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Constructing fair round robin tournaments with a minimum number of breaks

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ABSTRACT

Given n clubs with two teams each, we show that, if n is even, it is possible to construct a schedule for a single round robin tournament satisfying the following properties: the number of breaks is $2n-2$, teams of the same club never play at home simultaneously, and they play against each other in the first round. We also consider a fairness constraint related to different playing strengths of teams competing in the tournament.

1. Introduction and problem specification

Sports scheduling is a well-established and important area of operations research with numerous practical applications. Although schedules for certain simple tournaments can easily be generated using combinatorial methods, the problem of finding a schedule becomes very hard when it has to satisfy additional constraints. In practice, sports schedules are required to satisfy more and more constraints in order to meet the increasing demands of sports clubs and associations, supporters' organizations, TV networks, and local communities. We refer to the extensive survey by Rasmussen and Trick [5] for an overview of many of these constraints.

We consider sports leagues having a set of $2n$ teams. A *single round robin tournament* (SRRT) is a tournament where each team plays a match against every other team exactly once. The matches of an SRRT are divided into rounds in such a way that each team plays at most one match per round. Throughout this paper, we assume that each team plays exactly one match per round, which means that there are exactly $2n-1$ rounds; such an SRRT is called *compact* in [5]. A *timetable* for an SRRT is a table whose rows correspond to the teams and whose columns correspond to the rounds, such that the entry in row i and column j represents the opponent of team i in round j. A well-known method for generating a timetable for an SRRT is the so-called Circle Method, dating back to 1883 [4] (see also Section 2). Each match is carried out at the venue of one of the two opponents. A *home–away pattern* for a team is a sequence of length $2n-1$ specifying for each round whether the team plays at home or away. The home–away patterns for all $2n$ teams together constitute a *home–away pattern set*, determining the home team for each match of the SRRT. A *schedule* for an SRRT consists of a timetable and a corresponding home–away pattern set. We say that a team has a *break* in round k if the team plays either at home in rounds $k-1$ and k, or plays away in both these rounds. It is well known that each home–away pattern set of a single round robin tournament with $2n$ teams yields at least $2n-2$ breaks, and that a home–away pattern set with exactly $2n-2$ breaks exists for any timetable constructed using the Circle Method; see for example [3]. Two home–away patterns that are different in each round are called *complementary*. We say that two teams *play complementarily* if their home–away patterns are complementary.

Schedules for SRRTs with a minimum number of breaks have been studied by many different researchers: Rasmussen and Trick [5] devote an entire section of their survey to break minimization. Since playing away in several consecutive rounds is seen as a disadvantage, a schedule with as few breaks as possible is considered fairer than a schedule where more breaks occur. Moreover, such a schedule allows supporters to see a home game every other round, and this guarantees regular earnings from home games for the owner of the venue. Complementary constraints often appear in practice when designing schedules for sports leagues; again, we refer to [5] for many references. For example, if two football clubs both have a stadium in the same city, then it is desirable that they do not play at home simultaneously in order to avoid traffic problems and conflicts between supporters. Sometimes, for example

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when two football clubs share the same stadium, it is not only desirable but also essential that they do not both play at home in any round. A more extreme example, which forms the original motivation for this paper, is a local billiards league in the Netherlands. Due to the large number of teams competing in the league, the teams play each other exactly once per year in a single round robin tournament. The teams are associated to pubs. Although most pubs have more than one team competing in the league, they typically have only one billiards table. Since teams of the same pub share this single table, at most one of them can play at home in each round. For fairness reasons, it is required that teams of the same pub play against each other as early as possible in the tournament, so that a team cannot deliberately lose a game at the end of the tournament to give another team of the same pub an unfair advantage.

Motivated by the billiards league described above, we consider in Section 2 a set of n clubs, having exactly two teams and one venue each. As we mentioned before, any schedule for an SRRT with $2n$ teams contains at least $2n-2$ breaks. The question is whether a schedule with exactly $2n-2$ breaks exists if the schedule has to satisfy the following two conditions: teams of the same club do not play at home in the same round, and they play against each other in the first round. We prove that for every even n such a schedule indeed exists by presenting a method for constructing a schedule for an SRRT satisfying the following properties.

