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2007 Supercond. Sci. Technol. 20 S6

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Theory of macroscopic quantum tunnelling and dissipation in high- T_c Josephson junctions

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Received 4 July 2006, in final form 1 August 2006

Published 4 December 2006

Online at stacks.iop.org/SUST/20/S6

Abstract

We have investigated macroscopic quantum tunnelling (MQT) in in-plane high- T_c superconductor Josephson junctions and the influence of the nodal-quasiparticle and zero energy bound states (ZES) on MQT. We have shown that the presence of ZES at the interface between the insulator and the superconductor leads to strong Ohmic quasiparticle dissipation. Therefore, the MQT rate is noticeably suppressed in comparison with c -axis junctions in which ZES are completely absent.

1. Introduction

A mesoscopic single Josephson junction is an interesting physical object for testing quantum mechanics at a macroscopic level. In current-biased Josephson junctions, measurements of macroscopic quantum tunnelling (MQT) are performed by switching the junction from its metastable zero-voltage state to a non-zero voltage state (see figure 1(d)). Until now, experimental investigations of MQT have been focused on s -wave (low- T_c) junctions only. This is due to a naive preconception that the existence of low energy quasiparticles in the d -wave order parameter of a high- T_c cuprate superconductor [1] may preclude the possibility of observing MQT.

Recently we have theoretically investigated the effect of the nodal-quasiparticle on MQT in d -wave c -axis junctions (e.g. Bi2212 intrinsic Josephson junctions [12, 13] and cross whisker junctions [14]) [2, 3]. We have shown that the effect of the nodal-quasiparticle gives rise to super-Ohmic dissipation [4, 5] and the suppression of MQT due to the nodal-quasiparticle is very weak.

The first experimental observation of MQT in high- T_c Josephson junction was made by Bauch *et al.*, using a YBCO

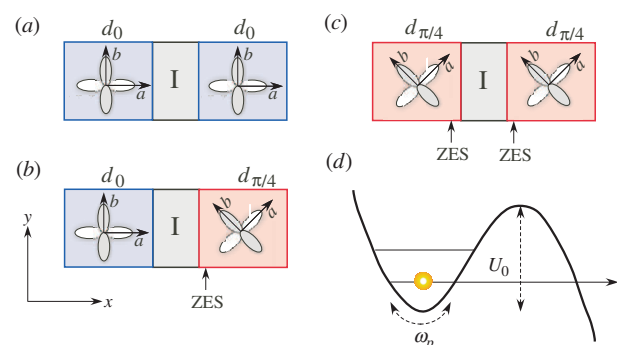


Figure 1. Schematic of an in-plane d -wave Josephson junction: (a) d_0/d_0 , (b) $d_0/d_{\pi/4}$ and (c) $d_{\pi/4}/d_{\pi/4}$. In the case of $d_0/d_{\pi/4}$ and $d_{\pi/4}/d_{\pi/4}$ junctions, zero energy bound states (ZES) are formed near the boundary between superconductor $d_{\pi/4}$ and insulating barrier I. (d) Potential $U(\phi)$ versus the phase difference ϕ between two superconductors. U_0 is the barrier height and ω_p is the Josephson plasma frequency.

(This figure is in colour only in the electronic version)

grain boundary bi-epitaxial junction [6, 7]. Recently, Inomata *et al* [8], Jin *et al* [9] and Kashiwaya *et al* [10, 11] have

experimentally observed MQT in c -axis (Bi2212 intrinsic) junctions. They reported that the effect of the nodal-quasiparticle on MQT is negligibly small and the thermal-to-quantum crossover temperature is relatively high (0.5–1 K) compared with the case of low- T_c and YBCO bi-epitaxial junctions. In Jin *et al*'s experiment, $O(N^2)$ (N being the number of the stacked junctions) enhancement of the MQT rate was reported. This enhancement is attributed to collective motion of the phase differences in the intrinsic junctions [15–17].

In this paper we will theoretically investigate MQT in d-wave in-plane junctions parallel to the ab -plane (see figure 1) [18]. In such junctions, ZES [19] are formed near the interface between the superconductor and the insulating barrier. ZES are generated by the combined effect of multiple Andreev reflections and the sign change of the d-wave order parameter symmetry, and are bound states for the quasiparticle at the Fermi energy. Below, we will show that ZES give rise to Ohmic-type strong dissipation so MQT is considerably suppressed compared with the c -axis and the d_0/d_0 junction cases.

