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Josephson supercurrent through a topological insulator surface state

M. Veldhorst, M. Snelder, M. Hoek, T. Gang, V. K. Guduru, X. L.Wang, U. Zeitler, W. G. van der Wiel, A. A. Golubov, H. Hilgenkamp and A. Brinkman

[1]*MESA+ Institute for Nanotechnology,*
University of Twente, 7500 AE Enschede, The Netherlands
[2]*High Field Magnet Laboratory, Institute for Molecules and Materials,*
Radboud University Nijmegen, 6525 ED Nijmegen, The Netherlands
[3]*Institute for Superconducting and Electronic Materials,*
University of Wollongong, Wollongong, NSW, 2522, Australia

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THEORETICAL MODELS FOR JOSEPHSON CURRENT

Any hybrid structure containing superconductors can be described on the basis of the Gor'kov equations [1]. In practice, these equations are typically simplified by a quasi-classical approximation, which is justified as long as the Fermi-wavelength is much smaller than other length scales in the problem. For superconductor - normal metal - superconductor (SNS) Josephson junctions Eilenberger quasi-classical equations [2] are used when the elastic mean free path l_e is larger than the length L and the coherence length ξ. The electronic transport in this clean limit is ballistic across the N layer. In the dirty limit of $l_e \ll L, \xi$, transport is diffusive and the Usadel equations [3] are used. When the transparency between the S and N layers is not unity, additional insulating barriers (I) are typically included.

Eilenberger theory fit

The clean limit theory on the basis of the Gor'kov equations for short SINIS junctions with arbitrary barrier transparency D [4] was generalized [5, 6] for arbitrary junction length on the basis of Eilenberger equations. The supercurrent density J is found to be [6]

$$J = \frac{2}{\pi} e k_F^2 k_B T \sin\chi \sum_{\omega_n > 0} \int_0^1 \mu d\mu \frac{t_1(\mu) t_2(\mu)}{Q^{1/2}(\chi, \mu)}, \tag{1}$$

where $\mu = k_x / k_F$, $t_{1,2} = D_{1,2}/(2 - D_{1,2})$, and

$$Q = \left[t_1 t_2 \cos\chi + \left(1 + (t_1 t_2 + 1)\frac{\omega_n^2}{\Delta^2}\right)\cosh\frac{2\omega_n L}{\mu\hbar v_F} + (t_1 + t_2)\frac{\omega_n \Omega_n}{\Delta^2}\sinh\frac{2\omega_n L}{\mu\hbar v_F}\right]^2 - \left(1 - t_1^2\right)\left(1 - t_2^2\right)\frac{\Omega_n^4}{\Delta^4}, \tag{2}$$

where the Matsubara frequency is given by $\omega_n = 2\pi k_B T(2n+1)$, and $\Omega_n = \sqrt{\omega_n^2 + \Delta^2}$. Δ is the gap in the S electrodes, χ the phase difference across the junction, while v_F is the Fermi velocity of the normal metal interlayer. The integral runs over all trajectory directions and can be adjusted to actual junction geometries.

Eq. (1) was evaluated as function of junction length and fitted to the measured critical current density. Since the prefactors in Eq. (1) implicitly contain the normal state resistance, which is not known for our junctions due to the bulk shunt, we left the overall scale of J free in the fit. Subsequently, the best fit to the data at 1.6 K was obtained for $\xi = \frac{\hbar v_F}{2\pi k_B T} = 75$ nm. It was found numerically that the value of the barrier transparencies in the symmetric case had no influence on the fitting value for ξ.

The temperature dependence of the critical current was calculated using Eq. (1) and the obtained coherence length. The fit to the measured data is excellent, considering that only the overall scale of J was free in this case.

