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Theory of Josephson effect in Sr_2RuO_4/diffusive normal metal/Sr_2RuO_4 junctions

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Abstract

We derive a generalized Nazarov's boundary condition for diffusive normal metal (DN)/chiral p-wave superconductor (CP) interface including the macroscopic phase of the superconductor. The Josephson effect is studied in CP/DN/CP junctions solving the Usadel equations under the above boundary condition. We find that, enhancement of a critical current at low temperature is small compared with that in p_x-wave /DN/p_x-wave junctions. As a result, temperature dependence of the critical current in these junctions is similar to that in conventional junctions. The result is consistent with the experiment in Sr_2RuO_4–Sr_3RuO_7 eutectic junctions. Similar feature is also found in current–phase relation.

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Keywords: Josephson effect; Proximity effect; Sr_2RuO_4

1. Introduction

Spin-triplet superconductivity is realized in Sr_2RuO_4 and paid much attention to [1]. Some experiments indicate that the pairing symmetry is chiral p-wave (CP) [2–4]. Motivated by these experiments, a lot of interesting phenomena are predicted in spin-triplet superconductor (TS)/diffusive normal metal (DN) hybrid junctions. In these junctions, mid gap Andreev resonant state (MARS) [5–9] is induced near the interface, and penetrates into the DN region. The proximity effect that the Cooper-pair penetrates from the superconductor, and MARS coexist in these junctions. The coexistence induces the zero bias conductance peak in TS/DN junctions [10], which was experimentally observed in Sr_2RuO_4 junctions [11]. Another example of the manifestation of the coexistence is that the Josephson current is enhanced strongly at low temperatures in TS/DN/TS junctions [12]. Then the Josephson current is proportional to sin $(\Psi/2)$ at low temperatures [13]. Here, Ψ is a macroscopic phase difference between left and right superconductors. Recently, the Josephson effect in Sr_2RuO_4–Sr_3RuO_7 eutectic junction was experimentally observed [14]. However, temperature dependence of critical current in this experiment is rather conventional, which is inconsistent with the above prediction.

In superconducting junctions, the quasi-classical Green's function theory is one of the useful methods for studying the transport phenomena. In diffusive regime, the quasi-classical Green's function obeys the Usadel equation [15]. The circuit theory [16] enables us to calculate the junction conductance at arbitral transparent interface for s-wave superconductor (S) junctions. This theory has been generalized for unconventional superconductor (US)

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junctions recently [17–20]. In these theories, the effect of making the MARS is included, and they reproduce the ballistic theories in no proximity limit. These theories are applicable to DN/US junctions without broken time reversal symmetry. However, these theories cannot be applied to calculate the Josephson current in CP/DN/CP junctions. Then, it is needed to generalize these theories to the interface between DN/CP.

In the present paper, we derive a generalized Nazarov's boundary condition for DN/CP interface including the macroscopic phase of the superconductor. The Josephson effect is studied numerically in CP/DN/CP junctions solving the Usadel equation under the above boundary condition. We find that the temperature dependence of the critical current is similar to that in S/DN/S junctions. Current–phase relation shows that the current is expressed by almost sinusoidal function. The enhancement of critical current at low temperature is small compared with that in p$_x$-wave superconductor (P$_x$)/DN/p$_x$-wave superconductor junctions, and then temperature dependence of the critical current in these junctions has a similar feature to that in S/DN/S junctions. The result is consistent with the experiment in Sr_2RuO_4–Sr_3RuO_7 eutectic junctions [14].

