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Josephson current between p-wave superconductors

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Abstract

Josephson current in p-wave superconductor/diffusive normal metal (DN)/p-wave superconductor junctions is calculated by solving the Usadel equation under the Nazarov's boundary condition extended to unconventional superconductors by changing the heights of the insulating barriers at the interfaces, the magnitudes of the resistance in DN, and the angles between the normal to the interface and the lobe directions of p-wave pair potentials. It is shown that the magnitude of the Josephson current strongly depends on the lobe directions of the p-wave pair potentials and the resulting magnitude of the Josephson current is large compared to that in the s-wave superconducting junctions due to the formation of the resonant states peculiar to p-wave superconductors.

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1. Introduction

Since the discovery of Josephson effect [1] in superconductor/insulator/superconductor (SIS) junctions, it has been studied in many situations [2]. In SIS or superconductor/diffusive normal metal/superconductor (S/DN/S) junctions the critical current increases monotonically with decreasing temperature [3–5]. In S/DN/S junctions, Josephson current flows by Cooper pairs tunneling through the DN. This phenomenon is interpreted as a result of proximity effect. On the other hand in d-wave superconductor/insulator/d-wave superconductor (DID) junctions, nonmonotonous dependence of critical current on temperature [6,7] occurs due to the formation of midgap Andreev resonant states (MARS) at the interface [8].

In order to study how proximity effect and MARS work together, Tanaka et al. have recently extended the circuit theory [9] to the systems with unconventional superconductors [10,11], which requires a conservation of matrix current instead of current. This enables us to use the generalized Kirchhoff's rules in unconventional superconducting junctions. In addition, this theory gives the boundary condition for the Usadel equation [12] which is widely used in diffusive superconducting junctions. Application of this theory to the DN/p-wave superconductor junctions has shown that the formation of the MARS strongly enhances the proximity effect in DN [11]. As a result, the zero-energy peak in the density of states and a giant zero bias conductance peak appear. Although this boundary condition is very general, the situation where the Josephson current is flowing is not considered. Since Josephson effect is one of the most important features in the physics of superconductivity, a theory for diffusive unconventional superconducting junctions in the presence of the Josephson current is interesting not only from the viewpoint of fundamental

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physics but also from that of future technologies, e.g., quantum computing.

In the present paper we extend the theory in [11] so that we can calculate Josephson current in p-wave superconductor/diffusive normal metal/p-wave superconductor (P/DN/P) junctions, solving the Usadel equation under the new boundary condition. This makes it possible to study the influence of the proximity effect and the formation of the MARS on Josephson current simultaneously. As a result we find a strong enhancement of the Josephson current.

2. Formulation

We consider a junction consisting of p-wave superconducting reservoirs (P) connected by a quasi-one-dimensional diffusive conductor (DN) with a resistance R_{d} and a length L much larger than the mean free path. The DN/P interface located at $x=0$ has the resistance R'_{b}, while the DN/P interface located at $x=L$ has the resistance R_{b}. We model infinitely narrow insulating barriers by the delta function model as $U(x)=H\delta(x-L)+H'\delta(x)$. The resulting transparencies of the junctions T_{m} and T'_{m} are given by $T_{\mathrm{m}}=4\cos^2\phi/(4\cos^2\phi+Z^2)$ and $T'_{\mathrm{m}}=4\cos^2\phi/(4\cos^2\phi+Z'^2)$, where $Z=2H/v_{\mathrm{F}}$ and $Z'=2H'/v_{\mathrm{F}}$ are dimensionless constants and ϕ is the injection angle measured from the interface normal to the junction and v_{F} is Fermi velocity.

We parametrize the quasiclassical Green's functions G and F by a function Φ:

$$G_\omega=\frac{\omega}{\sqrt{\omega^2+\Phi_\omega\Phi^*_{-\omega}}},\quad F_\omega=\frac{\Phi_\omega}{\sqrt{\omega^2+\Phi_\omega\Phi^*_{-\omega}}}$$

with Matsubara frequency ω. Then Usadel equation reads [12]

$$\xi^2\frac{\pi T_{\mathrm{C}}}{\omega G_\omega}\frac{\partial}{\partial x}\left(G^2_\omega\frac{\partial}{\partial x}\Phi_\omega\right)-\Phi_\omega=0$$

with $\xi=\sqrt{D/2\pi T_{\mathrm{C}}}$, diffusion constant D and critical temperature T_{C}. To derive a boundary condition we have to calculate the matrix current as in Ref. [10]. We consider the case of the interfaces with low transparencies ($T_{\mathrm{m}}\ll 1$ and $T'_{\mathrm{m}}\ll 1$) for simplicity. In this case the boundary conditions are given by [13]

$$\frac{G_\omega}{\omega}\frac{\partial}{\partial x}\Phi_\omega=-\frac{R_{\mathrm{d}}}{R'_{\mathrm{b}}L}\left(-\frac{\Phi_\omega}{\omega}I'_1+\mathrm{e}^{-\mathrm{i}\varphi}(I'_2-\mathrm{i}I'_3)\right)$$

