Comment on "Surface-impedance approach solves problems with the thermal Casimir force between real metals"

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In a recent paper Geyer, Klimchitskaya, and Mostepanenko [Phys. Rev. A **67**, 062102 (2003)] proposed the final solution of the problem of temperature correction to the Casimir force between real metals. The basic idea was that one cannot use the dielectric permittivity in the frequency region where a real current may arise leading to Joule heating of the metal. Instead, the surface impedance approach is proposed as a solution of all contradictions. The purpose of this comment is to show that (i) the main idea contradicts to the fluctuation dissipation theorem, (ii) the proposed method to calculate the force gives the wrong value of the temperature correction since the contribution of low frequency fluctuations is calculated with the impedance which is not applicable at low frequencies.

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The problem of temperature dependence of the Casimir force between real metals gave rise to a vivid discussion in the literature [1–11] (see Ref. [11] for more extensive reference list). The Casimir force between two metallic plates is calculated with the help of the Lifshitz formula [12]. The optical properties of real materials enter into the formula via the reflection coefficients for two polarization states, which, in their turn, depend on the dielectric functions of the metals. For the first time the problem with the thermal force between ideal metals was discussed many years ago [13]. At finite temperature the Lifshitz formula seemed did not give the same result as that found with the different method [14]. For the ideal metals the problem was settled by introducing the rule to calculate the reflection coefficients in the low frequency limit: One has to take first the limit of infinite permittivity before allowing frequency go to zero. It is so called Schwinger, DeRaad, and Milton prescription. Application of the Lifshitz formula for real metals raised the problem again. The force calculation showed [1] that the transverse electric mode does not contribute to the force in the zero frequency limit. The reflection coefficients in this limit was found to be independent on the metal properties:

$$r_{\parallel}^2(0,q)=1, \quad r_{\perp}^2(0,q)=0, \tag{1}$$

where $\mathbf{q}$ is the wave vector along the plates and the indexes $\parallel$ and $\perp$ stand for parallel (transverse magnetic) and perpendicular (transverse electric) polarizations. The problem is that for the ideal metal both polarizations contribute equally, $r_{\parallel}^2=r_{\perp}^2=1$, and there is no continuous transition between real and ideal metals. Difference in the reflection coefficients provides difference in the temperature corrections to the Casimir force. For the ideal metal this correction is negligible at small separations between bodies but for real metals with the coefficients (1) it becomes large and disagrees with the torsion pendulum experiment [15].

Different research groups took part in the discussion of the problem but after a few years still there is no satisfactory solution. It can be a signal that we do not understand something important in good old electrodynamics and it makes the problem even more interesting.

In the commented paper [11] B. Geyer, G. Klimchtskaya, and V. Mostepanenko claim that they have found the final solution of the problem. Before going into details let us present the structure of the Lifshitz formula [12] that will be used in the discussion. The Casimir free energy per unit area for two metallic semispaces separated by the distance a is

$$F=\frac{kT}{2\pi}\sum_{n=0}^{\infty}{}'\int_0^{\infty}[\ln(1-r_{\parallel}^2(\zeta_n,q)\exp(-2k_0a))+(r_{\parallel}\to r_{\perp})]qdq, \tag{2}$$

where $\zeta_n=2\pi kTn/\hbar$ are the Matsubara frequencies. The reflection coefficients for two polarizations are defined via the dielectric function $\varepsilon(i\zeta_n)$ at imaginary frequencies in the following way:

$$r_{\parallel}=\frac{\varepsilon(i\zeta)k_0-k_m}{\varepsilon(i\zeta)k_0+k_m}, \quad r_{\perp}=\frac{k_m-k_0}{k_m+k_0}, \tag{3}$$

where $k_m=\sqrt{\zeta^2/c^2\varepsilon(i\zeta)+q^2}$ and $k_0=\sqrt{\zeta^2/c^2+q^2}$ are the normal to the plates components of the wave vector in metal and vacuum, respectively.

The driving idea of Ref. [11] is that the electromagnetic fluctuations resulting in Eq. (2) cannot be used with the Drude dielectric function

$$\varepsilon(\omega)=1-\frac{\omega_p^2}{\omega(\omega+i\omega_\tau)}, \tag{4}$$

where ω_p and ω_τ are the plasma and relaxation frequencies, respectively, because nonzero imaginary part of $\varepsilon(\omega)$ will give rise to a real current inside of metal and will produce heating of the metal.