In fact, the overall scaling factor of the critical current in Eq. (1) can be estimated as well. The transparency of the interfaces between the topological insulator and the superconductor are important in this respect. The high transparency of our interfaces can be determined from the $I(V)$ characteristic. The excess current in the $I(V)$ characteristic is about 67$\%$ of the critical current I_c. In the Blonder-Tinkham-Klapwijk model [7], this gives a barrier strength of about $Z = 0.6$. For these high transparencies, at the lowest temperatures and for the 50 nm junction, Eq. (1) provides an I_cR_N product of the order of 1-2 mV. However the 3D bulk shunt will strongly reduce this value by decreasing R_n. From the Shubnikov de Haas oscillations we can estimate the surface to bulk resistance ratio. The surface resistance can be found through $R_{SDH} = R_cR_tR_d$. With R_tR_D = 0.77 Ω, we estimate a surface resistance of about 2$\%$ of the total resistance. Thus, the estimated surface resistance is approximately 29 Ω, which together with I_c = 32 μA results in I_cR_N = 1 mV, agreeing with the Eilenberger model. This agreement between model and measurements underlines the conclusion that supercurrent is flowing through the ballistic channels of the topological surfaces states, shunted by a normal state bulk conduction. Small quantitative differences in prefactors can be expected when including the TI in the junction model, such as a calculated prefactor of 2 in Ref. [8].

Usadel theory fit

The Usadel equation [3] for the S and N layers in a diffusive SNS junction can be written as

$$\Phi_{S,N} = \Delta_{S,N} + \xi_{S,N}^2 \frac{\pi k_B T_c}{\omega_n G_{S,N}} \frac{d}{dx} \left(G_{S,N}^2 \frac{d}{dx} \Phi_{S,N} \right), \tag{3}$$

where Φ is defined in terms of the normal Green's function G and the anomalous Green's function F by $\Phi G = \omega_n F$. The normalization condition $FF^* + G^2 = 1$ then gives

$$G_{S,N} = \frac{\omega_n}{\sqrt{\omega_n^2 + \Phi_{S,N}\Phi_{S,N}^*}} \tag{4}$$

The coherence length is given by $\xi = \sqrt{\frac{\hbar D}{2\pi k_B T}}$ where $D = v_F l_e/3$ is the diffusion constant.

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The pair potentials $\Delta_{S,N}$ are given by

$$\Delta_{S,N} \ln \frac{T}{T_{cS,N}} + 2\pi k_B T \sum_{\omega_n > 0} \frac{\Delta_{S,N} - \Phi_{S,N} G_{S,N}}{\omega_n} = 0 \tag{5}$$

In the dirty limit, Zaitsev's effective boundary conditions for quasi-classical Green's functions [9] were simplified by Kupriyanov and Lukichev [10]. When Φ and G are found using these boundary conditions, finally the supercurrent density can be obtained from

$$J = \frac{2\pi k_B T}{e\rho_N} \mathrm{Im} \sum_{\omega_n > 0} \frac{G_N^2}{\omega_n^2} \Phi_N \frac{d}{dx} \Phi_N, \tag{6}$$

where ρ_N is the N layer resistivity.

For junctions with arbitrary length and arbitrary barrier transparency, no analytical expressions exist for the Green's functions. Therefore a numerical code was used to fit the data. In the effective boundary conditions [10], two parameters play a role, $\gamma = \frac{\rho_S \xi_S}{\rho_N \xi_N}$ and $\gamma_B = \frac{2l_e}{3\xi_N} \left\langle \frac{1-D}{D} \right\rangle$, where the average of the transparencies takes place over all trajectory angles. For Nb as S electrode and Bi_2Te_3 as N interlayer, $\gamma \ll 1$ because of the lower resistivity ρ of Nb as compared to Bi_2Te_3. The junctions transparency is not known a priori, but from the voltage drop over a barrier in the normal state, as well as the large amount of excess current (more than 50% of the critical current) in the current-voltage characteristics of the superconducting state, a conservative estimate gives $D \gtrsim 0.5$, which implies $\gamma_B \lesssim 1$. Within this parameter range (or even outside the range) no consistent fit could be made to the data. Figure 6a shows the fit to the temperature dependence of the critical current for $\gamma = 0.1$, $\gamma_B = 1$ and $\xi(T_c) = \sqrt{\frac{\hbar D}{2\pi k_B T_c}} = 21$ nm, the latter value as obtained from fitting the length dependence of the junction.

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* Also at Leiden Institute of Physics, Leiden University, P.O. Box 9506, 2300 RA Leiden, The Netherlands

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