$$I'_1=\langle T'_{\mathrm{m}}g'_{\mathrm{S}}\rangle',\quad I'_2=\langle T'_{\mathrm{m}}\bar{f}'_{\mathrm{S}}\rangle',\quad I'_3=\langle T'_{m}f'_{\mathrm{S}}\rangle',$$

$$g'_{\mathrm{S}}=\frac{g_++g_-}{1+g_+g_-+f_+f_-},\quad \bar{f}'_{\mathrm{S}}=\frac{f_++f_-}{1+g_+g_-+f_+f_-},$$

$$f'_{\mathrm{S}}=\frac{\mathrm{i}(f_+g_--f_-g_+)}{1+g_+g_-+f_+f_-},\quad g_\pm=\frac{\omega}{\sqrt{\omega^2+\Delta^2_\pm}},$$

$$f_\pm=\frac{\Delta_\pm}{\sqrt{\omega^2+\Delta^2_\pm}},$$

with pair potentials $\Delta_\pm=\pm\Delta(T)\cos(\phi\mp\alpha)$ at a temperature T and $x=0$, and

$$\frac{G_\omega}{\omega}\frac{\partial}{\partial x}\Phi_\omega=\frac{R_{\mathrm{d}}}{R_{\mathrm{b}}L}\left(-\frac{\Phi_\omega}{\omega}I_1+(I_2+\mathrm{i}I_3)\right)$$

$$I_1=\langle T_{\mathrm{m}}g_{\mathrm{S}}\rangle I_2=\langle T_{\mathrm{m}}\bar{f}_{\mathrm{S}}\rangle I_3=\langle T_{\mathrm{m}}f_{\mathrm{S}}\rangle$$

at $x=L$ with $g_{\mathrm{S}},\bar{f}_{\mathrm{S}}$ and f_{S} defined as those with removing $'$, exchanging '+' for '−', putting $\varphi=0$ and substituting β into α in I'_1, I'_2 and I'_3 respectively. Here φ is the external phase difference across the junctions, and α and β denote the angles between the normal to the interface and the lobe directions of p-wave pair potentials for $x\leqslant 0$ and $x\geqslant L$ respectively. This expression is very general because it is applicable to any unconventional superconductors with $S_z=0$ by replacing $\Delta_\pm$. Here S_z denotes the z-component of the total spin of a Cooper pair. The average over the various angles of injected particles at the interface is defined as

$$\langle B(\phi)\rangle^{(\prime)}=\frac{\int_{-\pi/2}^{\pi/2}\mathrm{d}\phi\cos\phi B(\phi)}{\int_{-\pi/2}^{\pi/2}\mathrm{d}\phi T^{(\prime)}(\phi)\cos\phi}\tag{1}$$

with $B(\phi)=B$ and $T^{(\prime)}(\phi)=T^{(\prime)}_{\mathrm{m}}$. It is important to note that the solution of the Usadel equation is invariant under the transformation $\alpha\to-\alpha$ or $\beta\to-\beta$. This is clear by manipulating the variable transformation $\phi\to-\phi$ in the angular averaging. The resistance of the interface $R^{(\prime)}_{\mathrm{b}}$ is given by

$$R^{(\prime)}_{\mathrm{b}}=R^{(\prime)}_0\frac{2}{\int_{-\pi/2}^{\pi/2}\mathrm{d}\phi T^{(\prime)}(\phi)\cos\phi}.\tag{2}$$

Here, for example, $R^{(\prime)}_{\mathrm{b}}$ denotes R_{b} or R'_{b}, and $R^{(\prime)}_0$ is Sharvin resistance, which in three-dimensional case is given by $R^{(\prime)-1}_0=\mathrm{e}^2k^2_{\mathrm{F}}S^{(\prime)}_{\mathrm{c}}/(4\pi^2)$, where k_{F} is the Fermi wave-vector and $S^{(\prime)}_{\mathrm{c}}$ is the constriction area. Josephson current is given by the expression

$$\frac{eIR}{\pi T_{\mathrm{C}}}=\mathrm{i}\frac{RTL}{2R_{\mathrm{d}}T_{\mathrm{C}}}\sum_\omega\frac{G^2_\omega}{\omega^2}\left(\Phi_\omega\frac{\partial}{\partial x}\Phi^*_{-\omega}-\Phi^*_{-\omega}\frac{\partial}{\partial x}\Phi_\omega\right)$$

with $R\equiv R_{\mathrm{d}}+R_{\mathrm{b}}+R'_{\mathrm{b}}$. Below we will fix the barrier transparency parameters $Z=Z'=10$ so that $T_{\mathrm{m}}\ll 1$ and $T'_{\mathrm{m}}\ll 1$ are satisfied.