2. Formulation

We use the model of CP/DN/CP junction. Here, R_b' is a resistance of insulating barrier located at $x = 0$, R_d is a resistance of the DN, R_b is a resistance of insulating barrier located at $x = L$, and the length of DN, L is much larger than the mean flee path. We model the infinitely narrow insulating barriers described as $U(x) = H'\delta(x) + H\delta(x - L)$. Then the transparency $T_m^{(\prime)}$ is expressed by $T_m^{(\prime)} = 4\cos^2\phi/(4\cos^2\phi + Z^{(\prime)}2)$ with $Z^{(\prime)} = 2H^{(\prime)}/\hbar v_F$. Here, ϕ is an injection angle measured from the direction perpendicular to the interface between DN and CP, and v_F is Fermi velocity. The pair potential is chosen as $\Delta(\phi) = \Delta e^{i}\phi$, i.e., chiral p-wave.

We parameterize the quasi-classical Green's functions G_ω and F_ω with a function Φ_ω, [21,22].

$$G_\omega = \frac{\omega}{\sqrt{\omega^2 + \Phi_\omega \Phi^*_{-\omega}}}, \quad F_\omega = \frac{\Phi_\omega}{\sqrt{\omega^2 + \Phi_\omega \Phi^*_{-\omega}}}, \tag{1}$$

where ω is Matsubara frequency. The Usadel equation is expressed by [15]

$$\xi^2 \frac{\pi T_C}{\omega G_\omega} \frac{\partial}{\partial x}\left(G_\omega^2 \frac{\partial}{\partial x}\Phi_\omega\right) - \Phi_\omega = 0, \tag{2}$$

with the coherence length $\xi = \sqrt{D/2\pi T_C}$, the diffusion constant D, and the transition temperature T_C. Here, the average over the various angles of injected particle at the interface is defined as

$$\langle I_m^{(\prime)} \rangle = \frac{\int_{-\pi/2}^{\pi/2} d\phi I_m^{(\prime)} T_m^{(\prime)} \cos\phi}{\int_{-\pi/2}^{\pi/2} d\phi T_m^{(\prime)} \cos\phi}. \tag{3}$$

The resistance at the interface $R_b^{(\prime)}$ is given by

$$R_b^{(\prime)} = \frac{2R_0}{\int_{-\pi/2}^{\pi/2} d\phi T_m^{(\prime)} \cos\phi}, \tag{4}$$

where $R_0^{(\prime)}$ is the Sharvin resistance. The boundary condition at $x = L$ is expressed by

$$\frac{G_\omega}{\omega}\frac{\partial}{\partial x}\Phi_\omega = \frac{R_d}{R_b L}\left\{-\frac{\Phi_\omega}{\omega} I_1 + ie^{-i\Psi} I_2\right\}\Bigg|_{x=L_-},$$

$$I_1 \equiv \left\langle \frac{2T_m g_s}{\Lambda_m} \right\rangle, \quad I_2 \equiv \left\langle \frac{2T_m f_s}{\Lambda_m} \right\rangle,$$

$$\Lambda_m = 2 - T_m + T_m\{g_s G_\omega + f_s(B\sin\Psi - C\cos\Psi)\},$$

$$B = \frac{G_\omega(\Phi_\omega + \Phi^*_{-\omega})}{2\omega}, \quad C = \frac{iG_\omega(\Phi_\omega - \Phi^*_{-\omega})}{2\omega}, \tag{5}$$

with $f_{1\pm} = \mathrm{Re}(f_\pm)$, $f_{2\pm} = \mathrm{Im}(f_\pm)$, $g = \omega/\sqrt{\omega^2 + \Delta^2}$, $f_\pm = \Delta_\pm/\sqrt{\omega^2 + \Delta^2}$, and the macroscopic phase the superconductor Ψ. Here, $\Delta_+ = \Delta(\phi)$ and $\Delta_- = \Delta(\pi - \phi)$ are the effective pair potential with injected angle ϕ and $\pi - \phi$, respectively. The boundary condition $x = 0$ is expressed by:

$$\frac{G_\omega}{\omega}\frac{\partial}{\partial x}\Phi_\omega = -\frac{R_d}{R_b L}\left\{-\frac{\Phi_\omega}{\omega} I_1' + iI_2'\right\}\Bigg|_{x=0_+} \tag{6}$$

