Optimal Item Discrimination and Maximum Information for Logistic IRT Models

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Items with the highest discrimination parameter values in a logistic item response theory model do not necessarily give maximum information. This paper derives discrimination parameter values, as functions of the guessing parameter and distances between person parameters and item difficulty, that yield maximum information for the three-parameter logistic item response theory model. An upper bound for information as a function of these parameters is also derived. An algorithm is suggested for the maximum information item selection criterion for adaptive testing and is compared with a full bank search algorithm. *Index terms: adaptive testing, discrimination parameter, information function, item selection, logistic IRT model.*

The item information function (IIF) in item response theory (IRT) can be used to select items from item banks. This can be done sequentially during test administration, e.g., in computerized adaptive testing (Lord, 1980; Wainer, 1990).

The maximum-information selection criterion (e.g., Lord, 1980) is one of the most commonly used methods of item selection for adaptive testing. For the two- and three-parameter logistic (2PL and 3PL) IRT models, increasing the item discrimination parameter a_i will cause information to increase. Lord (1980, Equation 10-6) showed that for the 2PL and 3PL models, the maximum obtainable item information is an increasing function of the squared item discrimination parameter when item difficulty b_i and person trait level (θ) are optimally matched. For the 2PL model, maximum information is obtained when $b_i = \theta$. It can also be shown that the area under the IIF in the 2PL model equals a_i. A similar relationship holds for the 3PL model (Birnbaum, 1968, Equations 20.4.26).

The guessing parameter (c_i) for the 3PL model contaminates the other two item parameters. c_i reduces the discrimination power of the item and the item is easier than b_i suggests (Samejima, 1984). For example, the maximum slope of the item response function reduces by a factor $(1 - c_i)$, and the probability of giving a correct answer for persons with $\theta = b_i$ increases by $c_i/2$. Samejima (1984) calculated her "discrimination shrinkage factor" and "difficulty reduction index" as functions of c_i.

c_i also affects the information of the item. First, information decreases as c_i increases. Second, it decreases more for low θs than for high θs: the IIF becomes asymmetric. Third, maximum information is obtained when $b_i = \theta$ minus a term that increases as c_i increases.

Information in the 2PL Model

Figure 1 shows IIFs on a $\theta - b_i$ scale for different values of a_i for the 2PL model. Increasing a_i leads to a higher and more peaked IIF. This phenomenon shows that the area under the IIF is concentrated in a smaller range of θ values, i.e., the width of the IIF becomes smaller as a_i increases.

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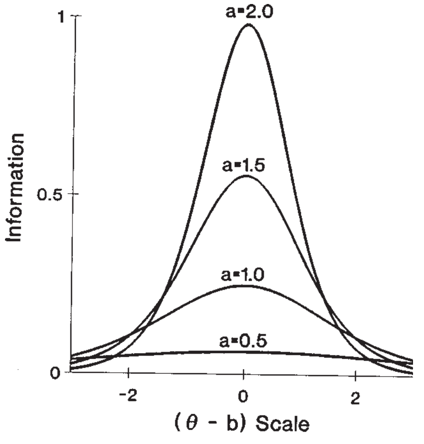
Figure 1
Item Information Functions

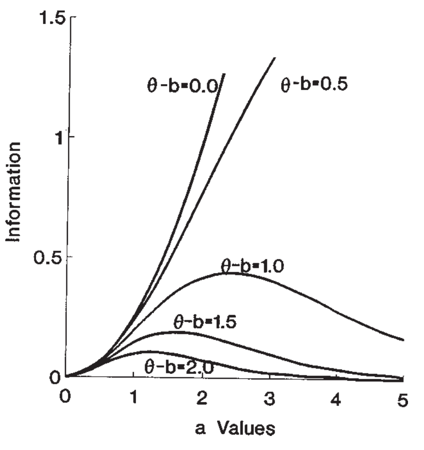


Samejima (1994) showed that the area under the square root of the IIF for the 2PL model equals $\pi(\approx 3.14)$, irrespective of a_i. This implies that in the 2PL model the IIFs of two items must cross at least once. For reasons of symmetry, the IIFs of two items with equal b_is but different a_is must cross twice. This is shown in Figure 1.