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In this connection we would like to stress that any fluctuating physical value in the system which is in thermal equilibrium is connected with the dissipation via the fluctuation dissipation theorem. The energy for fluctuation is taken from the heat bath and returned back with the dissipation of fluctuation. No heat is produced in the process. The fluctuations of electromagnetic field are not an exception to this fundamental theorem of statistical physics. The simplest example is the Johnson-Nyquist noise. Quite real fluctuating in time current can be measured in a wire without application any external force. Spectral density of this current is proportional to the temperature and wire conductivity. Lifshitz has used the theory of electromagnetic fluctuations [16] to get his famous formula for the Casimir force. The fluctuation dissipation theorem lies in the heart of this theory. It is used to connect the correlators of the vector potential $A_i(i=1,2,3)$ with the retarded Green function D_{ij}^R inside the gap [12]

$$(A_i(\mathbf{r}_1)A_j(\mathbf{r}_2))_\omega = -\coth\frac{\hbar\omega}{2kT} Im D_{ij}^R(\omega;\mathbf{r}_1,\mathbf{r}_2), \tag{5}$$

where $(\ldots)_\omega$ means the Fourier component of the correlator. For the technical reason (faster convergence of integrals) it is more convenient to express the correlators via the thermal Green function $\mathcal{D}_{ij}$ using the relation

$$\mathcal{D}_{ij}(\zeta;\mathbf{r}_1,\mathbf{r}_2) = \mathcal{D}_{ij}^R(i|\zeta|;\mathbf{r}_1,\mathbf{r}_2). \tag{6}$$

That is why the force is defined by the material dielectric function at imaginary frequencies $\varepsilon(i\zeta)$. This function cannot be measured directly but can be expressed via the imaginary part of $\varepsilon(\omega)=\varepsilon'(\omega)+i\varepsilon''(\omega)$ using the dispersion relation

$$\varepsilon(i\zeta) = 1 + \frac{2}{\pi}\int_0^\infty \frac{\omega\varepsilon''(\omega)}{\omega^2+\zeta^2}d\omega. \tag{7}$$

In this sense one can say that the force itself is defined by the dissipation [12]. It is true even for the plasma model, for which ω_τ in (4) is going to zero. It follows from (4) that

$$\varepsilon''(\omega) = \frac{\omega_p^2\omega_\tau}{\omega(\omega^2+\omega_\tau^2)}. \tag{8}$$

If we put here $\omega_\tau=0$ the dispersion relations between $\varepsilon'(\omega)$ and $\varepsilon''(\omega)$ will be broken because $\varepsilon''=0$ but $\varepsilon'=1-\omega_p^2/\omega^2$. We have to remember that ω_τ is finite but can keep it arbitrary small. Using the fact that in the limit $\omega_\tau\to 0$ the right hand side of (8) is proportional to the δ-function one can write formally the expression for $\varepsilon_p''(\omega)$ in the plasma model

$$\varepsilon_p''(\omega) = \pi\frac{\omega_p^2}{\omega}\delta(\omega). \tag{9}$$

The fact that $\varepsilon_p''(\omega)$ is nonzero means nothing else than finite dissipation in the plasma model too. It is also known that for the fluctuations in a lossless medium [12] dissipation has to be kept arbitrary small but finite.

We can conclude that the main idea of the commented paper contradicts to the physical nature of fluctuations. Without dissipation any fluctuation once appeared will persist forever and, therefore, thermodynamical equilibrium will not be possible.

However, the surface impedance approach the authors consider as an alternative has its right to exist. It has been already discussed in a number of papers [9,17,18] and it is definitely preferable at least in the range of anomalous skin effect [9]. In this approach the optical properties of a metal are described with the surface impedance $Z(\omega)$ instead of $\varepsilon(\omega)$. The boundary conditions connect the tangential components of electric $\mathbf{E}_t$ and magnetic $\mathbf{H}_t$ fields on the surface. For this reason the electromagnetic problem is much simpler to solve since there is no need to solve it inside of metal. Of course, in this case the Casimir force is also connected with the dissipation of surface current that authors seem did not notice. This current is defined as

$$\mathbf{j}_S = [\mathbf{H}_t\ \mathbf{n}], \tag{10}$$

where $\mathbf{n}$ is the normal unit vector directed inside of metal. Via the impedance boundary condition this current is connected with the surface electric field

$$\mathbf{j}_S = \frac{\mathbf{E}_t}{Z(\omega)}. \tag{11}$$

The real part of $Z(\omega)$ is responsible for dissipation.

The Lifshitz approach is applicable in this case also. The fluctuation dissipation theorem expressed by the relation (5) refers to the fields and the Green function inside the gap. The metal properties enter in this relation via the boundary conditions on the Green function. Therefore, the change in the boundary condition does not influence the relation (5) but changes the Green function and, as a result, the reflection coefficients. In the Lifshitz approach it is straightforward to show that the reflection coefficients expressed via the impedance will be

$$r_\parallel = \frac{k_0 - \dfrac{\zeta}{c}Z(i\zeta)}{k_0 + \dfrac{\zeta}{c}Z(i\zeta)}, \quad r_\perp = \frac{\dfrac{\zeta}{c} - k_0 Z(i\zeta)}{\dfrac{\zeta}{c} + k_0 Z(i\zeta)}. \tag{12}$$

The expression for the Casimir free energy (2) via the reflection coefficients is not changed at all.