Property I. *The number of breaks equals* $2n-2$.

Property II. *The teams of the same club play complementarily.*

Property III. *The teams of the same club meet in the first round.*

Note that Property II implies that teams of the same club do not play at home in the same round. In fact, the reverse implication also holds. After all, in every round, n teams play at home and n teams play away. If both teams of a club play away in round k, then the n teams that play at home in round k must belong to the other $n-1$ clubs. This implies that there is at least one club, both teams of which play at home in round k. Hence demanding teams of the same club not to play at home in the same round is equivalent to demanding teams of the same club to play complementarily.

In Section 3, we consider another fairness constraint. It is considered unfair if one team plays against strong teams in several consecutive matches, whereas another team has a match against a weak team following each match against a strong team. In order to deal with fairness issues arising from teams of different playing strengths, Briskorn [2] introduced the concept of *strength groups*. The basic idea is that teams with equal or similar playing strengths are contained in the same strength group. If there are s different strength groups, each containing the same number of teams, then the goal is to find a schedule in which no team plays against two teams of the same strength group within any s consecutive rounds. An SRRT having such a schedule is called *group-balanced*. Briskorn [2] studies the case where $2n/s$ is an integer and each strength group contains exactly $2n/s$ teams. He proves that a group-balanced schedule exists if and only if both s and $2n/s$ are even. In Section 3, we consider the case where $2n$ teams are divided into n strength groups, each containing two teams. We present a method for constructing a schedule for a group-balanced SRRT with $2n-2$ breaks, for every even n.

Section 4 contains the conclusions and mentions some open problems.

2. Sports schedules with multiple teams per club

One of the oldest and easiest ways of constructing a timetable for an SRRT with $2n$ teams is the so-called Circle Method, described by Lucas [4] in 1883. SRRTs constructed using the Circle Method are also known as *Lucas leagues*; see for example [1]. The Circle Method can be presented in algebraic form as follows.

Circle Method

(a) For $i, j < 2n$ and $i \neq j$, the teams i and j play in round k if $i + j - 1 \equiv k (\mathrm{mod}\ 2n-1)$.
(b) For $i < 2n$, the teams i and $2n$ play in round k if $2i - 1 \equiv k (\mathrm{mod}\ 2n-1)$.

For the remainder of this section we consider the case of n clubs, where n is even. Each club has exactly two teams, and the teams are numbered from 1 to $2n$ in such a way that for $i < n$ the teams i and $i+n-1$ belong to the same club, and the same holds for the teams $2n-1$ and $2n$.

The purpose of this section is to prove the existence of a schedule for an SRRT satisfying Properties I, II and III, specified in Section 1, for every even n. We do this by an explicit construction of such a schedule. We first describe a method, closely resembling the Circle Method, for generating a timetable for an SRRT with $2n$ teams; we call this method the Adapted Circle Method. We then show how to construct a home–away pattern set that, together with the timetable constructed by the Adapted Circle Method, constitutes a schedule for an SRRT satisfying Properties I, II and III.

Adapted Circle Method

(a) In round 1, each team plays against the other team of the same club.
(b) For $i, j \leq 2n-2$, the teams i and j of different clubs play in round $k \geq 2$ if $i + j \equiv k (\mathrm{mod}\ 2n-2)$.

It remains to describe the matches involving the teams $2n-1$ and $2n$. It follows from rules (a) and (b) that for team i these must be played in round k, where either $k \equiv 2i (\mathrm{mod}\ 2n-2)$ or $k \equiv 2i + n - 1 (\mathrm{mod}\ 2n-2)$.