Figure 1 also shows that an extreme increase in a_i can lead to a decrease of item information when b_i is not close to θ. This effect is called the *attenuation paradox* (Loevinger, 1954) in IRT by Lord & Novick (1968, p. 368) and Birnbaum (1968, p. 465).

Figure 2 illustrates IIFs for the 2PL model as a function of a_i for different values of the distance between θ and b_i. The fact that an item with a high a_i is not necessarily the most informative item and that, therefore, selection of items in an adaptive test should not solely be based on the a_i, can also be seen in this figure.

Figure 2
Item Information as a Function of a_i



Purpose

This paper shows which values of a_i give maximum information and the magnitude of that information. The optimal discrimination and the maximum attainable information are functions of the distance between b_i and θ for logistic IRT models. The results of this paper are implemented in a maximum information item selection algorithm for adaptive testing, and a small simulation study shows that this algorithm is faster than a full bank search.

Derivation of Optimal Item a_is

The item response function, or the probability of a correct response to item i for a person with θ, of the 3PL IRT model is (e.g., Lord, 1980, Equation 4-37)

$$P_i(\theta) = c_i + (1 - c_i)\frac{\exp(L_i)}{1 + \exp(L_i)} \ , \tag{1}$$

where
$L_i = a_i(\theta - b_i)$,
$a_i \in \mathcal{R}^+$ is the item discrimination parameter,
$b_i \in \mathcal{R}$ is the item difficulty parameter,
$c_i \in [0,1)$ is the guessing parameter,
$\theta \in \mathcal{R}$ is the person trait parameter, and
$\mathcal{R}$ and $\mathcal{R}^+$ are sets of real and positive real numbers, respectively. The corresponding IIF is

$$I_i(\theta) = \frac{a_i^2(1 - c_i)}{\left[c_i + \exp(L_i)\right]\left[1 + \exp(-L_i)\right]^2} \tag{2}$$

(e.g., Lord, 1980, Equation 4-43).
If $L_i = 0$, then

$$I_i(\theta) = \frac{1}{4}\, a_i^2\, \frac{1 - c_i}{1 + c_i} \ . \tag{3}$$

If $(\theta - b_i) = 0$, then $I_i(\theta)$ increases as a_i increases. If, however, $a_i = 0$, the minimum of the information as a function of a_i is reached: $I_i(\theta) = 0$.

Hereafter it is assumed that $L_i \neq 0$. Therefore the natural logarithm of the IIF,

$$\log\left[I_i(\theta)\right] = 2\log(a_i) + \log(1 - c_i) - \log\left[c_i + \exp(L_i)\right] - 2\log\left[1 + \exp(-L_i)\right] \ , \tag{4}$$

is defined because $I_i(\theta) > 0$ for $L_i \neq 0$.

L_is for which the information, as a function of L_i, reaches a maximum or minimum for fixed values of c_i and $(\theta - b_i) \neq 0$ are found by setting the derivative of $\log[I_i(\theta)]$ with respect to L_i equal to 0, i.e.,

$$\frac{\partial \log[I_i(\theta)]}{\partial L_i} = \frac{2}{L_i} - \frac{\exp(L_i)}{c_i + \exp(L_i)} + 2\,\frac{\exp(-L_i)}{1 + \exp(-L_i)} = 0 \ . \tag{5}$$

Using the fact that $L_i \neq 0$, this equation can be reduced to

$$2c_i(1 + L_i) + [2(c_i + 1) + L_i]\exp(L_i) + (2 - L_i)\exp(2L_i) = 0 \ . \tag{6}$$

The second derivative of $\log[I_i(\theta)]$ with respect to L_i,

$$\frac{\partial^2 \log[I_i(\theta)]}{\partial L_i^2} = -\frac{2}{L_i^2} - \frac{\exp(L_i)}{c_i + \exp(L_i)}\left[1 - \frac{\exp(L_i)}{c_i + \exp(L_i)}\right] - 2\left\{\frac{\exp(-L_i)}{1+\exp(-L_i)}\left[1 - \frac{\exp(-L_i)}{1+\exp(-L_i)}\right]\right\}^2 \ , \tag{7}$$

is negative for all $L_i \neq 0$ and all values of $c_i \in [0, 1)$ because in the sum on the right-hand side of Equation 7 the first and last terms are always negative and the second term is never positive (it is equal to 0 for c_i = 0.)