2. Effective action

By using the method developed by Eckern *et al* [20] the partition function of the system can be described by a functional integral over the macroscopic variable (the phase difference ϕ):

$$Z = \int \mathcal{D}\phi(\tau) \exp\left(-\frac{S_{\text{eff}}[\phi]}{\hbar}\right). \quad (1)$$

In the high barrier limit, i.e. $z_0 \equiv mw_0/\hbar^2k_F \gg 1$ (m is the mass of the electron, w_0 is the height of the delta function potential I and k_F is the Fermi wavelength), the effective action S_{eff} is given by

$$S_{\text{eff}}[\phi] = \int_0^{\hbar\beta} d\tau \left[\frac{M}{2} \left(\frac{\partial\phi(\tau)}{\partial\tau} \right)^2 + U(\phi) \right] + S_\alpha[\Phi],$$

$$S_\alpha[\Phi] = - \int_0^{\hbar\beta} d\tau \int_0^{\hbar\beta} d\tau' \alpha(\tau - \tau') \cos \frac{\phi(\tau) - \phi(\tau')}{2}. \quad (2)$$

In this equation $\beta = 1/k_B T$, $M = C(\hbar/2e)^2$ is the mass (C is the capacitance of the junction) and the potential $U(\phi)$ is described by

$$U(\phi) = \frac{\hbar}{2e} \left[\int_0^1 d\lambda \phi I_J(\lambda\phi) - \phi I_{\text{ext}} \right], \quad (3)$$

where I_J is the Josephson current and I_{ext} is the external bias current. The dissipation kernel $\alpha(\tau)$ is related to the quasiparticle current I_{qp} under constant bias voltage V by

$$\alpha(\tau) = \frac{\hbar}{e} \int_0^\infty \frac{d\omega}{2\pi} e^{-\omega\tau} I_{\text{qp}} \left(V = \frac{\hbar\omega}{e} \right), \quad (4)$$

at zero temperature.

Below, we will derive the effective action for the three types of d-wave junction (d_0/d_0 , $d_0/d_{\pi/4}$ and $d_{\pi/4}/d_{\pi/4}$) in order to investigate the effect of the nodal-quasiparticles and

ZES on MQT. In the case of the d_0/d_0 junction, node-to-node quasiparticle tunnelling can contribute to the dissipative quasiparticle current. However, ZES are completely absent. These behaviours are qualitatively identical with that for c -axis Josephson junctions [2, 3]. On the other hand, in the case of $d_0/d_{\pi/4}$ and $d_{\pi/4}/d_{\pi/4}$ junctions, ZES are formed around the surface of the superconductor $d_{\pi/4}$. Therefore node-to-ZES ($d_0/d_{\pi/4}$) and ZES-to-ZES ($d_{\pi/4}/d_{\pi/4}$) quasiparticle tunnelling becomes possible.

First, we will calculate the potential energy U in the effective action (2). As mentioned above, U can be described by the Josephson current through the junction in the high barrier limit. In order to obtain the Josephson current we start from the Bogoliubov–de Gennes (BdG) equation [19]:

$$\int d\mathbf{r}' \begin{pmatrix} \delta(\mathbf{r} - \mathbf{r}')h(\mathbf{r}') & \Delta(\mathbf{r} - \mathbf{r}')e^{i\varphi} \\ \Delta^*(\mathbf{r} - \mathbf{r}')e^{-i\varphi} & -\delta(\mathbf{r} - \mathbf{r}')h^*(\mathbf{r}') \end{pmatrix} \begin{pmatrix} u(\mathbf{r}) \\ v(\mathbf{r}) \end{pmatrix} = E \begin{pmatrix} u(\mathbf{r}) \\ v(\mathbf{r}) \end{pmatrix}, \quad (5)$$

where φ is the phase of the order parameter, $u(v)$ is the amplitude of the wavefunction for the electron (hole)-like quasiparticle, $h(\mathbf{r}) = -\hbar^2\nabla^2/2m - \mu + w_0\delta(x)$, and $\Delta(\mathbf{r} - \mathbf{r}') = \Omega^{-1} \sum_k \Delta_k \exp[i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')] is the order parameter (Ω is the volume of the superconductor). In the superconductor regions (d_0 and $d_{\pi/4}$), the BdG equation (5) can be transformed into the eigenequation$

$$\begin{pmatrix} \xi_k & \Delta_k e^{i\varphi} \\ \Delta_k e^{-i\varphi} & -\xi_k \end{pmatrix} \begin{pmatrix} u_k \\ v_k \end{pmatrix} = E \begin{pmatrix} u_k \\ v_k \end{pmatrix}, \quad (6)$$

