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Macroscopic quantum tunneling in π Josephson junctions with insulating ferromagnets and its application to phase qubits

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Abstract

We theoretically investigate macroscopic quantum tunneling (MQT) in a current-biased π junction with a superconductor (S) and an insulating ferromagnet (IF). By using the functional integral method and the instanton approximation, the influence of the quasiparticle dissipation on MQT is found to be very weak. This feature makes it possible to realize a quantum information devices with high coherency. We also discuss how to make use of S–IF–S π junctions in the construction of a phase qubit.

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1. Introduction

If two superconductors are weakly coupled via a thin insulating barrier, a direct current can flow even without bias voltage. The driving force of this supercurrent is the phase difference in the macroscopic wave function. The supercurrent I and the phase difference ϕ across the junction have a relation $I = I_C \sin\phi$ with $I_C > 0$ being the critical current. If the weak link consists of a thin metallic ferromagnet (MF) layer, the result can be a Josephson junction with a built-in phase difference of π. This is a consequence of the phase change of the order parameter induced in the MF by the proximity effect [1]. Superconductor (S)–MF–S Josephson junctions presenting a negative coupling or a negative I_C are usually called π junctions and such behavior has been observed experimentally [2–4].

Recently, *quiet* qubits consisting of a superconducting loop with a S–MF–S π junction have been proposed [5–7]. In the quiet qubits, a quantum two level system (qubits) is spontaneously generated and therefore it is expected to be robust to the decoherence by the fluctuation of the external magnetic field. From the viewpoint of quantum dissipation, however, the structure of S–MF–S junctions is inherently identical with S–N–S junctions (N is a normal nonmagnetic metal). Therefore a gapless quasiparticle excitation in the MF layer is inevitable. This feature gives a strong Ohmic dissipation [8] and the coherence time of S–MF–S quiet qubits is bound to be very short. On the other hand, as was shown by Tanaka and Kashiwaya, [9] the π junction can also be formed in Josephson junctions with an insulating ferromagnet insulator (IF). In S–IF–S junctions, the quasiparticle excitation in the IF is expected to be very weak as in the case of S–I–S junctions [10] (I is a nonmagnetic insulator).

In this paper, we will investigate the influence of the quasiparticle dissipation on the macroscopic quantum tunneling (MQT) in S–IF–S π junctions in order to understand the quasiparticle dissipation mechanism in these system. By using the functional integral method, we will show that the quasiparticle dissipation in these systems

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is considerably weaker than in S–MF–S junctions. We will also propose a phase qubit using a current-biased S–IF–S π junction.

2. Effective action

First, we will calculate the effective action for S–IF–S Josephson junctions which consist of two superconductors (L and R) and a thin IF barrier (Fig. 1). Examples of IFs include the *f*-electron systems EuX (X = O, S, and Se) [11], and rare-earth nitrides (e.g., GdN [12]). Multiferroic materials (MM) [13] can serve as an IF. The IF barrier can be described by a spin σ-dependent delta-function potential, $V_\sigma(\boldsymbol{r}) = \rho_\sigma V \delta(x)$, where $\rho_\uparrow = 1$ and $\rho_\downarrow = -1$ [9,14]. The spin dependence of V_σ is essential for the formation of π coupling.

By using the functional integral method [10,15–17], the partition function of S–IF–S Josephson junctions can be written as $Z = \int D\phi(\tau) \exp(-S_{\rm eff}[\phi]/\hbar)$, where the effective action $S_{\rm eff}$ is given by

$$S_{\rm eff}[\phi] = \int_0^{\hbar\beta} d\tau \left[\frac{C}{2} \left(\frac{\hbar}{2e} \frac{\partial \phi(\tau)}{\partial \tau} \right)^2 - E_J \cos\phi(\tau) \right] + S_\alpha[\phi], \tag{1}$$

$$S_\alpha[\phi] \equiv -\sum_\sigma \int_0^{\hbar\beta} d\tau\, d\tau'\, \alpha_\sigma(\tau - \tau')\, e^{i\rho_\sigma \frac{\phi(\tau)-\phi(\tau')}{2}}. \tag{2}$$