For exactly one value of L_i between −3 and −1, the first derivative of $\log[I_i(\theta)]$ with respect to L_i (Equation 5) equals 0. Substituting $L_i = -3$ and $L_i = -1$, respectively, into Equation 5 yields

$$\left.\frac{\partial \log[I_i(\theta)]}{\partial L_i}\right|_{L_i=-3} = \frac{2}{-3} - \frac{e^{-3}}{c_i + e^{-3}} + 2\frac{e^3}{1+e^3} \geq -\frac{5}{3} + \frac{2e^3}{1+e^3} > 0 \ \ \forall\, c_i \in [0, 1) \ , \tag{8}$$

and

$$\left.\frac{\partial \log[I_i(\theta)]}{\partial L_i}\right|_{L_i=-1} = -2 - \frac{e^{-1}}{c_i + e^{-1}} + \frac{2e}{1+e} < -\frac{e^{-1}}{1+e^{-1}} - \frac{2}{1+e} < 0 \ \ \forall\, c_i \in [0, 1) \ , \tag{9}$$

respectively. Equation 7 indicates that $\partial \log[I_i(\theta)]/\partial L_i$ strictly decreases as L_i increases. Combining this fact with Equations 8 and 9 shows that for each c_i there exists exactly one value of $L_i < 0$ for which $\partial \log[I_i(\theta)]/\partial L_i$ equals 0, and that this value of L_i lies between −3 and −1. Furthermore, such value of L_i corresponds with a maximum of $\log[I_i(\theta)]$ and $I_i(\theta)$, because

$$\frac{\partial^2 \log[I_i(\theta)]}{\partial L_i^2} < 0 \ . \tag{10}$$

Similar reasoning applies for $L_i > 0$. For $L_i = 1$ and $L_i = 3$, $\partial \log[I_i(\theta)]/\partial L_i$ is always positive and always negative, respectively. This proves that $I_i(\theta)$ has exactly one maximum for $L_i > 0$ and that this maximum is reached when L_i is between 1 and 3.

The two optimal L_i values can be found for each c_i value by solving Equation 6 iteratively, substituting real numbers for L_i between −3 and −1, and 1 and 3. Doing so for c_i values ranging from 0.0 to .9 with steps of .1 results in finding the values of L_i given in Table 1.

The corresponding optimal a_i values can also be derived from these optimal L_i values. If, for example, $c_i = .1$ and $(\theta - b_i) = -2$, then the optimal a_i value will be −1.816/−2 = .908. This value is depicted as an "X" in Figure 3. The optimal a_i values for c_i = 0.0 and .9 are shown in Figure 3 as functions of $(\theta - b_i)$. All values for $0.0 < c_i < .9$ lie between the values for c_i = 0.0 and c_i = .9.

The maximum values of $I_i(\theta)(\theta - b_i)^2$ in Table 1 can be obtained by substituting the values for c_i and the optimal values for a_i and L_i in Equation 2. Upper bounds for $I_i(\theta)$ can be found by dividing these maxima by $(\theta - b_i)^2$. For items with c_i = .1, for example, the information at $\theta < b_i$ can be, at most,

$$\frac{[-1.816/(\theta - b_i)]^2}{e^{-1.816}(1 + e^{1.816})^2} < \frac{0.222}{(\theta - b_i)^2} \ . \tag{11}$$

This means that information for items with c_i = .1 and $(\theta - b_i) = -2$ is always less than .055. This value is shown as an "X" in Figure 4 with the upper-bound IIFs for c_i = 0 and .6. Similar plots can

Table 1
Optimal L_i Values and Corresponding Maxima for $I_i(\theta)(\theta - b_i)^2$ (Rounded Up) for c_i Values Given Fixed $(\theta - b_i)$ Values