where $\xi_k = \hbar^2k^2/2m + \hbar^2p^2/2m - \mu$ ($p = 2\pi n/D$ and D is the width of the junction). The amplitude of the order parameter Δ_k is given by $\Delta_0 \cos 2\theta \equiv \Delta_{d_0}(\theta)$ for d_0 and $\Delta_0 \sin 2\theta \equiv \Delta_{d_{\pi/4}}(\theta)$ for $d_{\pi/4}$, where $\cos \theta = k/k_F$. The Andreev reflection coefficient for the electron (hole)-like quasiparticle r_{he} (r_{eh}) is calculated by solving the eigenequation (6) together with the appropriate boundary conditions. By substituting r_{he} (r_{eh}) into the formula of the Josephson current for unconventional superconductors (the Tanaka–Kashiwaya formula) [19]

$$I_J = \frac{e}{\hbar} \sum_p \frac{1}{\beta} \sum_{\omega_n} \left(\frac{\Delta_+}{\Omega_+} r_{\text{he}} - \frac{\Delta_-}{\Omega_-} r_{\text{eh}} \right), \quad (7)$$

we can obtain ϕ dependence of the Josephson current. Here $\Delta_\pm = \Delta_{(\pm k, p)}$, $\Omega_\pm = \sqrt{(\hbar\omega_n)^2 - |\Delta_\pm|^2}$, $\omega_n = (2n+1)\pi/\beta\hbar$ is the fermionic Matsubara frequency. In the case of low temperatures ($\beta^{-1} \ll \Delta_0$) and the high barrier limit ($z_0 \gg 1$), we get

$$I_J(\phi) \approx \begin{cases} I_1 \sin \phi & \text{for } d_0/d_0 \\ -I_2 \sin 2\phi & \text{for } d_0/d_{\pi/4} \\ I_3 \sin \frac{\phi}{2} & \text{for } d_{\pi/4}/d_{\pi/4} \end{cases} \quad (8)$$

where $I_1 \equiv 3\pi\Delta_0/10eR_N$, $I_2 \equiv \pi^2\hbar\beta\Delta_0^2/35e^3N_cR_N^2$ and $I_3 \equiv 3\pi z_0\Delta_0/4eR_N$ ($R_N = 3\pi\hbar z_0^2/2e^2N_c$ is the normal state resistance of the junction and N_c is the number of channels at the Fermi energy).

By substituting the Josephson current into equation (3), we can obtain the analytical expression of the potential U , i.e.

$$U(\phi) \approx \begin{cases} -\frac{\hbar I_1}{2e} \left(\cos \phi + \frac{I_{\text{ext}}}{I_1} \phi \right) & \text{for } d_0/d_0 \\ -\frac{\hbar I_2}{4e} \left(-\cos 2\phi + 2\frac{I_{\text{ext}}}{I_2} \phi \right) & \text{for } d_0/d_{\pi/4} \\ -\frac{\hbar I_3}{e} \left(\cos \frac{\phi}{2} + \frac{1}{2} \frac{I_{\text{ext}}}{I_3} \phi \right) & \text{for } d_{\pi/4}/d_{\pi/4}. \end{cases} \quad (9)$$

As in the case of the s -wave and the c -axis junctions [2], U can be expressed as a tilted washboard potential (see figure 1(d)).

3. Dissipation due to nodal-quasiparticles and ZES

Next we will calculate the dissipation kernel $\alpha(\tau)$ in the effective action (2). In the high barrier limit, the quasiparticle current is given by [19]

$$I_{\text{qp}}(V) = \frac{2e}{h} \sum_p |t_N|^2 \int_{-\infty}^{\infty} dE N_L(E, \theta) N_R(E + eV, \theta) \times [f(E) - f(E + eV)], \quad (10)$$

where $t_N \approx \cos \theta / z_0$ is the transmission coefficient of the barrier I , $N_{L(R)}(E, \theta)$ is the quasiparticle surface density of states ($L = d_0$ and $R = d_0$ or $d_{\pi/4}$) and $f(E)$ is the Fermi–Dirac distribution function. The explicit expression of the surface density of states was obtained by Matsumoto and Shiba [21]. In the case of d_0 , there are no ZES. Therefore the angle θ dependence of $N_{d_0}(E, \theta)$ is the same as the bulk and is given by

$$N_{d_0}(E, \theta) = \text{Re} \frac{|E|}{\sqrt{E^2 - \Delta_{d_0}(\theta)^2}}. \quad (11)$$