3. Results

Let us first study the current-phase relation. Fig. 1 shows the current-phase relation for $T/T_{\mathrm{C}}=0.1$ and $E_{\mathrm{Th}}/\Delta(0)=1$ with various α and β at $R_{\mathrm{d}}/R_{\mathrm{b}}=R_{\mathrm{d}}/R'_{\mathrm{b}}=0.1$ in (a) and $R_{\mathrm{d}}/R_{\mathrm{b}}=R_{\mathrm{d}}/R'_{\mathrm{b}}=1$ in (b). In this case current-phase relation has a sinusoidal form. Josephson current is suppressed as $\alpha\to\pi/2$ or $\beta\to\pi/2$ because of the suppression of the proximity effect [11]. As a reference, we also plot the current-phase relation for s-wave superconductor with the same parameters. Although line nodes exist for p-wave superconductors, the magnitude of Josephson current for s-wave superconductors is small compared to that of p-wave superconductors. This stems from the formation of the resonant states peculiar to p-wave superconductors. Note that for $\alpha=\pi/2$ or $\beta=\pi/2$, the resulting Josephson

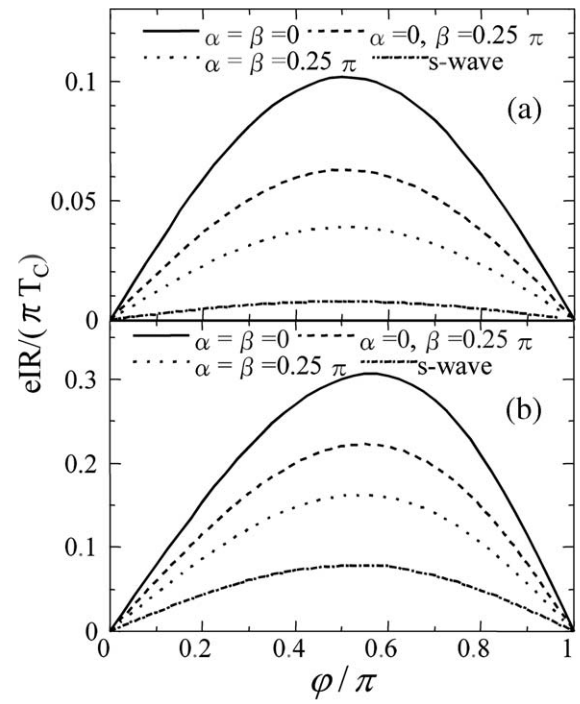

Fig. 1. Current-phase relation with $T/T_C = 0.1$ and $E_{Th}/\Delta(0) = 0.1$. (a) $R_d/R_b = R_d/R'_b = 0.1$ and (b) $R_d/R_b = R_d/R'_b = 1$.

current vanishes. But this does not mean that Josephson current can not flow because our results give an ensemble-averaged value and each ensemble has a nonzero value because of the fluctuation [14].

The corresponding plot for the critical current is shown in Fig. 2 where I_C denotes the critical current. The critical current is suppressed as $\alpha \to \pi/2$ or $\beta \to \pi/2$. The magnitude of Josephson current for p-wave superconductors is large compared to that of s-wave superconductors at low temperatures. The difference becomes clear as decreasing temperature. This is because the formation of the resonant states becomes pronounced with decreasing temperature.

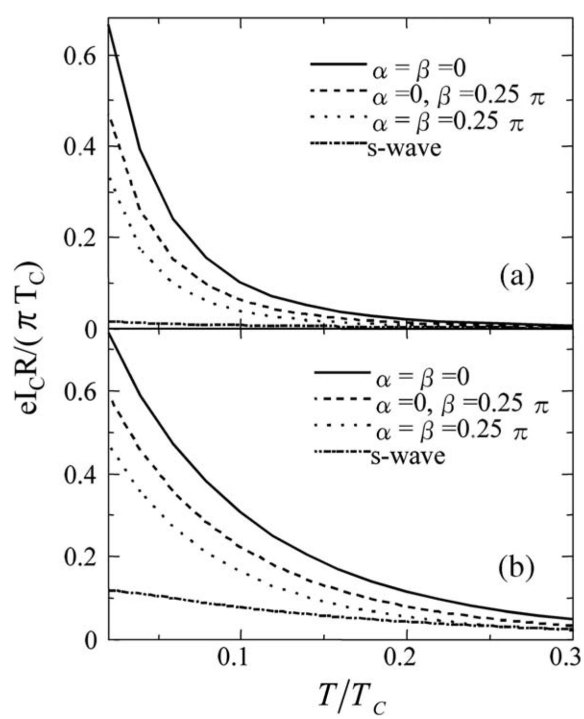

Fig. 2. Critical current with $E_{Th}/\Delta(0) = 0.1$. (a) $R_d/R_b = R_d/R'_b = 0.1$ and (b) $R_d/R_b = R_d/R'_b = 1$.

4. Conclusions

In the present paper, we have studied the Josephson current in p-wave superconductor/diffusive normal metal/p-wave superconductor junctions by solving the Usadel equation under the Nazarov's boundary condition extended to unconventional superconductors by changing the heights of the insulating barriers at the interfaces, the magnitudes of the resistance in DN and the angles between the normal to the interface and the lobe directions of p-wave pair potentials. It is shown that Josephson current strongly depends on the lobe directions of the p-wave pair potentials and the magnitude of the Josephson current is large compared to the s-wave superconducting junctions due to the formation of the resonant states.

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