	$(\theta - b_i) < 0$		$(\theta - b_i) > 0$	
c_i	L_i	$I_i(\theta)(\theta - b_i)^2$	L_i	$I_i(\theta)(\theta - b_i)^2$
0.0	−2.399	.440	2.399	.440
.1	−1.816	.222	2.417	.392
.2	−1.669	.145	2.434	.346
.3	−1.591	.101	2.451	.300
.4	−1.541	.073	2.467	.255
.5	−1.505	.052	2.482	.211
.6	−1.478	.037	2.497	.168
.7	−1.457	.025	2.512	.125
.8	−1.440	.015	2.526	.083
.9	−1.427	.007	2.540	.041

be drawn for c_i = .1, .2, .3, .4, and .5. These values all lie between the two plots in Figure 4. For c_i = .7, .8, and .9, the values lie below those of c_i = .6.

For the c_is in Table 1, the upper-bound values of $I_i(\theta)(\theta - b_i)^2$, given the value of $(\theta - b_i)$, decreases as c_i increases. This is also the case for other values of c_i.

Suppose $(\theta - b_i) > 0$, and $c_{2i} > c_{1i}$. L_{1i} and L_{2i} are defined as the optimal positive L_is for $c_i = c_{1i}$ and $c_i = c_{2i}$, respectively. The upper-bound value of $I_i(\theta)(\theta - b_i)^2$ for $c_i = c_{1i}$ is

$$\frac{L_{1i}^2(1 - c_{1i})}{\left[c_{1i} + \exp(L_{1i})\right]\left[1 + \exp(-L_{1i})\right]^2} . \tag{12}$$

Because L_{1i} is defined as the value of L_i for which $I_i(\theta)(\theta - b_i)^2$ reaches its upper bound,

$$\frac{L_{1i}^2(1 - c_{1i})}{(c_{1i} + \exp(L_{1i})\,(1 + \exp(-L_{1i})^2} \geq \frac{L_{2i}^2(1 - c_{1i})}{(c_{1i} + \exp(L_{2i})\,(1 + \exp(-L_{2i})^2} . \tag{13}$$

When $c_{2i} > c_{1i}$,

$$\frac{L_{2i}^2(1 - c_{1i})}{\left[c_{1i} + \exp(L_{2i})\right]\left[1 + \exp(-L_{2i})\right]^2} > \frac{L_{2i}^2(1 - c_{2i})}{\left[c_{2i} + \exp(L_{2i})\right]\left[1 + \exp(-L_{2i})\right]^2} . \tag{14}$$

Figure 3
Optimal a_i Value as a Function of $(\theta - b)$

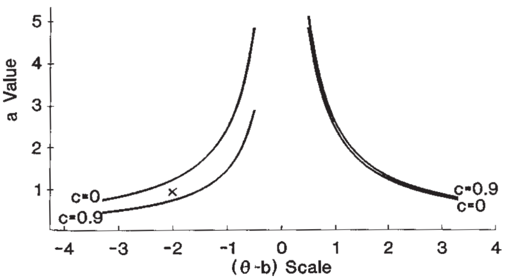
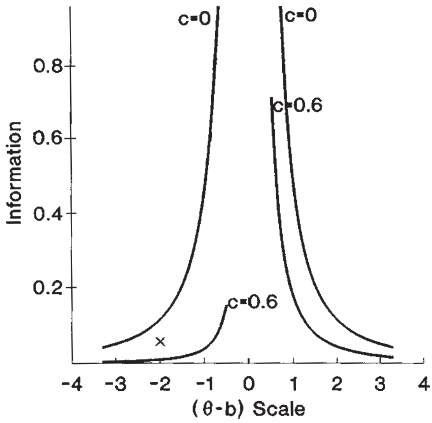


Figure 4
Upper-Bound Information as a Function of $(\theta - b)$



The right-hand side of Equation 14 is the upper bound of $I_i(\theta)(\theta - b_i)^2$ for $c_i = c_{2i}$. A similar reasoning applies for $(\theta - b_i) < 0$, which completes the proof.

