MR2567276 (2011g:68003) 68-02 (20M35 37B15 68Q45 68Q70)
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★**Elements of automata theory.**

Translated from the 2003 French original by Reuben Thomas.
Cambridge University Press, Cambridge, 2009. *xxiv*+758 *pp*. $145.00. *ISBN* 978-0-521-84425-3

Since its early days the theory of finite automata has been related to certain subjects from algebra. Initially [cf. e.g., A. Ginzburg, *Algebraic theory of automata*, Academic Press, New York, 1968; MR0242679 (39 #4009)], attention focused on the underlying structure of automata and the relation to semigroups and groups (syntactic monoid, Krohn-Rhodes decomposition theory). Then the two-volume treatise of S. Eilenberg [*Automata, languages, and machines. Vol. A*, Academic Press, New York, 1974; MR0530382 (58 #26604a), *Automata, languages, and machines. Vol. B*, Academic Press, New York, 1976; MR0530383 (58 #26604b)] presented a major unifying approach to automata theory. It was based on the concepts of semiring and formal power series over a semiring (in order to deal with "multiplicities") and it allowed for many new results, particularly on the characterization of certain subfamilies of recognizable languages in terms of varieties of finite semigroups. Nowadays "multiplicities" are replaced by "weights" and the study of "weighted" finite-state devices is a flourishing research topic in theoretical computer science [cf. *Handbook of weighted automata*, Springer, Berlin, 2009; Zbl 1200.68001].

The monograph under review is an extensive treatise of weighted automata theory in an algebraic setting; its core—divided into two parts—consists of Chapters I–V preceded by Chapter 0 that contains preliminaries on relations, monoids, words and languages, semirings, matrices, graphs, complexity and decidability. Chapters I–IV are each in their turn subdivided into two parts: the first 4–5 sections are labeled by "Contents of the chapter" [*sic*], the remaining, supplementary 3–4 sections by "Deeper & broader".

Part I ("The three stages of rationality"; Chapters I–III) deals with automata as accepting devices.

Chapter I ("The simplest possible machine . . . —automata on a free monoid") surveys the classic, or "naive", theory of finite automata (i.e., automata with weights from $\{0,1\}$ over an alphabet, or rather over a finitely generated free monoid): automata as labeled graphs, rational operations, Kleene's theorem, rational expressions, star (or pumping) lemma and its variants (particularly, a necessary and sufficient version), Arden's lemma, deterministic automata, minimal automata, constructing an automaton from an expression and vice versa, quotient of a language, derivative of an expression and its calculus, pattern matching in strings, and a proof that the star height of a rational language can be arbitrarily large.

Chapter II ("The power of algebra—automata over an arbitrary monoid") is devoted to automata in which the edges are labeled by elements from an arbitrary monoid rather than by symbols (i.e., generators of a freely generated monoid). This generalization requires a careful distinction between rational and recognizable subsets of a monoid, which are induced by the notions of automaton and action (of a monoid on a finite set), respectively. Central in this approach are: monoid morphism, morphism of an automaton, representation of an automaton by Boolean matrices, Myhill-Nerode's theorem, syntactic monoid, Schützenberger's covering of automata, well quasi-order, and a construction due to J. H. Conway to obtain the "universal automaton" of a language. The supplementary sections are devoted to rational subsets of the free group (and their relation to pushdown automata and Büchi's regular canonical systems), to rational subsets of free commutative monoids (which are applied in verification processes and "timed automata"), and to McNaughton's lemma on the star height of group languages.

In Chapter III ("The pertinence of enumeration—weighted automata") the automaton model of the previous chapter is generalized by providing each edge with a weight taken from a semiring $\mathbb{K}$. For such automata, languages become (formal power) series over the monoid M with coefficients in $\mathbb{K}$, and actions become matrix representations.

