



The effect of transverse modes in a wave-guide resonator on the resonance condition of a Compton FEL¹

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Abstract

In a free electron laser where the resonator has a wave-guide structure the phase velocity of the light has a significant impact on the wavelength of the output radiation. This effect is well known in Raman-type FEL's. It is however less known that this effect can also play an important role in Compton-type FEL's. This effect has been experimentally verified in our 6 MeV Compton-type FEL, having a wave-guide resonator with a hole in the outcoupling mirror. Successive transverse modes, which have distinct phase velocities, show up at different wavelengths in the output spectrum. The difference appeared to be as large as 5% of the resonance wavelength. In normal lasers this difference is not more than about 0.1%.

1. Introduction

In a free electron laser (FEL) the radiation production always takes place in vacuum because the electron beam can only propagate over long distances in vacuum. Furthermore in calculating the frequency of the output radiation usually a one-dimensional model is used in which the radiation is considered as a plane wave. According to this model and assuming vacuum the resonance condition for a relativistic electron beam is given by [1]:

$$\lambda = \frac{\lambda_u}{2\gamma^2} (1 + K^2), \quad (1)$$

where λ is the wavelength of the FEL radiation; $K = eB_{av} \lambda_u / 2\pi m_0 c$ is the undulator parameter; λ_u

and B_{av} are the period and the average magnetic field strength of the undulator respectively; γ , e and m_0 are the relativistic factor, the charge and the rest mass of the electron respectively. However this equation is not accurate enough if the three-dimensional character of the laser beam is taken into account. This is well known in Raman-type FEL's where the optical cavity has a wave-guide structure because of the long laser wavelength [2–4]. The correct resonance condition can be found by using the fact that an electron has to see the same phase of the laser beam each time it reaches a new undulator period. This means that during the time that the electron travels one undulator period, the light has to travel the undulator period plus a whole number of wavelengths. For the fundamental frequency this means:

$$\frac{\lambda_u}{v_e} = \frac{\lambda_u + \lambda}{v_{ph}}, \quad (2)$$

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where v_e is the component in the z -direction of the electron velocity and v_{ph} is the phase velocity of light. Because in a wave-guide structure this phase velocity is larger than the light velocity c , it is easy to see that for a fixed electron energy and a fixed undulator period the wavelength λ must increase for an increasing phase velocity. In a Compton-type FEL usually the phase velocity is approximated by c . Then Eq. (1) follows for $\gamma \gg 1$. We will show that also for Compton-type FEL's this approximation can lead to a wrong value for the resonance wavelength. While the principle is well known, it is seldom realised that the effect can be significantly large for highly relativistic electrons. That is because in that case λ is always much smaller than λ_u so that even small deviations of the phase velocity from c will lead to a large shift in wavelength. This effect is especially important if high-quality electron beams are used.

2. The effect of the wave guide mode pattern on the resonance condition of an FEL

If the laser beam of the FEL is bounded by circular metallic side walls the pattern of the modes are given by the well-known circular electric TE_{nm}

and magnetic TM_{nm} wave-guide modes, as can be found in a number of textbooks (see e.g. Ref. [5]). All those modes have a space dependent part in the expressions for the electric and magnetic field strength, containing Bessel functions, and a phase factor $\exp [j(\omega t - \gamma_{nm} z)]$, where γ_{nm} is the complex propagation constant

$$\gamma_{nm} = \beta_{nm} + j\alpha_{nm}. \tag{3}$$

The real part of this propagation constant is given by:

$$\beta_{nm} = k \left[1 - \frac{1}{2} (u_{nm}/ka)^2 \right] \tag{4}$$

for TM_{nm} modes,

$$\beta'_{nm} = k \left[1 - \frac{1}{2} (u'_{nm}/ka)^2 \right] \tag{5}$$

for TE_{nm} modes,

where a is the radius of the circular wave guide, u_{nm} is the m th root of the equation:

$$J_n(x) = 0, \tag{6}$$

and u'_{nm} the m th root of the equation

$$\frac{d}{dx} J_n(x) = 0, \tag{7}$$

with J_n the Bessel function of order n .

