Tanaka, S. Kashiwaya, Phys. Rev. B 70 (2004) 012507.
- [21] K.D. Usadel, Phys. Rev. Lett. 25 (1970) 507.