Conclusions

Three factors determine the value of the upper-bound information. It was just shown that it is a decreasing function of c_i for a fixed value of $(\theta - b_i)$. It also decreases for increasing values of $(\theta - b_i)^2$, for fixed values of c_i, and the sign of $(\theta - b_i)$, because then the upper bound of $I_i(\theta)(\theta - b_i)^2$ is constant. Finally, for fixed values of $|\theta - b_i|$ and c_i, it is higher for $(\theta - b_i) > 0$ than for $(\theta - b_i) < 0$.

For example, for items with $c_i > .1$ and $(\theta - b_i) < -2$, the maximum information is less than the maximum information for items with $c_i = .1$ and $(\theta - b_i) = -2$, which in turn is less informative than items with $c_i = .1$ and $(\theta - b_i) = 2$. As a consequence, when items with $c_i \geq .1$ exist, which are above a person's θ by at least two units [i.e., $(\theta - b_i) \leq -2$], then $I_i(\theta) \leq .055$. For items with the same c_is, but with $(\theta - b_i) \geq 2$, $I_i(\theta) \leq .392/4 = .098$.

Finally, in Figure 5 the upper-bound information multiplied by $(\theta - b_i)^2$ is shown as a function of c_i. These results are based on the results given in Table 1. Interpolation was used for all c_i values not shown in Table 1. Note that the exact shapes of the lines were thus not proven.

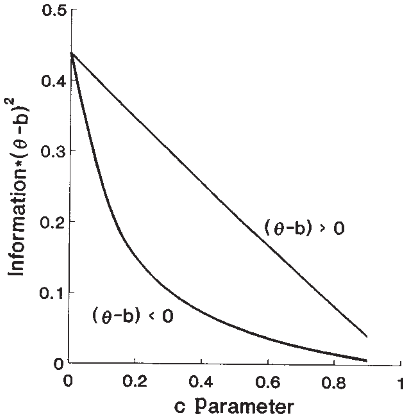
An Item Selection Algorithm

The relationship between θ, b_i, and item information at θ can be used for item selection in adaptive testing. Suppose the task is to select items from an item bank in an adaptive test with the maximum information criterion. This criterion selects the item with the highest value of IIF at a value on the θ scale, usually some provisional estimate, e.g., θ_0. One way to select the most informative item at θ_0 is a full bank search, i.e., to calculate the information of all items at value θ_0.

An Alternative to a Full Bank Search

This algorithm is based on the fact that it is unnecessary to compute the information of all items in an item bank to determine which item has maximum information. Let $c_{\min}$ be the smallest c_i value among all items in the item bank. Let $I_{\max,+}(c_{\min})$ and $I_{\max,-}(c_{\min})$ be the upper bounds of $I_i(\theta_0)(\theta_0 - b_i)^2$ for positive and negative values of $(\theta_0 - b_i)$, respectively, and for $c_i = c_{\min}$.

Figure 5
Upper-Bound Information Times $(\theta - b)^2$ as a Function of c_i



These upper bounds are shown in Table 1 under $I_i(\theta_0)(\theta_0 - b_i)^2$ and the values of $c_{\min}$ are under c_i, e.g., $I_{\max,+}(.1)$ = .392 and $I_{\max,-}(.1)$ = .222. For c_i values other than those shown in Table 1, the upper bounds can be determined from Equation 6. For $c_i = c_{\min}$, the two upper bounds of $I_i(\theta_0)(\theta_0 - b_i)^2$ are the highest c_i values in the item bank, because these upper bounds decrease as c_i increases. The two upper-bound values that correspond to the lowest c_i values are in some sense the two upper-bound values for the entire item bank, one for negative and one for positive values of $(\theta_0 - b_i)$.