Here I_1' and I_2' are obtained by adding superscript $'$, changing ϕ to $\pi - \phi$, and putting $\Psi = 0$ for I_1 and I_2 at $x = L$. Then the macroscopic phase differences, left and right superconductor becomes Ψ. The Josephson current is expressed as:

$$\frac{eIR}{\pi T_C} = i\frac{RTL}{2R_d T_C}\sum_\omega \frac{G_\omega^2}{\omega^2}\left(\Phi_\omega \frac{\partial}{\partial x}\Phi^*_{-\omega} - \Phi^*_{-\omega}\frac{\partial}{\partial x}\Phi_\omega\right) \tag{7}$$

with temperature T, and total resistance of the junction $R = R_b + R_b' + R_d$. In the following discussions, we fix $Z = 1$, $R_b' = R_b$ and $T_m' = T_m$.

3. Result

Fig. 1 shows the temperature dependence of a critical current. Here I_C is critical current and Δ_0 is defined as $\Delta_0 = \Delta(T = 0)$. The critical current is enhanced for large E_{Th}/Δ_0, large R_d/R_b, and low temperatures in both (a) and (b). These results indicate that the critical current gets enhanced as the degree of the proximity effect becomes strong, which is a conventional result similar to that in S/DN/S junctions. Fig. 2 shows the current–phase relation for $R_d/R_b = 1$. We find that the peak is slightly shifted to $\Psi > \pi/2$ at low temperatures, and its effect becomes strong in particular, for large E_{Th}/Δ_0. However, the effect of coexistence of MARS and the proximity effect on current phase relation does not appear clearly, in contrast to P$_x$ junctions. In order to compare our results with the past theories, we also calculate the critical current in S/DN/S junction and P$_x$/DN/P$_x$ junctions [20,23]. Fig. 3 is the result in CP/DN/CP, P$_x$/DN/P$_x$, and S/DN/S junctions for $R_d/R_b = 1$. Here, the pair potential of P$_x$, and S are

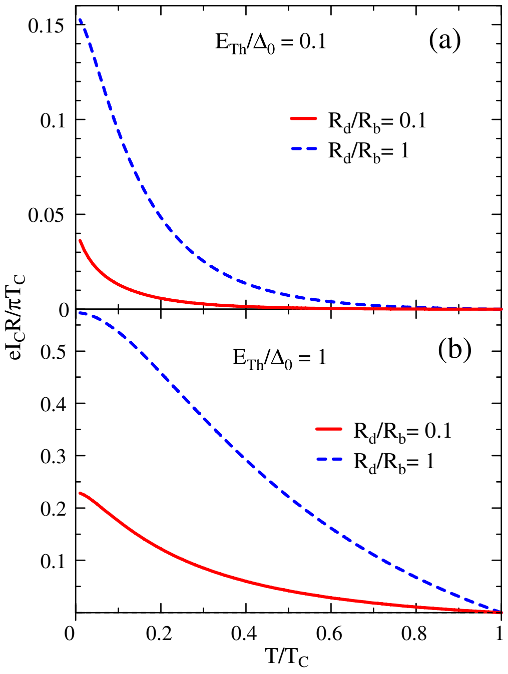

Fig. 1. Temperature dependence of the critical current. Solid lines are results for $R_d/R_b = 0.1$ and broken lines are results for $R_d/R_b = 1$. (a) Is result for $E_{Th}/\Delta_0 = 0.1$ and (b) is result for $E_{Th}/\Delta_0 = 1$.

