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The onset of full coupling in multi-filament superconducting tapes exposed to an alternating external magnetic field

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Abstract

At high amplitudes and frequencies of the external magnetic field, the superconducting filaments in a composite conductor are fully coupled. This causes a high magnetisation loss. Methods to decouple the filaments are well known for the low-T_c superconductors and have recently been applied also to high-T_c composites. In this paper, expressions are derived for the magnetic-field amplitude and frequency where full coupling sets in, as well as for the twist length required for decoupling the filaments in a general rectangular composite. A comparison is made with other expressions found in the literature. The filaments in a typical Bi-2223/Ag tape in parallel magnetic fields are decoupled just by twisting. In perpendicular fields a high transverse resistivity is required to decouple the filaments and decrease the magnetisation loss.

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1. Introduction

AC loss is a critical issue for the design of power-engineering devices based on high-T_c superconductors. The alternating magnetic field of the device causes a magnetisation loss, which consists of hysteresis and coupling-current losses. The hysteresis loss is decreased by dividing the superconductor into many thin filaments. However, the filaments in a composite conductor are coupled by currents flowing across the normal-conducting matrix. At high frequencies and amplitudes of the external magnetic field, the filaments are fully coupled. The composite conductor then behaves like a single large filament with a high magnetisation loss. Full coupling is avoided by twisting the filaments and increasing the matrix resistivity. In this paper expressions are derived for the magnetic-field amplitude and frequency where full coupling sets in, as well as for the twist length necessary to decouple the filaments. The expressions are valid for composites with a rectangular cross-section of arbitrary aspect ratio. The results are applied to typical multi-filament Bi-2223 superconducting tapes.

At low amplitudes and frequencies of the external magnetic field B, the coupling currents

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are well described with a model presented by Campbell [1]. The model is based on the assumption of a uniform magnetic field B_{in} inside the composite conductor, averaged over a volume that contains many filaments. The time constant τ of the composite is defined as

$$B - B_{in} = \tau \frac{dB_{in}}{dt}. \quad (1)$$

The coupling-current loss density (in Joule per field cycle per m^3 of composite material) is given by

$$Q_c = \frac{B_a^2}{\mu_0} \frac{\pi n \omega \tau}{1 + \omega^2 \tau^2}, \quad (2)$$

where ω is the angular frequency and B_a is the amplitude of the sinusoidal external magnetic field. The factor n depends on the shape of the composite cross-section. The coupling-current loss in a given magnetic field is reduced by making the product $\omega\tau$ much smaller than 1. The time constant is decreased by twisting the filaments and increasing the transverse resistivity of the matrix.

The Campbell coupling-current model is valid if the longitudinal coupling-currents flow only in the outer layer of filaments. For increasing magnetic-field amplitudes and frequencies the outer filaments become saturated and coupling currents must flow also in the inner filaments. There is a transition from 'Campbell coupling' towards full coupling, where all filaments are saturated. No adequate analytical model has been found to describe the magnetisation loss in the transition regime. The Campbell model itself is used here to calculate the magnetic-field amplitude and frequency where the transition begins.

2. Magnetic field for the onset of full coupling

Fig. 1 displays the coupling currents in a general rectangular composite conductor. The external magnetic field is oriented along the y-axis. The filamentary region has dimensions $2a$ in the x-direction and $2b$ in the y-direction. Dashed arrows in the figure indicate the coupling currents. They follow the outer filaments (marked in grey) for about one-half twist length. Then they cross over from top to bottom in the y-direction parallel to the external magnetic field. Fig. 3 in Ref. [1] contains an expression for the electric field E_y in the y-direction, in the symmetry plane with $y = 0$

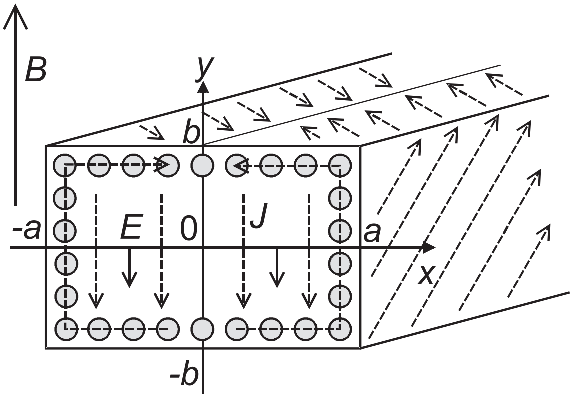

Fig. 1. Coupling currents between the filaments in a composite conductor with a rectangular cross-section. The outer layer of filaments is marked in grey. Dashed arrows indicate the coupling currents. They follow the outer filaments for about one-half twist length, then cross the matrix in a direction parallel to the magnetic field.

$$E_y = \frac{L_p a}{4(a+b)} \frac{dB_{in}}{dt}, \quad (3)$$

where L_p is the twist length: the length of one full turn of the filaments around the tape axis. The transverse current density J_y in the composite is given by E_y/ρ_{eff}. Here ρ_{eff} is the effective transverse resistivity of the inner region, averaged over a volume containing many filaments.