On the other hand, $N_{d_{\pi/4}}(E, \theta)$ is given by

$$N_{d_{\pi/4}}(E, \theta) = \text{Re} \frac{\sqrt{E^2 - \Delta_{d_{\pi/4}}(\theta)^2}}{|E|} + \pi |\Delta_{d_{\pi/4}}(\theta)| \delta(E). \quad (12)$$

The delta function peak at $E = 0$ corresponds to ZES. Because of the bound state at $E = 0$, the quasiparticle current for the $d_0/d_{\pi/4}$ and $d_{\pi/4}/d_{\pi/4}$ junctions is drastically different from that for the d_0/d_0 junctions in which no ZES are formed. By substituting equations (11) and (12) into equation (10), we can obtain the analytical expression of the quasiparticle current $I_{\text{qp}}(V)$. In the limit of low bias voltages ($eV \ll \Delta_0$) and low temperatures ($\beta^{-1} \ll \Delta_0$), this can be approximated as

$$I_{\text{qp}}(V) \approx \begin{cases} \frac{3^2 \pi^2}{2^8 \sqrt{2}} \frac{eV^2}{\Delta_0 R_N} & \text{for } d_0/d_0 \\ \frac{3\pi^2}{2^4 \sqrt{2}} \frac{V}{R_N} & \text{for } d_0/d_{\pi/4} \\ \frac{2^5 \pi}{35} \left(\frac{\Delta_0}{\epsilon} \right)^2 \frac{V}{R_N} & \text{for } d_{\pi/4}/d_{\pi/4}. \end{cases} \quad (13)$$

In the calculation of I_{qp} for the $d_{\pi/4}/d_{\pi/4}$ junctions, we have replaced the ZES delta function $\delta(E)$ in equation (12) with the Lorentz type function, i.e.

$$\delta(E) \rightarrow \frac{1}{\pi} \frac{\epsilon}{\epsilon^2 + E^2}, \quad (14)$$

in order to avoid a mathematical difficulty and model the real systems (which include, for example, disorder and many-body effects). It is apparent from equation (13) that, in the case of d_0/d_0 junctions, the dissipation is of the super-Ohmic type as in the case of c -axis junctions [2]. This can be attributed to the effect of the node-to-node quasiparticle tunnelling. Thus the quasiparticle dissipation is very weak. On the other hand, in the case of $d_0/d_{\pi/4}$ junctions, node-to-ZES quasiparticle tunnelling gives the Ohmic dissipation which is similar to that in normal junctions [20]. Therefore the dissipation for $d_0/d_{\pi/4}$ junctions is stronger than that for d_0/d_0 junctions. Moreover, in the case of $d_{\pi/4}/d_{\pi/4}$ junctions, ZES-to-ZES quasiparticle tunnelling dominates the quasiparticle dissipation. The broadening of the ZES peak ϵ is typically one order of magnitude smaller than Δ_0 . Therefore, due to the prefactor $(\Delta_0/\epsilon)^2$ in equation (12), the quasiparticle dissipation in the $d_{\pi/4}/d_{\pi/4}$ junctions becomes enormously stronger than that for the d_0/d_0 and $d_0/d_{\pi/4}$ cases.

From equation (4), the asymptotic form of the dissipation kernel is given by

$$\alpha(\tau) \approx \begin{cases} \frac{3^2 \hbar^2}{2^7 \sqrt{2}} \frac{R_Q}{\Delta_0 R_N} \frac{1}{|\tau|^3} & \text{for } d_0/d_0 \\ \frac{3\hbar}{2^4 \sqrt{2}} \frac{R_Q}{R_N} \frac{1}{|\tau|^2} & \text{for } d_0/d_{\pi/4} \\ \frac{2^5 \hbar}{35\pi} \left(\frac{\Delta_0}{\epsilon} \right)^2 \frac{R_Q}{R_N} \frac{1}{|\tau|^2} & \text{for } d_{\pi/4}/d_{\pi/4}. \end{cases} \quad (15)$$

The result for the d_0/d_0 junction is in agreement with previous works [4, 5, 22, 23].