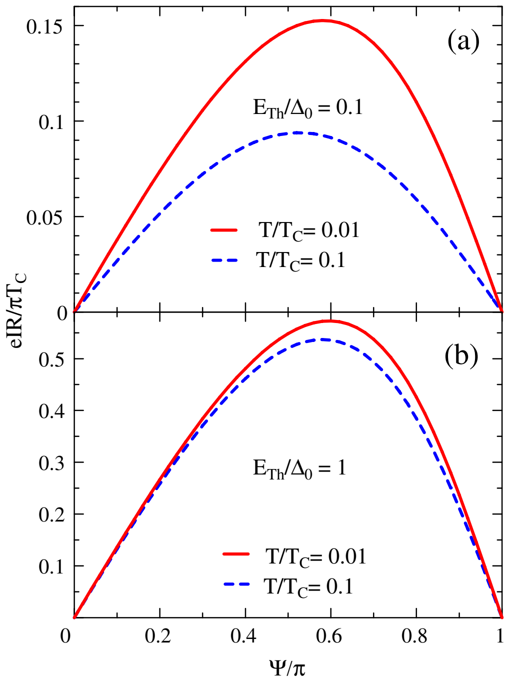

Fig. 2. Current–phase relation for $R_d/R_b = 1$. Solid lines are results for $T/T_C = 0.01$ and broken lines are results for $T/T_C = 0.1$. (a) Is result for $E_{Th}/\Delta_0 = 0.1$ and (b) is result for $E_{Th}/\Delta_0 = 1$.

expressed as $\Delta(\phi) = \Delta\cos\phi$, and $\Delta(\phi) = \Delta$, respectively. We find the enhancement of critical current at low temperature is small compared with that in P_x/DN/P_x junctions. The obtained results stem from the fact that the coexistence of MARS and proximity effect is induced only for the injection angle $\phi = 0$ in CP/DN/CP junctions. As a result, temperature dependence of the critical current in CP/DN/CP junction is similar to that in the S/DN/S junctions. The result is consistent with the experiment in Sr_2RuO_4–Sr_3RuO_7 eutectic junctions [14].

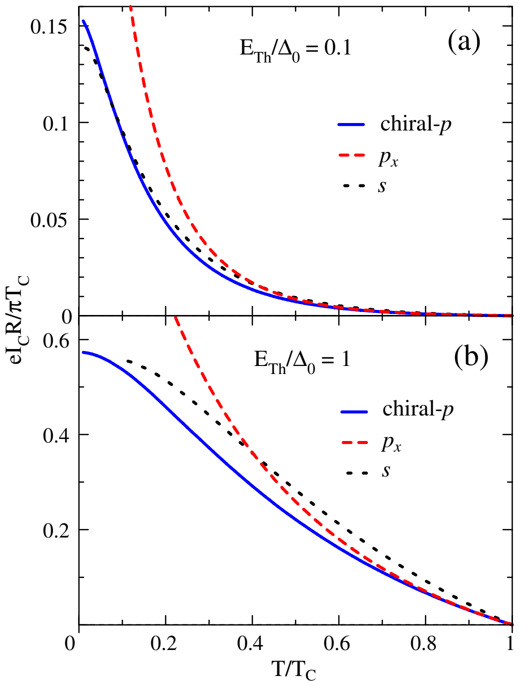

Fig. 3. Temperature dependence of the critical current for $R_d/R_b = 1$. Solid lines are results for CP/DN/CP, broken lines are results for P_x/DN/P_x, and dotted lines are results for S/DN/S junctions. (a) Is result for $E_{Th}/\Delta_0 = 0.1$ and (b) is result for $E_{Th}/\Delta_0 = 1$.

4. Conclusion

We have derived the generalized Nazarov's boundary condition for DN/CP interface including the macroscopic phase of the superconductor. The Josephson effect has been studied in CP/DN/CP junctions, solving the Usadel equation under the above boundary condition.

We have found that temperature dependence of the critical current and current–phase relation in CP/DN/CP junctions have similar forms to those in S/DN/S junctions. The enhancement of the critical current at low temperature is small compared with P_x/DN/P_x junctions. Our results are consistent with the experiment in Sr_2RuO_4–Sr_3RuO_7 eutectic junctions.

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