Fig. 2 displays the lower plane $y = -b$ of the composite conductor. Inclined lines represent the outer filaments. These filaments collect the transverse current and then carry it upward along the sides of the composite. The grey area with length L_p and width a is traversed exactly one time by each of the outer filaments. Therefore the total current coming down in the grey area must at no time exceed the total critical current of all the outer filaments. From Eq. (1) the amplitude $B_{a,in}$ of the internal magnetic field is found

$$B_{a,in} = \frac{B_a}{\sqrt{1 + \omega^2 \tau^2}}. \quad (4)$$

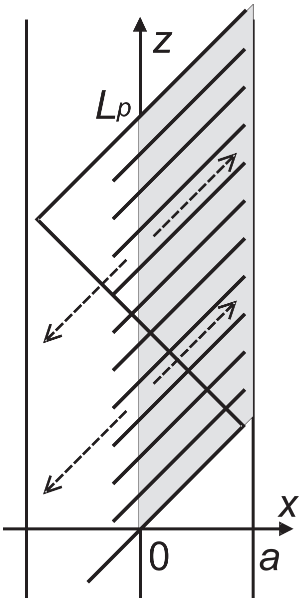

Fig. 2. Lower plane at $y = -b$ of the composite conductor displayed in Fig. 1. The outer filaments are displayed here as inclined lines. They collect the coupling currents coming down into the plane of the drawing. The grey area is traversed exactly one time by each of the outer filaments. The Campbell coupling model is valid if the amplitude of the coupling currents coming down in the grey area is lower than the total critical current of the outer filaments.

The amplitude $I_{\mathrm{a,coup}}$ of the coupling current coming down in the grey area is

$$I_{\mathrm{a,coup}} = \frac{aL_{\mathrm{p}}}{\rho_{\mathrm{eff}}} E_y = \frac{a^2 L_{\mathrm{p}}^2}{4(a+b)\rho_{\mathrm{eff}}} \omega B_{\mathrm{a,in}} = \frac{a^2 L_{\mathrm{p}}^2}{4(a+b)\rho_{\mathrm{eff}}} \frac{\omega B_{\mathrm{a}}}{\sqrt{1+\omega^2\tau^2}}. \quad (5)$$

This result has been published earlier in Ref. [2].

The coupling-current amplitude should be lower than the total critical current $I_{\mathrm{c,out}}$ of the outer filaments. This requirement leads to

$$\frac{\omega B_{\mathrm{a}}}{\sqrt{1+\omega^2\tau^2}} \leqslant \frac{4(a+b)\rho_{\mathrm{eff}} I_{\mathrm{c,out}}}{a^2 L_{\mathrm{p}}^2} \quad (6)$$

which gives the maximum value of ωB_{a} where the Campbell model is still valid, for any rectangular composite geometry. In order to decrease the coupling-current loss, $\omega\tau$ must be much smaller than 1. In that case Eq. (6) can be simplified by omitting the square-root term.

Kwasnitza gives a criterion for the onset of full coupling in tapes oriented perpendicularly to the magnetic field. The criterion is listed as Eq. (12) in Ref. [3]

$$\omega B_{\mathrm{a}} \leqslant 48 \frac{J_{\mathrm{c}} \rho_{\mathrm{eff}} d_{\mathrm{fil}} b}{L_{\mathrm{p}}^2 a} \frac{1}{\sqrt{1+(4a/L_{\mathrm{p}})^2}}, \quad (7)$$

where J_{c} is the critical-current density in the filaments and d_{fil} is the filament thickness. This criterion is compared to Eq. (6). When the outer filaments are rectangular and close together, their total critical current $I_{\mathrm{c,out}}$ is approximately given by $4ad_{\mathrm{fil}}J_{\mathrm{c}}$ if the tape is oriented perpendicularly to the magnetic field. Then Eq. (7) can be rewritten as

$$\omega B_{\mathrm{a}} \leqslant \frac{4(a+b)\rho_{\mathrm{eff},\perp} I_{\mathrm{c,out}}}{a^2 L_{\mathrm{p}}^2} \times \frac{1}{\sqrt{1+(4a/L_{\mathrm{p}})^2}} \frac{3b}{a+b}. \quad (8)$$