In the following, $I_{\max}$ will be $I_{\max,+}(c_{\min})$ or $I_{\max,-}(c_{\min})$. For two items i and j, the following can be derived:

$$\text{If } (\theta_0 - b_i)^2 > \frac{I_{\max}}{I_j(\theta_0)}\ , \quad \text{then } I_i(\theta_0) < I_j(\theta_0)\ . \tag{15}$$

This follows from

$$(\theta_0 - b_i)^2 > \frac{I_{\max}}{I_j(\theta_0)} \Leftrightarrow I_j(\theta_0) > \frac{I_{\max}}{(\theta_0 - b_i)^2} \tag{16}$$

and

$$I_i(\theta_0)(\theta_0 - b_i)^2 < I_{\max} \Leftrightarrow I_i(\theta_0) < \frac{I_{\max}}{(\theta_0 - b_i)^2}\ . \tag{17}$$

The left-hand side of Equation 9 follows from the definition of $I_{\max}$ as an upper bound of $I_i(\theta_0)(\theta_0 - b_i)^2$.

When it is determined that an item j has a certain information $I_j(\theta_0)$, then all items i with positive $\theta_0 - b_i$ and $(\theta_0 - b_i)^2 > I_{\max,+}(c_{\min})/I_j(\theta_0)$ or negative $\theta_0 - b_i$ and $(\theta_0 - b_i)^2 > I_{\max,-}(c_{\min})/I_j(\theta_0)$ will have less information than item j, regardless of the values of a_i and $c_i \geq c_{\min}$.

The Algorithm

The algorithm has the following initialization steps:

1. Order the N items in the item bank according to their difficulties: $b_1 < b_2 < \ldots < b_N$.
2. Determine the smallest value of c_i in the item bank: $c_{\min}$.

3. Compute the constants $I_{\max,+}(c_{\min})$ and $I_{\max,-}(c_{\min})$.

If θ_0 is a provisional estimate after the administration of a set of items in an adaptive test, then the selection of the next item with maximum information on θ_0 from the set of items not already included in the test consists of the following steps:

1. Search for item j with b_j equal to the smallest positive difference $(\theta_0 - b_j)$ among all items. If $\theta_0 < b_1$ or $\theta_0 > b_N$, then item j is the easiest or the most difficult item, respectively.
2. Search for items i in increasing order of $|\theta_0 - b_i|$, for positive values of $(\theta_0 - b_i)$. If $(\theta_0 - b_i)^2 > I_{\max,+}(c_{\min})/I_j(\theta_0)$, then continue. Otherwise, compute $I_i(\theta)$. If $I_i(\theta_0) > I_j(\theta_0)$, then set j equal to i, and continue the search.
3. Search for items i in increasing order of $|\theta_0 - b_i|$, for negative values of $(\theta_0 - b_i)$. If $(\theta_0 - b_i)^2 > I_{\max,-}(c_{\min})/I_j(\theta_0)$, then end the search. Otherwise, compute $I_i(\theta)$. If $I_i(\theta_0) > I_j(\theta_0)$, then set j equal to i, and continue the search with Step 1.

Step 1 can be made more efficient by beginning the search for item j with an item that is expected to have a difficulty close to b_j. Note that in Steps 2 and 3, the index j represents the most informative item that is eventually administered to the examinee. The answer to this item is scored, and the examinee's θ is re-estimated. Item j is then removed from the item bank.

This search process uses only a part of the item bank, i.e., information is computed only for items with relatively small values of $(\theta_0 - b_i)^2$. For that reason, item selection will be much faster with this algorithm than with a full bank search.

A Simulation Study

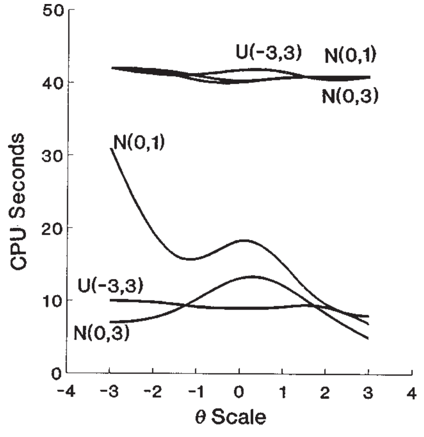
To establish the relative speed of this item selection algorithm, a simulation study was performed in which the algorithm was compared to the algorithm with straightforward calculation of the information of all items in an item bank. The maximum information criterion was used to select items for both algorithms. This means that exactly the same items, in the same order, were selected by the two algorithms for each test. The algorithms differed only in their computational (CPU) times. The simulations were performed using a program written in Borland Pascal 7.0 and were run on a 486-DX2 66 MHz computer.