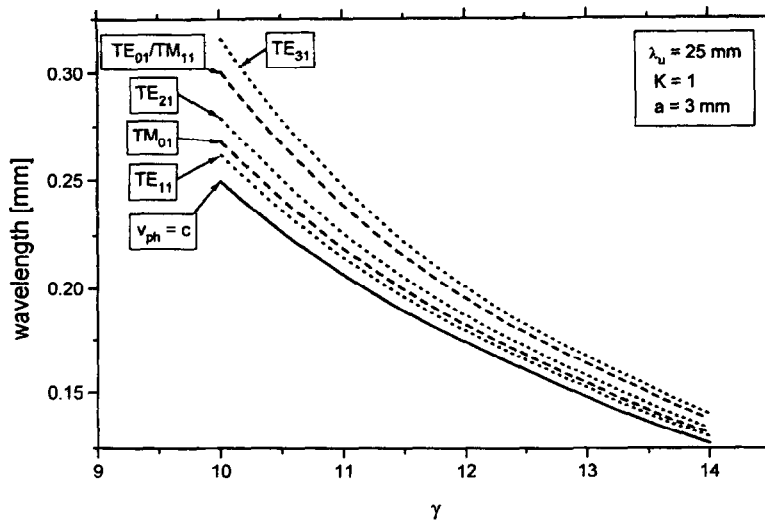


Fig. 1. The FEL resonance wavelength as a function of the relativistic factor of the electrons γ for a number of wave-guide modes.

From Eqs. (4) and (5) an expression can be found for the phase velocity of the beam:

$$v_{\text{ph}} = \omega / \beta_{nm}. \quad (8)$$

It can be derived that for wave guide modes the following relation holds:

$$v_{\text{gr}} v_{\text{ph}} = \omega^2 / k^2. \quad (9)$$

As can be seen both group and phase velocity depend on the mode pattern. Since the phase velocity in a wave guide is always larger than c the resonance wavelength is longer than the plane-wave value. Higher-order modes have longer wavelengths than lower-order modes. If the radiation is coupled out by means of a central hole in the mirror higher-order modes will dominate the mode pattern giving a large shift in the phase velocity and consequently in the wavelength.

To have an idea of the magnitude of the effect, in Fig. 1 a plot has been made of the FEL resonance wavelength as a function of the relativistic factor of the electrons for a number of wave-guide modes. For this plot a wave-guide radius of 3 mm, an undulator wavelength of 25 mm and an undulator K -parameter of unity has been used.

3. Experimental verification of the FEL resonance condition

At Twente University a Compton FEL has been built [6]. The system consists of a 6 MeV photocathode RF linac, which can produce a very bright electron beam. This beam consists of a 10 μs long train of 20 ps pulses. The repetition frequency of those pulses f_{eb} is 81.25 Mhz, which is the 16th subharmonic of the 1.3 GHz RF frequency of the linac. The beam line of the system has an inner tube diameter of 25 mm. In this beam line several diagnostic elements like current monitors, button monitors and viewing screens are present. Two quadrupole triplets as well as the accelerator solenoid and a number of beam steering coils are used to focus the electron beam on the entrance of the undulator. The undulator [7] has 50 periods with a period length of 25 mm. The undulator has a gap of 8 mm and a peak field of 0.7 Tesla, which means a K -value of about unity. The beam line inside the undulator has an inner diameter of only 6 mm so that it will act as an overmoded wave guide for the radiation. After the undulator there is a tapered section in the vacuum tube which is 40 cm long and brings the 6 mm

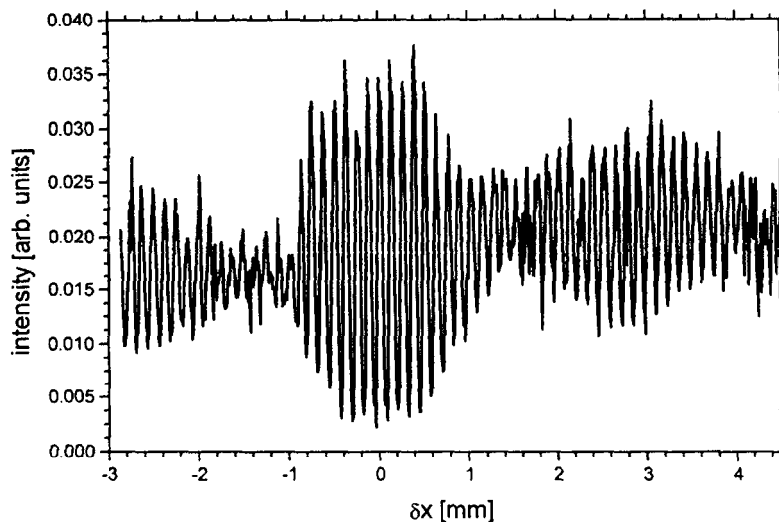


Fig. 2. A typical scan made with our Michelson interferometer. δx stands for the change in length of one of the interferometer arms. For $\delta x = 0$ both arms have equal length.