Here, the Josephson coupling energy $E_{\rm J} = (\hbar/2e) I_{\rm C}$ is given in terms of the anomalous Green's function in the left (right) superconductor $F_{\rm L(R)}$:

$$E_{\rm J} = \frac{2}{\hbar} \int_0^{\hbar\beta} d\tau \sum_{\boldsymbol{k},\boldsymbol{k}'} T_\downarrow^*(\boldsymbol{k},\boldsymbol{k}') T_\uparrow(\boldsymbol{k},\boldsymbol{k}') F_{\rm L}(\boldsymbol{k},\tau) F_{\rm R}(\boldsymbol{k}',-\tau). \tag{3}$$

In the high-barrier limit ($h \equiv mV/\hbar^2 k_{\rm F} \gg 1$), the tunneling matrix element is given by $T_\sigma(\boldsymbol{k},\boldsymbol{k}') = i\rho_\sigma k_x/(k_{\rm F} h)\delta_{k_y,k_y'}\delta_{k_z,k_z'}$, where m is the electron mass and $k_{\rm F}$ is the Fermi wave number. Then, we can obtain $E_{\rm J} \approx -\Delta_0 R_{\rm Q}/4\pi R_{\rm N} < 0$, where Δ_0 is the gap of S, $R_{\rm Q} = h/4e^2$ is the resistance quantum, and $R_{\rm N}$ is the normal state resistance of the junction. As expected, $E_{\rm J}$ becomes negative. Therefore, S–IF–S junctions can serve as π junctions similar to S–MF–S junctions.

The dissipation action S_α describes the tunneling of quasiparticles which is the origin of the quasiparticle dissipation. In Eq. (2), the dissipation kernel $\alpha_\sigma(\tau)$ is given by

$$\alpha_\sigma(\tau) = -\frac{2}{\hbar} \sum_{\boldsymbol{k},\boldsymbol{k}'} |T_\sigma(\boldsymbol{k},\boldsymbol{k}')|^2 G_{\rm L}(\boldsymbol{k},\tau) G_{\rm R}(\boldsymbol{k}',-\tau), \tag{4}$$

where $G_{\rm L(R)}$ is the diagonal component of the Nambu Green's function. In the high-barrier limit, we obtain $\alpha_\sigma(\tau) \approx (\Delta_0^2/4\pi^2 e^2 R_{\rm N}) K_1(\Delta_0|\tau|/\hbar)^2$, where K_1 is the modified Bessel function. For $|\tau| \gg \hbar/\Delta_0$ the dissipation kernel decays exponentially as a function of the imaginary time τ, i.e., $\alpha_\sigma(\tau) \sim \exp(-2\Delta_0|\tau|/\hbar)$. If the phase varies only slowly with the time scale given by $\hbar/\Delta_0$, we can expand $\phi(\tau) - \phi(\tau')$ in Eq. (2) about $\tau = \tau'$. This gives $S_\alpha[\phi] \approx (\delta C/2) \int_0^{\hbar\beta} d\tau [(\hbar/2e)\partial\phi(\tau)/\partial\tau]^2$, so the effect of the quasiparticles results in an increase of the capacitance, $C \to C + \delta C \equiv C_{\rm ren}$. This indicates that the quasiparticle dissipation in S–IF–S junctions is qualitatively weaker than that in S–MF–S junctions in which the strong Ohmic dissipation appears [8]. At the zero temperature, the capacitance increment δC is given by $\delta C = 3e^2 R_{\rm Q}/32\pi\Delta_0 R_{\rm N}$. As will be shown in next section, $\delta C/C \ll 1$. Therefore the effect of the quasiparticle dissipation on the macroscopic quantum dynamics of S–IF–S junctions is very small.