The first term of Eq. (8) is equal to Eq. (6). The square-root term is 1 for long twist lengths and decreases to $1/\sqrt{2}$ for a twist length equal to $4a$. Then the outer filaments are twisted at an angle of 45° to the tape axis (see Fig. 2) and shorter twist lengths are impractical. Therefore the difference between Eqs. (6) and (7) is mainly determined by the geometrical factor $3b/(a+b)$. The factor is about 0.15 for typical Bi-2223 tapes in a perpendicular magnetic field. There are three possible causes for the difference:

1. Eq. (7) is based on a model used to describe a superconducting cable, comprising two layers of strands separated by an insulating ceramic barrier [4]. Due to the barrier, the coupling currents cannot cross over between the strands in the direction of the magnetic field, as displayed in Fig. 1. Instead they must follow a longer path from strand to strand along the sides of the cable.
2. Eq. (7) is derived for a cable with a relatively small number of strands. This expression is therefore expected to describe the onset of full coupling in Bi-2223 tapes with a low number

of filaments, especially when there are two filament layers separated by a ceramic barrier as described in Ref. [5]. However, a significant decrease in magnetisation loss requires a large number of decoupled filaments. A large number of filaments is assumed in the Campbell model, which is used to derive Eq. (6). For typical Bi-2223 tapes with 50–150 filaments Eq. (6) is therefore expected to describe well the onset of full coupling.

3. In several of the equations in Ref. [3] the symbol ρ_e is used for the mean transverse resistivity between two crossing outer filaments. This resistivity may be different from ρ_{eff} used in the Campbell model, which would make the comparison in Eq. (8) invalid. The parameter ρ_{eff} can be calculated using materials properties and the shape and structure of the inner filaments [6,7]. It is not made clear in Ref. [3] how ρ_e can be quantitatively calculated.

For these three reasons Eq. (6) is expected to describe the onset of full coupling in typical Bi-2223 tapes in a perpendicular magnetic field more accurately than Eq. (7). Furthermore Eq. (6) is valid for composite conductors with an arbitrary rectangular cross-section. It can be applied also to a tape whose wide side is oriented parallel to the magnetic field.

Eq. (6) is used to calculate the magnetic-field amplitude where the transition to full coupling begins

$$B_{a,tfc} = \frac{a+b}{a^2}\frac{4\rho_{eff}I_{c,out}}{\omega L_p^2}. \tag{9}$$

This result has been published earlier in Ref. [8]. At the magnetic-field amplitude $B_{a,tfc}$ the coupling currents become too large for the outer ring of filaments. The Campbell model then becomes invalid but the filaments are not yet fully coupled. The transition amplitude $B_{a,tfc}$ is increased most effectively by decreasing the twist length L_p and the dimension a of the composite conductor perpendicular to the magnetic field. A higher critical current and a higher transverse resistivity also increase $B_{a,tfc}$.

The transition amplitude is calculated for a typical Bi-2223 tape, assuming a critical current $I_{c,out} = I_c/4 = 10$ A in the outer filaments. The twist length is 10 mm and the transverse resistivity has the value 0.27 μΩ cm for silver at 77 K. A 50 Hz magnetic field is oriented parallel to the wide dimension of the tape. The filamentary region has dimensions $2a = 0.2$ mm and $2b = 3.0$ mm. Then Eq. (9) gives a transition amplitude $B_{a,tfc} = 0.55$ T. In parallel magnetic fields at power frequencies, full coupling can be avoided up to relatively high field amplitudes. In perpendicular fields the dimensions are $2a = 3.0$ mm and $2b = 0.2$ mm which gives $B_{a,tfc} = 2.4$ mT. The filaments are fully coupled already at low magnetic-field amplitudes. In reality the inner filaments make the effective transverse resistivity lower than the resistivity of silver [6,7]. The Campbell model including Eq. (5) with a lower ρ_{eff} agrees with measurement results obtained in a parallel magnetic field [9].