(c1) For $1 \leq i \leq \frac{1}{2}n$, team i plays against team

$$\begin{cases} 2n-1 & \text{in round } 2i+n-1 \\ 2n & \text{in round } 2i. \end{cases}$$

(c2) For $\frac{1}{2}n + 1 \leq i \leq n-1$, team i plays against team

$$\begin{cases} 2n-1 & \text{in round } 2i \\ 2n & \text{in round } 2i-(n-1). \end{cases}$$

(c3) For $n \leq i \leq \frac{3}{2}n - 1$, team i plays against team

$$\begin{cases} 2n-1 & \text{in round } 2i-2(n-1) \\ 2n & \text{in round } 2i-(n-1). \end{cases}$$

(c4) For $\frac{3}{2}n \leq i \leq 2n-2$, team i plays against team

$$\begin{cases} 2n-1 & \text{in round } 2i-3(n-1) \\ 2n & \text{in round } 2i-2(n-1). \end{cases}$$

It is not difficult to verify that every team i plays against each of the $2n-1$ other teams for $i \leq 2n-2$. For teams $2n-1$ and $2n$, the opponents follow from rules (c1)–(c4). Team $2n-1$ plays against team $2n$ in round 1, and plays against teams $1, 2, \ldots, 2n-2$ in rounds

$$\begin{aligned} &n+1, n+3, \ldots, 2n-3, 2n-1, \\ &n+2, n+4, \ldots, 2n-4, 2n-2, \\ &2, 4, \ldots, n-2, n, \\ &3, 5, \ldots, n-3, n-1, \end{aligned} \tag{1}$$

respectively. For team $2n$, the analysis is similar. We conclude that the Adapted Circle Method indeed yields a timetable for an SRRT with $2n$ teams. See Table 1 for an example of a timetable for 12 teams generated using the Adapted Circle Method.

Table 1
A schedule for an SRRT with 12 teams generated using the Adapted Circle Method. The grey entries form the band.

Round:	1	2	3	4	5	6	7	8	9	10	11
Team 1	6	−12	−2	3	−4	5	−11	7	−8	9	−10
Team 2	7	−10	1	−12	−3	4	−5	6	−11	8	−9
Team 3	8	−9	10	−1	2	−12	−4	5	−6	7	−11
Team 4	9	−8	12	−10	1	−2	3	11	−5	6	−7
Team 5	10	−7	8	−9	12	−1	2	−3	4	11	−6
Team 6	−1	11	7	−8	9	−10	12	−2	3	−4	5
Team 7	−2	5	−6	11	8	−9	10	−1	12	−3	4
Team 8	−3	4	−5	6	−7	11	9	−10	1	−2	12
Team 9	−4	3	−11	5	−6	7	−8	−12	10	−1	2
Team 10	−5	2	−3	4	−11	6	−7	8	−9	−12	1
Team 11	12	−6	9	−7	10	−8	1	−4	2	−5	3
Team 12	−11	1	−4	2	−5	3	−6	9	−7	10	−8

Note that in rule (b) of the Adapted Circle Method there is a shift of one round as compared to rule (a) of the Circle Method, due to the special first round. Also note that calculations are done modulo $2n-2$ in the Adapted Circle Method, as opposed to modulo $2n-1$ calculations in the Circle Method. This is due to the fact that the Circle Method has only one exceptional team, namely team $2n$ (sometimes called team ∞ in the literature), whereas the Adapted Circle Method has two exceptional teams, namely teams $2n-1$ and $2n$. We point out that the Adapted Circle Method cannot be applied when n is odd, since in that case the aforementioned rounds in (1) are not all different.

To define a home–away pattern set satisfying Properties I and II we introduce the *band*.

Definition 1. Given a timetable constructed using the Adapted Circle Method, the *band* consists of all pairs (i, k) of team $i \le 2n-2$ and round k, for which

$$\frac{1}{2}(k+\delta_k) \le i < \frac{1}{2}(k+\delta_k) + n - 1,$$

where

$$\delta_k = \begin{cases} 1 & \text{if } k \le n \\ 0 & \text{if } k > n. \end{cases}$$

The grey entries of the schedule in Table 1 form the band of the corresponding timetable for 12 teams, constructed using the Adapted Circle Method. Using the band, we construct a home–away pattern set as follows.