In order to define the star operation for series by $s^\star = \sum_{n\in\mathbb{N}} s^n$, the author requires that M be *graded*, i.e., M is equipped with an additive length: there exists a function $\varphi\colon M \to \mathbb{N}$ such that $\varphi(m) = 0$ only if $m = 1_M$ and for all m and n in M, $\varphi(mn) = \varphi(m) + \varphi(n)$ holds. Then it is possible to define a topology on the set $\mathbb{K}\langle\langle M\rangle\rangle$ of all series, whenever $\mathbb{K}$ possesses a topology. By a generalization of the previous chapters, we obtain the notions of $\mathbb{K}$-automaton, $\mathbb{K}$-rational power series, $\mathbb{K}$-rational operation, rational $\mathbb{K}$-expression, $\mathbb{K}$-covering, $\mathbb{K}$-quotient, culminating in the Fundamental Theorem of Finite Automata: a series of $\mathbb{K}\langle\langle M\rangle\rangle$ is $\mathbb{K}$-rational iff it is the behavior of some finite $\mathbb{K}$-automaton over M. Restricting to finitely generated monoids, the author focuses on the Kleene-Schützenberger Theorem (i.e., a series of $\mathbb{K}\langle\langle A^\star\rangle\rangle$ is $\mathbb{K}$-rational iff it is $\mathbb{K}$-recognizable), derivation of rational $\mathbb{K}$-expressions, and (assuming $\mathbb{K}$ is a field) the algebraic theory of the minimization of automata via their matrix representation.

Part II ("Rationality in relations"; Chapters IV–V) treats automata with outputs: finite-state transducers, and in particular, the realization of functions by finite automata.

In Chapter IV ("The richness of transducers—relations realised by finite automata") relations between monoids M and N are viewed either as subset of $M \times N$ or as a mapping from M to the power set of N. Either view enables one to apply approaches from previous chapters which yield the Evaluation Theorem (let τ be a rational relation from M to N; then the image under τ of a recognizable subset of M is a rational subset of N), the Composition Theorem (the composition of two rational relations is a rational relation), and their generalizations to power series. Next, attention is paid to the equivalence problem for finite $\mathbb{K}$-transducers: for transducers without weights the problem is undecidable (even in the case of a unary alphabet), but on the other hand, whenever $\mathbb{K}$ is a field, it happens to be decidable. The second part of this chapter deals with deterministic rational relations, synchronized transducers, and Malcev-Neumann series.

Chapter V ("The simplicity of functional transducers—functions realised by finite automata") starts by showing that functionality of a rational relation is a decidable property. Most of this chapter is devoted to structural results in terms of sequential and co-sequential functions which are analogues of deterministic and co-deterministic automata, respectively. Using Schützenberger's coverings the author arrives via the Rational Uniformization Theorem (every rational relation from $A^\star$ to $B^\star$ is uniformized by an unambiguous rational function) at Elgot and Mezei's Theorem: every rational function from $A^\star$ to $B^\star$ is the composition of a pure sequential function and a pure co-sequential function. The chapter concludes with the decidability of sequentiality and two characterizations of sequential functions: one in terms of "translations of a function" (which resemble quotients of a language), and one which is based on topological properties.

Each chapter contains a rich collection of exercises (many of which are provided with a solution) and concludes with historical remarks, additional notes and references.

{Reviewer's remarks: This is a splendid monograph that will be a reference text for the next few decades. It is a pleasure to read; it is written with skill, with an eye for motivation, details and language, and sometimes with a touch of humor. But it is not perfect: apart from a few misprints, this reviewer has some reservations with respect to two points. Firstly, the numbering of items is rather nonstandard: it results, e.g., on pp. 100–101 in: Lemma 2.9, Example 2.6 and Remark 2.5 (in that order). Fortunately, the margin frequently contains the appropriate page number when a reference is made. Secondly, the notation is sometimes idiosyncratic. Viz. identifying a singleton with its element yields $S \cup s$ instead of $S \cup \{s\}$; but the author still writes $S \cup \{s, t\}$ rather than $S \cup s \cup t$. A more serious example is occasionally writing $\{S\} \cup \{T\}$ for the union $S \cup T$ of sets S and T as in, e.g., (I.4.3), (I.4.7), (I.5.1), (I.5.2) and (II.1.2). Neither is this monograph complete; in its preface the author lists subjects that he omitted: variety theory, automata on trees, infinite words, alternating automata, pushdown automata. And one may add fuzzy finite automata as well. But at several spots in the text the author vaguely promises to write a sequel to cover these subjects too. The translation is close to the French original; the only deviation consists of additional footnotes where recent results—particularly, solutions to problems that were open in 2003—are mentioned.}

Reviewed by *Peter R. J. Asveld*