3. Twist length for decoupling the filaments

For a given amplitude and frequency of the magnetic field, filaments with a large twist length are fully coupled while tightly twisted filaments may be decoupled. The onset of full coupling occurs at twist lengths larger than the critical twist length $L_{p,c}$. From Eq. (9) one obtains

$$L_{p,c} = \frac{2}{a}\sqrt{\frac{(a+b)\rho_{eff}I_{c,out}}{\omega B_a}}. \tag{10}$$

At a twist length of $L_{p,c}$ the outer ring of filaments is just saturated by coupling currents; the filaments are not fully coupled.

In a simple model with two non-twisted slab-like filaments separated by a normal-conducting material [10], full coupling also occurs at a critical length

$$L_c = 4\sqrt{\rho_m J_{c,fil} d_{fil}/2\omega B_a}, \tag{11}$$

where ρ_m is the resistivity of the normal-conducting material. Eq. (11) is commonly used also for twisted round wires, where the critical twist length for full coupling of the filaments is approximately equal to $2L_c$. Eq. (11) is compared to Eq. (10), which describes the onset of full coupling in a rectangular composite. In the two-filament model both filaments are placed on the outside, at $x = \pm a$ in Fig. 1. They are fully coupled when the coupling-current amplitude is equal to their combined critical current $I_{c,out}$. In this case $I_{c,out}$ is equal to $4bd_{fil}J_c$ which gives

$$L_{p,c} = \frac{1}{a}\sqrt{\frac{\rho_{eff}}{\rho_m}2b(a+b)}4\sqrt{\frac{\rho_m J_c d_{fil}}{2\omega B_a}} = \sqrt{\frac{\rho_{eff}}{\rho_m}}\sqrt{\frac{2b}{a}+\frac{2b^2}{a^2}}L_c. \quad (12)$$

Assuming ρ_{eff} is equal to ρ_m and setting $a = b$ for a square wire, one obtains $L_{p,c} = 2L_c$ which is similar to the well-known result for a round wire. The filamentary region of a typical Bi-2223 tape has an aspect ratio of 15. In that case one finds $L_{p,c} = 21.9L_c$ in a parallel magnetic field and $L_{p,c} = 0.377L_c$ in a perpendicular field. This explains why the authors of Refs. [11,12] found decoupled filaments also for twist lengths longer than L_c in parallel magnetic fields, and full coupling for twist lengths shorter than L_c in perpendicular fields.

Using the same tape parameters as before, with a magnetic-field amplitude of 0.1 T parallel to the tape, Eq. (10) yields $L_{p,c} = 23$ mm. At this twist length the coupling currents are confined to the outer ring of filaments. Then the Campbell model is valid and Eq. (2) may be used to estimate the magnitude of the coupling loss. A twist length of 23 mm is technically quite possible and is also short enough to make $\omega\tau$ lower than 1. Therefore if the magnetic field is oriented parallel to the wide side of the tape, full coupling can be avoided by twisting the filaments. The coupling-current loss can then be reduced even if the matrix is pure silver.

The tapes in devices are usually arranged in such a way that the magnetic-field components perpendicular to the tape are minimised. However, even for a magnetic-field amplitude of 0.01 T perpendicular to the tape, Eq. (10) gives a critical twist length $L_{p,c} = 5$ mm. Even shorter twist lengths are required in that case to make $\omega\tau$ smaller than 1, which is necessary to significantly decrease the coupling-current loss. In tapes of a few mm wide, twist lengths shorter than 5 mm are difficult to achieve without a serious decrease of the critical current. The present tapes have a high aspect ratio due to the necessity of texturing the Bi-2223 material. Therefore an effective transverse resistivity about 10 times the resistivity of silver is required in order to decrease the AC loss below the full-coupling level in a perpendicular magnetic field [2,8]. Inserting ceramic barriers between the filaments increases the effective resistivity [3,5,8].

4. Conclusions

We have derived an expression for the magnetic-field amplitude where the transition to full coupling begins. This amplitude is effectively increased by decreasing the twist length and the dimension of the composite perpendicular to the magnetic field. The twist length necessary for decoupling the filaments is different from the commonly used critical coupling length, which is based on a model with non-twisted filaments. When a typical Bi-2223 tape is oriented with its wide side parallel to the external magnetic field, the filaments are decoupled at rather high magnetic-field amplitudes just by twisting. In perpendicular magnetic fields an effective transverse resistivity much higher than that of silver is required to decouple the filaments and decrease the magnetisation loss.

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