Lemma 1. *Given a timetable, with band B, constructed using the Adapted Circle Method, a home–away pattern set is obtained as follows. For $i \le 2n-2$, team i plays at home in round k if and only if one of the following holds:*

$$\begin{cases} (i, k) \in B & \text{and } k \text{ is odd} \\ (i, k) \notin B & \text{and } k \text{ is even}. \end{cases}$$

In round 1, team $2n-1$ plays at home against team $2n$. In every other round, teams $2n-1$ and $2n$ play complementarily to their opponents.

Proof. It is clear that each team i plays either at home or away in each round. Hence, in order to prove that the rules described in Lemma 1 define a proper home–away pattern set, it suffices to prove that, if two teams meet in round k, then one plays at home and the other plays away. For all matches involving teams $2n-1$ and $2n$, this immediately follows from the formulation of the lemma. So suppose now that teams i and j meet in round k, and assume without loss of generality that $i < j \le 2n-2$. We have to prove that exactly one of the pairs (i, k) and (j, k) is in the band. For $k = 1$, this is the case, since $j = i + n - 1$, which implies that $(i, 1)$ is in the band, and $(j, 1)$ is outside. For $k > 1$, it follows from rule (b) of the Adapted Circle Method that either $i + j = k$ or $i + j = k + 2n - 2$. We consider both cases below.

In the first case, $i < \frac{1}{2}k$, so $(i, k) \notin B$. On the other hand, $j > \frac{1}{2}k$ implies that $\frac{1}{2}(k+\delta_k) \le j$. To prove that $(j, k) \in B$, it remains to prove that $j < \frac{1}{2}(k+\delta_k) + n - 1$. For this, we note that $j < k$ and $k \le 2n-1$ imply that

$$\begin{aligned} j \le k-1 &= \frac{1}{2}k + \frac{1}{2}k - 1 \le \frac{1}{2}k + \frac{1}{2}(2n-1) - 1 \\ &= \frac{1}{2}k + n - \frac{3}{2} < \frac{1}{2}(k+\delta_k) + n - 1. \end{aligned}$$

Hence we have that $j < \frac{1}{2}(k+\delta_k) + n - 1$, and we have proved that $(j, k) \in B$.

In the second case, $j > \frac{1}{2}k + n - 1$, which implies that $j \ge \frac{1}{2}(k+\delta_k) + n - 1$, and hence $(j, k) \notin B$. At the same time, we have that $i < \frac{1}{2}k + n - 1$, so surely $i < \frac{1}{2}(k+\delta_k) + n - 1$. To prove that $(i, k) \in B$, it remains to prove that $i \ge \frac{1}{2}(k+\delta_k)$. To get a contradiction, we assume that $i < \frac{1}{2}(k+\delta_k)$. This implies that $i \le \frac{1}{2}k$. If $i = \frac{1}{2}k$ (implying that k is even), team i plays against team $2n-1$ or $2n$; cases we already discarded. If $i < \frac{1}{2}k$, then the opponent is $k-i$, implying that $j < k$. This is in contradiction with $j > \frac{1}{2}k + n - 1$. Hence $(i, k) \in B$. $\square$

Table 1 contains a schedule for an SRRT with 12 teams; the timetable has been constructed using the Adapted Circle Method, and the home–away pattern set is generated as described in Lemma 1. The entries belonging to the band are colored grey, and a positive (respectively negative) entry in row i and column k indicates that team i plays at home (respectively away) in round k. For other values of n a Java script is available: see http://wwwhome.math.utwente.nl/~postgf/RoundRobinWithTwoTeamsPerClub.html.

It is easy to verify that the schedule in Table 1 satisfies Properties I, II and III. We now prove that this is the case for every schedule constructed using the Adapted Circle Method and the home–away pattern set defined in Lemma 1.

Theorem 1. *For every even n there exists a single round robin tournament with $2n$ teams, satisfying Properties I, II and III specified in Section 1.*

Proof. We show that any timetable generated using the Adapted Circle Method, together with a home–away pattern set defined in Lemma 1, constitutes a schedule for an SRRT satisfying Properties I, II and III.

Table 2
A schedule for a group-balanced SRRT with 12 teams, obtained from the schedule in Table 1. The grey entries formed the band of the original schedule.