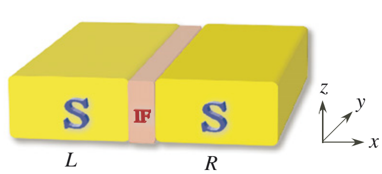

Fig. 1. (Color online) Schematic view of the superconductor–insulating ferromagnet–superconductor (S–IF–S) Josephson junction.

3. Theory of MQT

Next, we will develop a theory of MQT in single S–IF–S junctions (Fig. 1). In order to observe MQT, an external bias current $I_{\rm ext}$ which is close to $I_{\rm C}$ is applied to the junction. This leads to an additional term $-(\hbar/2e)\int_0^{\hbar\beta} d\tau\, I_{\rm ext}\phi(\tau)$ in Eq. (1) [10]. The resultant action $S_{\rm eff}[\phi] = \int_0^{\hbar\beta} d\tau [(C_{\rm ren}/2)\{(\hbar/2e)(\partial\phi(\tau)/\partial\tau)\}^2 - U(\phi)]$ describes the macroscopic quantum dynamics of a fictive particle (the phase difference ϕ) with mass $M = C_{\rm ren}(\hbar/2e)^2$ moving in the tilted washboard potential $U(\phi) = -E_{\rm J}[\cos\phi(\tau) - \eta\phi(\tau)]$, where $\eta \equiv I_{\rm ext}/|I_{\rm C}|$. The MQT escape rate from this metastable potential at the zero temperature is defined by $\Gamma = \lim_{\beta\to\infty}(2/\beta)\,{\rm Im}\ln Z$ [18]. By using the instanton approximation [19], the MQT rate can be approximated as $\Gamma(\eta) = [\omega_{\rm p}(\eta)/2\pi]\sqrt{120\pi B(\eta)}\,e^{-B(\eta)}$, where $\omega_{\rm p}(\eta) = \sqrt{\hbar I_{\rm C}/2eM}(1-\eta^2)^{1/4}$ is the Josephson plasma frequency and $B(\eta) = S_{\rm eff}[\phi_{\rm B}]/\hbar$ is the bounce exponent, that is, the value of the action $S_{\rm eff}$ evaluated along the bounce trajectory $\phi_{\rm B}(\tau)$. The analytic expression for the bounce exponent is given by $B(\eta) = (12/5e)\sqrt{(\hbar/2e) I_{\rm C} C_{\rm ren}}(1-\eta^2)^{5/4}$. In actual MQT experiments, the switching current distribution $P(\eta)$ is measured. $P(\eta)$ is related to the MQT rate $\Gamma(\eta)$ as $P(\eta) = v^{-1}\Gamma(\eta)\exp(-v^{-1}\int_0^\eta \Gamma(\eta')\,d\eta')$, where $v \equiv |d\eta/dt|$ is the sweep rate of the external bias current. The average value of the switching current is expressed by $\langle\eta\rangle \equiv \int_0^1 d\eta' P(\eta')\eta'$. At high-temperature regime, the thermally activated decay dominates the escape process. Below the crossover temperature T^*, the escape process is dominated by MQT. The crossover temperature T^* is defined by $T^* = \hbar\omega_p(\eta = \langle\eta\rangle)/2\pi k_B$ [20]. Importantly, T^* is depressed in the presence of a dissipation [19].

In order to check explicitly the effect of the quasiparticle dissipation on MQT, we numerically estimate T^*. Currently no experimental data are available for S–IF–S junctions. Therefore we estimate T^* by using the parameters for a high-quality $Nb/Al_2O_3/Nb$ junction [21] ($\Delta_0 = 1.30\,\mathrm{meV}$, $C = 1.61\,\mathrm{pF}$, $|I_C| = 320\,\mu\mathrm{A}$, $R_N = \Delta_0/4e|I_C|$, $v|I_C| = 0.245\,\mathrm{A/s}$). By using these data, we obtain $\delta C/C = 0.0145 \ll 1$. Then, we get the crossover temperature $T^* = 281\,\mathrm{mK}$ for the dissipationless case ($C_{ren} = C$) and $T^* = 280\,\mathrm{mK}$ for the dissipation case ($C_{ren} = C + \delta C$). We find that, due to the existence of the quasiparticle dissipation, T^* is reduced, but T^* reduction is negligibly small.