Design. Adaptive tests of 30 items were simulated for seven values, $\theta = -3, -2, -1, 0, 1, 2,$ and 3. These simulations were repeated for three different item banks. Each item bank contained 200 items. The distributions of the a_is and c_is were uniform for each bank. The a_is were uniformly distributed between .5 and 2, i.e., $a_i \sim U(.5,2)$, and the c_is between .1 and .3, i.e., $c_i \sim U(.1,.3)$. The three item banks differed only in their distributions of b_i. One bank had a uniform $U(-3,3)$ distribution of b_i; the other two were normally distributed with a variance of 1 or 3. Thus, the distributions of b_is were $U(-3,3)$, $N(0,1)$, and $N(0,3)$, respectively. For each combination of θ and item bank, there were 100 replications.

Results. Figure 6 shows the CPU time used to select 30 items in 100 tests, as a function of θ, for the two algorithms and the three item banks. The three lines at the top of Figure 6 show the CPU time needed to calculate the information of all items in the bank not already included in the test. The algorithm improved item selection by a factor between 1.5 and 6. For a uniform distribution of the b_is, the relative speed did not depend on θ. But for the item banks with normally distributed b_is, speed was dependent on θ. For $\theta = 0$, CPU time was relatively high because there were relatively many items with a b_i in that region. Therefore, in these cases the information had to be computed for relatively many items.

The high CPU times for extreme negative θs for the $N(0,1)$ distribution of b_is can be explained as follows. The b_is were thinly spread around -3. The information of the most informative item j in the search process was usually not very high, because most of the time $(\theta_0 - b_j) < 0$ and

Figure 6
Total CPU Time for Two Maximum Information Item Selection Algorithms for 100 30-Item Adaptive Tests Using Three 200-Item Banks



$|\theta_0 - b_j|$ was large. Therefore, the range of b_is that were more informative than item j was rather large. On the other hand, for extreme positive θs, the CPU times were much lower because the information for items j with large distances between θ_0 and b_j was higher for $(\theta_0 - b_j) > 0$ than for $(\theta_0 - b_j) < 0$. Note that for the $N(0,3)$ distribution, the b_j values were spread out much more than for the $N(0,1)$distribution; thus, this effect did not occur.

Discussion

One application of the algorithm proposed here is an adaptive testing procedure in which the test is constrained with respect to its content. In such cases, the use of information tables (Thissen & Mislevy, 1990, pp. 116–117) is virtually impossible because items are generally selected from changing subsets in the bank. Stocking & Swanson (1993), for example, have developed a weighted deviations model for constrained item selection in adaptive testing. Swanson & Stocking (1993) present a heuristic for solving this model for very large numbers of constraints that gives a suboptimal solution because of the complexity of the selection procedure. Furthermore, exercising control of item exposure can lead to the practice of searching for a group of nearly optimal items each time an item is selected for administration. Because time often plays a critical role in constrained item selection procedures, it may be worthwhile to investigate the possibility of incorporating the algorithm proposed here into Swanson and Stocking's heuristic or other heuristics for item selection procedures for constrained adaptive testing. Another possible need for quick alternatives to a full bank search may arise in applications of adaptive testing with frequently changing banks, due to the replacement of obsolete items or to the use of rotating banks introduced to secure item content. In such cases, a quick item selection algorithm is a good alternative to the practice of frequently recalculating and replacing information tables.

Conclusions

When b_i and θ are not optimally matched, the optimal a_i in logistic item response models is not its maximum value. It has been shown here that the optimal a_i is inversely related to the distance between b_i and θ, and it was determined that a_i provides the most information at certain points on the $b_i - \theta$ scale. The corresponding maximum information is inversely related to the squared distance between θ and b_i.

The relation between this distance and an upper bound on information was used in an algorithm for the maximum information item selection criterion for adaptive testing. In a small simulation study, this algorithm was 1.5 to 6 times faster than a full bank search. Because much faster algorithms than a full bank search are being used in practice, further research is needed to compare this algorithm with those methods.

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