4. MQT in in-plane d-wave junctions

Let us move to calculation of the MQT rate Γ for d-wave Josephson junctions based on the standard Caldeira and Leggett theory [24]. At zero temperature Γ is given by

$$\Gamma \approx A \exp\left(-\frac{S_B}{\hbar}\right), \quad (16)$$

where $S_B \equiv S_{\text{eff}}[\phi_B]$ and ϕ_B is the bounce solution. Following the procedures above, we obtain the analytical formulae of the MQT rate for in-plane d-wave junctions as

$$\frac{\Gamma}{\Gamma_0} \approx \begin{cases} \exp\left[-\left(c_0 \frac{3^5 \pi}{2^7 \sqrt{2}} \frac{\hbar \eta}{\Delta_0} + \frac{18}{5} \frac{\delta M}{\hbar}\right) \frac{U_0}{M \omega_p}\right] & \text{for } d_0/d_0 \\ \exp\left[-\frac{3^4 \zeta(3)}{2^5 \sqrt{2} \pi^2} \eta (1-x^2)\right] & \text{for } d_0/d_{\pi/4} \\ \exp\left[-\frac{2^8 3^3 \zeta(3)}{35 \pi^3} \left(\frac{\Delta_0}{\epsilon}\right)^2 \eta (1-x^2)\right] & \text{for } d_{\pi/4}/d_{\pi/4}, \end{cases} \quad (17)$$

where $c_0 = \int_0^\infty dy y^4 \ln(1+1/y^2) / \sinh^2(\pi y) \approx 0.0135$, $\zeta(3)$ is the Riemann zeta function, $\eta = R_Q/R_N$ is the dissipation parameter, U_0 is the barrier height of the potential U , ω_p is the

Josephson plasma frequency, $x = I_{\text{ext}}/I_i$ ($i = 1, 2, 3$), and

$$\Gamma_0 = 12\omega_p \sqrt{\frac{3U_0}{2\pi\hbar\omega_p}} \exp\left(-\frac{36U_0}{5\hbar\omega_p}\right) \quad (18)$$

is the MQT rate without the dissipation. In equation (17)

$$\delta M = \frac{3}{2^4\sqrt{2}} \frac{\hbar^2\eta}{\Delta_0} \int_{-1}^1 dy y^2 \frac{1+y}{\sqrt{1-y}} \int_0^{\frac{\Delta_0}{\hbar\omega_p}} dz z^2 K_1(z|y|)^2 \quad (19)$$

is the renormalized mass due to the high frequency components ($\omega \geq \omega_p$) of the quasiparticle dissipation.

In order to compare the influence of ZES and the nodal-quasiparticle on MQT more clearly, we will estimate the MQT rate (17) numerically. For a mesoscopic bi-crystal YBCO Josephson junction [25] ($\Delta_0 = 17.8$ meV, $C = 20 \times 10^{-15}$ F, $R_N = 100 \Omega$, $x = 0.95$), the MQT rate is estimated as

$$\frac{\Gamma}{\Gamma_0} \approx \begin{cases} 83\% & \text{for } d_0/d_0 \\ 25\% & \text{for } d_0/d_{\pi/4} \\ 0\% & \text{for } d_{\pi/4}/d_{\pi/4}. \end{cases} \quad (20)$$

As expected, the node-to-ZES and ZES-to-ZES quasiparticle tunnelling in $d_0/d_{\pi/4}$ and $d_{\pi/4}/d_{\pi/4}$ junctions gives strong suppression of the MQT rate compared with the d_0/d_0 junction cases. Moreover in the $d_{\pi/4}/d_{\pi/4}$ cases, MQT is almost completely depressed.

5. Summary

In conclusion, MQT in in-plane high- T_c superconductors has been theoretically investigated and the formulae for the MQT rate, which can be used to analyse experiments, have been analytically obtained. Node-to-node quasiparticle tunnelling in d_0/d_0 junctions gives rise to weak super-Ohmic dissipation as in the case of c -axis junctions [2]. For $d_0/d_{\pi/4}$ junctions, on the other hand, we have found that node-to-ZES quasiparticle tunnelling leads to Ohmic dissipation. Moreover, in the case of $d_{\pi/4}/d_{\pi/4}$ junctions, ZES-to-ZES quasiparticle tunnelling gives very strong Ohmic dissipation so MQT is drastically suppressed.

In this paper we have only considered the case of a high barrier limit ($z_0 \gg 1$). In low barrier cases, the ZES become split into two finite energy Andreev levels due to ZES resonance [19]. Moreover, the energy of the split Andreev levels depends on the phase difference ϕ and the influence of the proximity effect becomes more important. To take

into account such effects, the present approach should be considerably modified. This issue will be investigated in future articles.

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