4. Proposal of π junction phase qubit

The results in previous section strongly indicate the high potentiality for the S–IF–S junctions as quantum information devices with high coherency. In this section, we will propose a phase qubit [22–24] using the S–IF–S π junctions. Note that the flux qubit using a superconducting loop with single S–IF–S π junction have been proposed recently [17,25].

The S–IF–S phase qubit is constructed from the S–IF–S π-junction with a fixed dc current source (Fig. 2). The ground state $|0\rangle$ and the first excited state $|1\rangle$ in a metastable potential are used as the qubit. Rabi oscillations between the stats $|0\rangle$ and $|1\rangle$ can be observed by irradiating the qubit with microwaves at a frequency ω_{01} and then measuring the occupation probability of being in the state $|1\rangle$. The measurement of the qubit state utilizes the escape from the cubic potential via MQT. To measure the occupation probability P_1 of state $|1\rangle$, we pulse microwaves at frequency ω_{12}, driving a $1 \to 2$ transition. The large tunneling rate $\Gamma_2$then causes state $|2\rangle$ to rapidly tunnel. After tunneling, the junction behaves as an open circuit, and a dc voltage of the order of the superconducting gap Δ_0 appears across the junction. Therefore, the occupation probability P_1 is equal to the probability of observing a voltage across the junction after the measurement pulse.

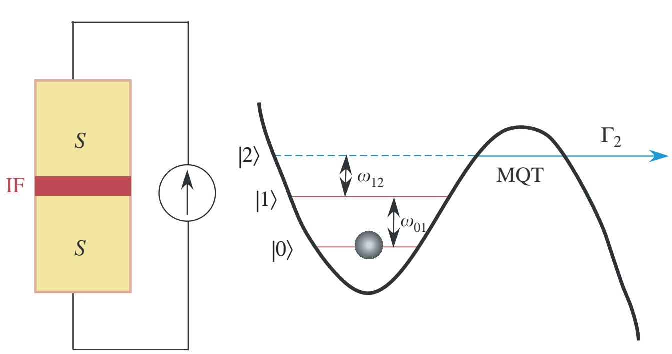

Fig. 2. (Color online) Schematic drawing of the phase qubit using the current-biased S–IF–S Josephson junctions.

The two-qubit gate (e.g., the controlled-NOT gate) can be performed by using the capacitive coupling between two adjacent qubits.

Finally, we will discuss the advantage of S–IF–S π-junctions as the phase qubit in comparison with usual S–I–S junctions. One of the possible example of IF is the MM which show both of the ferroelectricity and ferromagnetism [13]. In such materials, the magnetism (V in our model) can be artificially controlled by applying the static electric field. On the other hand, in order to realize the quantum computer with large number of phase qubits, we have to tune ω_p of each qubit independently. In usual S–I–S junctions, however, precise control of ω_p, e.g., by changing the thickness of the insulating barrier, is very difficult. In the case of S–IF–S junctions, we can artificially and precisely control the qubit parameter $\omega_p \propto \sqrt{I_C} \propto V^{-1}$ only by applying the electric field to the MM layer. Therefore, high controllability of qubit parameters is an advantage of S–IF–S π-junctions for realizing scalable quantum computers.

5. Summary

To summarize, we have theoretically investigated the effect of the quasiparticle dissipation on MQT, and showed that this effect is considerably smaller compared with S–MF–S junction cases. Moreover, we have proposed a phase qubit which consists of the single S–IF–S π junction. The weak quasiparticle-dissipation feature of this system will make it possible to realize the qubit with long coherence time.

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