Round:	1	2	3	4	5	6	7	8	9	10	11
Team 1	−11	7	−8	9	−10	6	−12	−2	3	−4	5
Team 2	−5	6	−11	8	−9	7	−10	1	−12	−3	4
Team 3	−4	5	−6	7	−11	8	−9	10	−1	2	−12
Team 4	3	11	−5	6	−7	9	−8	12	−10	1	−2
Team 5	2	−3	4	11	−6	10	−7	8	−9	12	−1
Team 6	12	−2	3	−4	5	−1	11	7	−8	9	−10
Team 7	10	−1	12	−3	4	−2	5	−6	11	8	−9
Team 8	9	−10	1	−2	12	−3	4	−5	6	−7	11
Team 9	−8	−12	10	−1	2	−4	3	−11	5	−6	7
Team 10	−7	8	−9	−12	1	−5	2	−3	4	−11	6
Team 11	1	−4	2	−5	3	12	−6	9	−7	10	−8
Team 12	−6	9	−7	10	−8	−11	1	−4	2	−5	3

In order to prove Property I, we note that team $i \leq 2n-2$ does *not* have a break in round k, $2 \leq k \leq 2n$, if and only if the pairs $(i, k-1)$ and (i, k) are both inside or both outside the band. Since for each $i \leq 2n-2$ the transition from inside to outside the band – or vice versa – happens exactly once, each of those teams has exactly one break. For every round k, if team $2n-1$ plays against team i, then the pair (i, k) is outside B. Hence the home–away pattern for team $2n-1$ is home–away–home–$\cdots$–away–home, which means that team $2n-1$ does not have a break. Similarly, the pair (i, k) corresponding to opponent i of team $2n$ in round k is always inside B. Hence the home–away pattern of team $2n$ is complementary to that of team $2n-1$, which means that team $2n$ does not have a break either.

In order to prove Property II, we note that for every fixed round exactly $n-1$ "consecutive" teams belong to the band. Hence for $i < n$ the teams i and $i+n-1$ play complementarily, since for each round k either (i, k) or $(i+n-1, k)$ is inside the band. As noted in the first part of the proof, teams $2n-1$ and $2n$ play complementarily as well.

Property III follows directly from the construction of the first round. □

3. Group-balanced schedules

The schedule constructed using the Adapted Circle Method as described in Section 2 is much more structured than is required by Properties I, II and III. In fact, a closer look at the schedule in Table 1 and rules (b) and (c1)–(c4) of the Adapted Circle Method reveals that any such schedule also has the following property.

Property IV. *If a team of club A plays against a team of club B, then the other teams of clubs A and B meet in the same round.*

For the matches defined by rule (b) this can be seen by considering $i, j \leq n-1$, and realizing that by adding $n-1$ to i and j we move to the teams of the same club. If $i+j \equiv k(\mathrm{mod}\ 2n-2)$, then also $(i+n-1)+(j+n-1) \equiv k(\mathrm{mod}\ 2n-2)$. Hence, if teams i and j of different clubs play each other in round k, then the other two teams of the same clubs meet in round k as well. For the matches defined by the rules (c1)–(c4) the same holds, as can be verified case by case.

Looking at rounds 2 to $2n-1$ at *club* level, which can be done due to Property IV, we see that the clubs play exactly a *double* round robin tournament in these $2n-2$ rounds. Moreover, the two matches between the same clubs are exactly $n-1$ rounds apart. This extra structure can be used to construct a schedule for a group-balanced single round robin tournament with n strength groups of size 2, when n is even.

Theorem 2. *For every even n and every set of $2n$ teams, divided into n strength groups containing two teams each, there exists a group-balanced single round robin tournament with $2n-2$ breaks.*

Proof. We consider a set of $2n$ teams, divided into n strength groups containing two teams each. It is easy to see that in a group-balanced SRRT a team plays against distinct teams j and j' from the same strength group in two rounds having absolute difference exactly n, and the teams from the same strength group meet in round n. We prove Theorem 2 by constructing a schedule for such a group-balanced SRRT using the Adapted Circle Method.