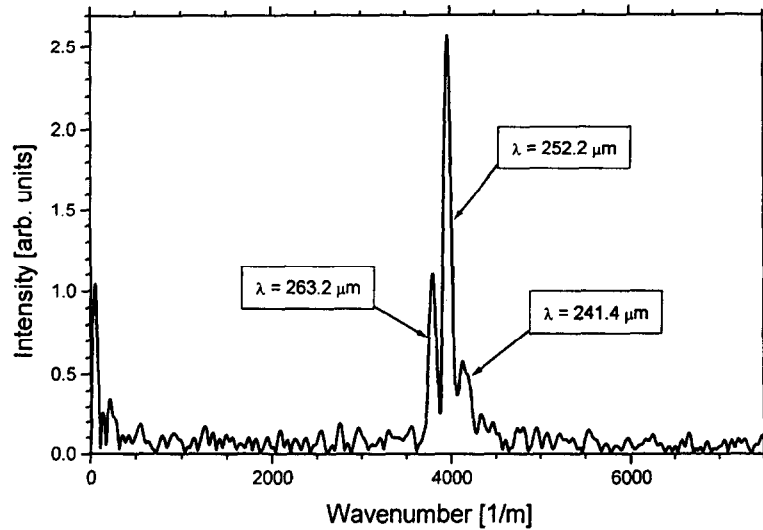


Fig. 3. The wavelength spectrum of the scan from Fig. 2 obtained by a Fourier transform.

diameter back to 25 mm. The optical cavity has a plane front mirror just before the undulator. It has a diameter of 6 mm with a 2 mm diameter hole in it through which the electron beam can enter the undulator section. The downstream mirror is also a plane one and has a diameter of 25 mm with a 12 mm hole in it. This hole is used by the electron beam to leave the resonator and for outcoupling of the radiation. The distance between the two mirrors L_{cav} is given by the repetition frequency of the electron beam pulses and the group velocity of the radiation:

$$L_{\text{cav}} \equiv v_{\text{gr}}/2f_{\text{eb}}. \quad (10)$$

The side walls of the resonator are formed by the vacuum tube, so that it can be considered as an overmoded wave guide with a tapered section in it. Further downstream the beam line there is a 90° spectrometer with a beam dump and a mirror at an angle of 45° with the beam line together with a window to couple the radiation out of the system.

The wavelength spectra of the radiation have been measured with a Michelson interferometer. An example of those measurements can be seen in Fig. 2, where the intensity in arbitrary units has been plotted against the change in length δx of one arm of the interferometer. For $\delta x = 0$ both arms have equal length. A Fourier analysis of this measurement is plotted in Fig. 3. In principle it is possible to calculate the resonance wavelength from a measurement

of the length of the optical cavity, which determines the group velocity, and Eqs. (9) and (2), and check this theoretical result with the measurements. However for our experimental circumstances the length of the optical cavity is not only determined by the group velocity of the radiation but also by the effect of slippage. Another complicating factor is the drastic change in mode pattern due to the outcoupling through a hole in the mirror. This is especially true for the TE_{1m} and TM_{1m} modes because they have the maximum intensity on the axis. So each mode will have a pattern determined by a certain mixture of the unperturbed wave-guide patterns. Less influenced by the outcoupling hole will be the phase difference between two successive modes. This phase difference will lead to a wavelength difference which can be compared with the experimental value. For instance it is possible to calculate the phase velocity of the four lowest order modes namely the TE_{11} , TM_{01} , TE_{21} , $\text{TE}_{01}/\text{TM}_{11}$ and TE_{31} modes from Eqs. (4) to (8). The roots of these modes respectively have the following values: 1.841, 2.405, 3.054, 3.832 and 4.201. This gives, assuming a radiation wavelength around 250 μm and a wave-guide radius of 3 mm, the following values for $\Delta\beta/\beta$: -0.00021 between the TE_{11} and TM_{01} modes, -0.00031 between the TM_{01} and TE_{21} modes, -0.00047 between the TE_{21} and $\text{TE}_{01}/\text{TM}_{11}$ modes and -0.00026 between the $\text{TE}_{01}/\text{TM}_{11}$ and TE_{31} modes.

Now from Eq. (2) it follows for the wavelength difference between the modes: $\Delta\lambda = -\lambda_u \Delta\beta/\beta = 5.3 \mu\text{m}$, $7.8 \mu\text{m}$, $11.8 \mu\text{m}$ and $6.5 \mu\text{m}$.

Comparing this result with the measurements from Fig. 3 the agreement is reasonably good.

4. Conclusions

We found experimentally that in a Compton FEL the wavelength difference between transverse modes can be as large as 5% whereas in normal lasers this difference is of the order of 0.1%. This can be explained by the fact that small changes in the phase velocity of the radiation, due to the transverse radiation profile in a wave guide, have a large influence on the resonance condition of an FEL and thus on the wavelength of the output radiation. This is especially important for high-quality electron beams because then wave-guide cavities can be used with a

small diameter. We found good agreement between the experimentally observed wavelength differences of successive modes and theoretical estimates.

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