To calculate the Casimir force, the authors are using the Lifshitz formula (2) with the reflection coefficients expressed via the impedance at imaginary frequencies $Z(i\zeta)$ (see Eq. (41), Ref. [11]), which coincide with (12). Unfortunately, the way they perform the calculations also cannot be considered as satisfactory. The function $Z(\omega)$ can be represented in different explicit forms depending on the frequency range. The authors separate the domains of normal skin effect, anomalous skin effect, and domain of infrared optics. To make calculations they use these pieces of the same function in the following peculiar way: " … at each separation distance between the plates one should, first, determine the characteristic frequency ω_c and, second, fix the proper impedance function. Thereafter the chosen impedance function can be used at all frequencies when performing the integration in Eq. (48). At zero temperature this prescription is optional. At $T \neq 0$, however, it takes on great significance." The characteristic frequency ω_c mentioned here is defined as $\omega_c=c/2a$, where a is the separation between plates, Eq. (48) gives the Casimir energy [free energy (2) in this text at $T \to 0$].

At this point we have to make a comment. The surface impedance knows nothing about the problem in which we are going to use it. It is one function pieces of which we know in explicit analytic form for different frequency domains. It depends only on the material properties. At any circumstances this function cannot change in dependence on distance between plates. One has to use it as a whole for the force calculation.

It is easy to understand what will happen if we proceed as the authors of the commented paper recommend. In the normal skin effect and infrared optics domains, where nonlocal effects are not important, the impedance [19] can be represented as

$$Z(i\zeta) = 1/\sqrt{\varepsilon(i\zeta)}, \tag{13}$$

with $\varepsilon(i\zeta)$ given by (4). Suppose ω_c corresponds to the infrared optics domain so that $\omega_c \gg \omega_\tau$. When $\zeta \gg \omega_\tau$ the impedance of the plasma model follows from (13)

$$Z(i\zeta) = \frac{\zeta}{\sqrt{\omega_p^2 + \zeta^2}}. \tag{14}$$

The frequencies $\zeta_n = 2\pi kTn/\hbar$ around ω_c give the main contribution in the Casimir force but not in the temperature correction under discussion. It is clear already from the fact that the first term $n=0$ in (2) corresponding to $\zeta \to 0$ is the reason of controversies with the temperature correction. However, the authors continue the function (14) to small frequencies $\zeta_n < \omega_\tau$, where Eq. (14) does not work any more. This procedure is equivalent to a definite prescription for the $n=0$ term in the Lifshitz formula (2). Really, the reflection coefficients at $\zeta=0$ with the impedance given by (14) are (see Eq. (58), Ref. [11])

$$r_\parallel^2(0,q) = 1, \quad r_\perp^2(0,q) = \left(\frac{\omega_p - cq}{\omega_p + cq}\right)^2. \tag{15}$$

Since (14) is the impedance of the plasma model, the coefficients (15) reproduce the prescription of the plasma model [3]. Negligible temperature correction at small separations between bodies is predicted with the coefficients (15) [3].

The right way to calculate the Casimir force is straightforward and obvious. As an example we will consider gold with the parameters $\omega_p = 1.37 \cdot 10^{16}$rad/s, $\omega_\tau = 5 \cdot 10^{13}$rad/s, and $v_F = 1.4 \cdot 10^8$cm/s, where v_F is the Fermi velocity. The contribution of fluctuations with a frequency ζ_n must be taken into account with the impedance $Z(i\zeta_n)$ (13) at a given frequency ζ_n. Suppose the separation between bodies is $a = 150$ nm. Then $\omega_c = 1 \cdot 10^{15}$rad/s is well in the infrared optics domain. At room temperature $T = 300$ K the Matsubara frequency quant is $\omega_T = 2\pi kT/\hbar \approx 2.5 \cdot 10^{14}$rad/s. Nonlocal effects at room temperature are very small so we can use the impedance (13) for all frequencies. For all terms $n \neq 0$ the impedance of the plasma model is really not bad approximation since $\omega_T > \omega_\tau$. But for the first term in (2), which correspond to $n=0$, the frequency $\zeta_0 \to 0$ falls out of the applicability range of the plasma model. If we do not make any prescription about the $n=0$ term (the authors [11] claim they do not need any *ad hoc* prescription) this term must be calculated with the impedance of the normal skin effect

$$Z(i\zeta) = \frac{\sqrt{\zeta\omega_\tau}}{\omega_p}, \tag{16}$$

which follows from (13) at $\zeta \ll \omega_\tau$. Actually we have to put here $\zeta_0 = 0$ to find $Z(\zeta_0) = 0$. It means the ideal reflection and instead of (15) for the reflection coefficients at $\zeta = 0$ we will find

$$r_\parallel^2(0,q) = 1, \quad r_\perp^2(0,q) = 1. \tag{17}$$

These coefficients coincide with that for the ideal metal. Temperature correction in this case will be small but not negligible [2].