We interpret each strength group as a club, containing exactly two teams. We then construct a schedule for an SRRT using the Adapted Circle Method and the home–away pattern set defined in Lemma 1, like we described in Section 2. It follows from the proof of Theorem 1 that this schedule satisfies Properties I, II and III. Finally, we modify this SRRT by rotating every round $n-1$ "slots" to the right; that is, round k becomes round $k+n-2(\mathrm{mod}\ 2n-2)+1$. In particular, the first round is moved to round n. Table 2 provides the schedule obtained in this way from the schedule in Table 1.

First, we observe that the obtained schedule is group-balanced. As we noted before, the Adapted Circle Method yields a schedule such that for each strength group the matches against the two teams in another strength group are carried out in two rounds that differ by $n-1$. Hence, after rotating, these matches are played in rounds that differ by exactly n. Thus, the schedule is group-balanced.

Second, we show that the number of breaks is $2n-2$. Note that, in an SRRT with a minimum number of breaks, for two teams the last entry of the home–away pattern equals the first entry (home–home or away–away). Hence we could say that these teams have a break in the first round, implying that each team has exactly one break. Note that by rotating rounds we do not destroy this property. Hence, the modified schedule has a minimum number of breaks if and only if two teams have a break in the first round. The Adapted Circle Method yields a timetable in which teams $\frac{1}{2}n$ and $\frac{3}{2}n-1$ have a break in round $n+1$, which becomes round 1 after rotating. Thus, the obtained schedule has the minimum number of breaks. In Table 2 we can see that for teams $\frac{1}{2}n$ and $\frac{3}{2}n-1$ either the first or the last match lies in the band, which illustrates their breaks in the first round. □

4. Conclusions

Our construction only works for an even number of clubs. Formulating the problem as an integer linear program and solving for small instances ($n = 3$ and $n = 5$) suggests that for an odd number of clubs an SRRT with Properties I, II and III does not exist. It is known that, in any schedule for an SRRT with $2n$ teams in which each team has at most one break, the $2n$ teams can be grouped into n disjoint pairs in such a way that each pair of teams plays complimentary [3]. This implies that any schedule for an SRRT with Property I automatically satisfies Property II [3]. It is remarkable that by adding Property III we seem to lose all odd n. An interesting question is what the minimum number of breaks is in a schedule for an SRRT satisfying Properties II and III when the number of clubs is odd, and how to construct such a schedule.

Although we do not know whether an SRRT with Properties I, II and III exists for some odd n larger than 5, we can easily see that no SRRT with Properties I, II, III and IV can exist for any odd n. In fact, there is no SRRT with Properties III and IV for any odd n. Satisfying Property III means that teams of the same club play against each other in the first round. Property IV implies that clubs can be grouped in pairs in each round other than the first round. In each of those rounds, this leaves a single club if the total number of clubs is odd. Since every team must play in every round according to the definition of a single round robin tournament, the two teams of this club must play against each other in two different rounds. This is not possible in a single round robin tournament.

The combination of break minimization and fairness with regard to strength groups raises interesting open questions for future research.

- What is the minimum number of breaks in a group-balanced SRRT with $2n$ teams and g strength groups?
- What is the computational complexity of the break minimization problem for a group-balanced SRRT?

Our result in Section 3 gives an answer to the first question for the special case where each strength group has size 2.

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References

[1] T. Barnard, A thousand million leagues, The Mathematical Gazette 77 (480) (1993) 329–336.
[2] D. Briskorn, Combinatorial properties of strength groups in round robin tournaments, European Journal of Operational Research 192 (3) (2009) 744–754.
[3] D. de Werra, Scheduling in sports, in: P. Hansen (Ed.), Studies on Graphs and Discrete Programming, North-Holland, Amsterdam, 1981, pp. 381–395.
[4] E. Lucas, Sixième récréation: Les jeux de demoiselles, in: Récréations mathématiques, vol. 2, Gauthier-Villars, Paris, 1883, pp. 161–197.
[5] R. Rasmussen, M. Trick, Round robin scheduling — a survey, European Journal of Operational Research 188 (2008) 617–636.