The essence of the problem with the temperature correction for real metals is the value of the coefficient $r_\perp^2(0,q)$. Together with M. Lokhanin the author of this comment provided a number of physical arguments [2,9] in favor of (17). However, as was emphasized in the conclusion of Ref. [9], we still unsure that the impedance approach gives the final solution. At room temperature the normal skin effect is applicable at low frequencies and it is unclear why the impedance boundary conditions should be preferable in comparison with the conditions of continuity of tangential components of electric and magnetic fields on the surface.

Our last comment is related to the range of anomalous skin effect. A good guiding principle to deal with the temperature correction was proposed in Ref. [8]. Any physically reasonable result must obey the Nernst heat theorem: the entropy must disappear in the zero temperature limit $T \to 0$. At low temperatures and frequencies most of good metals are in the range of anomalous skin effect. This is because the relaxation frequency $\omega_\tau(T)$ decreases fast with temperature and the mean free path for electrons becomes the largest length scale at frequencies $\zeta < v_F \omega_p / c$. In our paper [9] it was noted for the first time that to check the Nernst theorem one has to use the impedance of anomalous skin effect for calculations. We also demonstrated that the entropy is going to zero in the limit $T \to 0$ only if the reflection coefficients are chosen as in (17). Additionally it was shown that the relative temperature correction to the free energy is not negligibly small.

The authors of commented paper came to a different conclusion that the temperature correction in the anomalous skin effect domain is very small. They connected disagreement with the improper range of application for the anomalous skin effect in our paper. In reality the difference comes again from the way the they calculated the Casimir force. At small separations $a = 100 - 500$ nm considered in our paper the characteristic frequency in the gap between plates is in the range of infrared optics $\omega_c = (0.3 - 1.5) \cdot 10^{15}$rad/s. For definiteness we will take the value $a = 150$ nm or $\omega_c = 1 \cdot 10^{15}$rad/s. The main contribution in the force comes again from the impedance of infrared optics domain (14). However, the situation is different from that at room temperature. Let us consider as an example the temperature $T = 4$ K. In this case the Matsubara frequency quant $\omega_T = 2\pi kT/\hbar \approx 3.3 \cdot 10^{12}$rad/s is much smaller than that at room temperature. For this reason the terms in the Lifshitz formula defining the force magnitude correspond to large $n \sim \hbar\omega_c/2\pi kT \approx 300 \gg 1$. However, much smaller n give the main contribution in the temperature correction. As we already saw the contribution in the correction comes from the first terms in the Lifshitz formula. It was a point of special concern in Ref. [9] [see discussion after Eqs. (16) and (19) there] to demonstrate that the main contribution in the correction is defined by the low frequency range $\zeta < \Omega = v_F\omega_p/c$, where $\Omega \approx 6.4 \cdot 10^{13}$rad/s is the characteristic frequency of anomalous skin effect. It means that the temperature correction is defined by the first $n < \hbar\Omega/2\pi kT \approx 20$ terms in the Lifshitz formula. All these terms are in the range of anomalous skin effect and, therefore, can be calculated with the impedance [20,21]

$$Z(i\zeta) = \frac{4}{3\sqrt{3}}\left(\frac{4}{3\pi}\frac{v_F}{c}\frac{\zeta^2}{\omega_p^2}\right)^{1/3}. \tag{18}$$

Instead of using (18) the authors [11] continued Eq. (14) to the frequencies $\zeta_n < \Omega$ where it is not applicable. Since (14) corresponds to the plasma model and it is known that the temperature correction in this model is very small, they got their result. However, this result has no relation with reality.

In conclusion, we see that the original claim [11] for the final solution of the problem with the temperature correction cannot be supported. The inconsistent way to calculate the Casimir force did not allow to derive the right result for the temperature correction. Actually the authors introduced a specific prescription from the very beginning.

However, the commented paper so as the previous works [9,18] draw our attention to the surface impedance approach. At low temperatures and low frequencies it is the only way to describe a metal. At higher temperatures and low frequencies it seems that both descriptions with the dielectric function and surface impedance are equally right but they give completely different results for the temperature correction. The impedance approach is in good correspondence with the ideal metal, but the usual boundary conditions do not give smooth transition to the ideal metal. As was pointed out in Ref. [10], the latter is a natural conclusion since in the static limit magnetic field penetrates freely into a real metal but does not penetrate into the ideal metal. We still do not know the solution of this problem.

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