INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **39** (2006) 6323–6331 doi:10.1088/0305-4470/39/21/S24

Spatial dispersion in Casimir forces: a brief review

R Esquivel-Sirvent[1], C Villarreal[1], W L Mochán[2], A M Contreras-Reyes[3] and V B Svetovoy[4]

[1] Instituto de Física, Universidad Nacional Autónoma de México, Apartado Postal 20-364, 01000 Distrito Federal, Mexico
[2] Centro de Ciencias Físicas, Universidad Nacional Autónoma de México, Apartado Postal 48-3, 62251 Cuernavaca, Morelos, Mexico
[3] Department of Physics and Astronomy, University of Sussex, Brighton, East Sussex BN1 9QH, UK
[4] MESA+ Research Institute, University of Twente, PO Box 217, 7500 AE Enschede, The Netherlands

E-mail: raul@fisica.unam.mx, carlos@fisica.unam.mx, mochan@fis.unam.mx, amc40@sussex.ac.uk and svetovoy@yandex.ru

Received 5 November 2005, in final form 22 December 2005
Published 10 May 2006

Abstract
We present the basic principles of non-local optics in connection with the calculation of the Casimir force between half-spaces and thin films. At currently accessible distances L, non-local corrections amount to about half a per cent, but they increase roughly as $1/L$ at smaller separations. Self-consistent models lead to corrections with the opposite sign as models with abrupt surfaces.

PACS numbers: 2.20.Ds, 42.50.Lc, 73.20.Mf, 78.68.+m

1. Introduction

The recent measurements of Casimir forces [1–5] and their comparison with theory have made it necessary to consider in detail the electromagnetic response of the involved materials. In this paper, we concentrate our attention on the spatial dispersion of the response which leads to the so-called non-local effects. The problem of non-locality in connection with Casimir forces was pointed out by Kats [6], who made a qualitative estimate of the effect and concluded that it was necessary to specify the correct dependence of the dielectric function on both the frequency and the wave vector. A more formal study was done by Heinrichs [7] and Buhl [8] who studied the Van der Waals interaction taking into account spatial dispersion using a hydrodynamic model for the electronic dynamics, showing that at large distances non-local effects were negligible. More recently, the study of non-local effects in Casimir forces was revived [9–13] showing the need for an accurate theoretical description of the system. It has also been shown that the correct understanding of spatial dispersion is fundamental in order to solve recent controversies regarding the behaviour of the Casimir force at finite temperatures

[14, 15]. In this paper, we present a brief review of non-local effects, their incorporation in the calculation of optical properties and their importance in Casimir forces at zero temperature, with particular attention to the importance of a proper optical model of the materials.

2. Non-local media

In a linear and causal time-independent system, the most general relation between the electric displacement $\vec{D}$ and the electric field $\vec{E}$ is

$$\vec{D}(\vec{r},t) = \int \mathrm{d}^3r' \int_{-\infty}^{t} \mathrm{d}t' \epsilon(\vec{r},\vec{r}\,',t-t') \cdot \vec{E}(\vec{r}\,',t'), \tag{1}$$

where $\epsilon(\vec{r},\vec{r}\,',t-t')$ is the dielectric response tensor of the material. The response of the system at time t depends on the excitation not only at t, but also at previous times $t' < t$, i.e., its response is not instantaneous. This fact, known as temporal dispersion, is closely related to the well-known dependence of the index of refraction on frequency ω. The time integration in equation (1) is readily recognized as a convolution, so that it may be eliminated through a temporal Fourier transform,

$$\vec{D}(\vec{r},\omega) = \int \mathrm{d}^3r' \epsilon(\vec{r},\vec{r}\,',\omega) \cdot \vec{E}(\vec{r}\,',\omega). \tag{2}$$

Equation (1) also shows that the response at a position $\vec{r}$ might depend not only on the excitation at the same point, but also on positions $\vec{r}\,'$ within some neighbourhood Ω of $\vec{r}$. This non-local dependence arises from the interaction among different parts of the system and is known as spatial dispersion due to its resemblance to temporal dispersion. The size of the neighbourhood Ω within which $\epsilon(\vec{r},\vec{r}\,',t-t')$ is non-negligible is called the range of non-locality. It is typically about an atomic distance, i.e., a few Å, as that is the only length scale that characterizes the microscopic response of a material, although it may become orders of magnitude larger, for example, in ultra-pure conductors at low temperatures. As the wavelength of light is typically much larger than the range of non-locality, it is common to assume that $\vec{E}(\vec{r}\,') \approx \vec{E}(\vec{r})$ within Ω, so that we may take the electric field out of the spatial integration in equations (1) and (2), yielding a local response $\vec{D}(\vec{r},\omega) = \epsilon(\vec{r},\omega) \cdot \vec{E}(\vec{r},\omega)$, where $\epsilon(\vec{r},\omega) \equiv \int \mathrm{d}^3r' \epsilon(\vec{r},\vec{r}\,',\omega)$ is the local dielectric response. An equivalent result is obtained if we replace $\epsilon(\vec{r},\vec{r}\,',\omega)$ by the local kernel $\epsilon(\vec{r},\omega)\delta(\vec{r}-\vec{r}\,')$ in equation (2). However, we remark that this approximation might fail close to the surface of a material where its dielectric properties change rapidly, as some components of the field have rapid variations in this region regardless of the frequency. Thus, the detailed study of the electromagnetic screening at surfaces requires a non-local approach. From the optical point of view, non-locality produces corrections to the optical coefficients, such as the reflection amplitudes, of order Λ/λ where Λ is the characteristic length scale of the selvedge region where non-locality has to be accounted for [16] and λ is the wavelength of light. As Λ is typically a few Å, while λ is on the order of hundreds or thousands of Å, non-locality may be safely ignored in many applications, although it has to be accounted for in any precise calculation. Note that within a non-local material we have to include the full tensorial character of the response even in the isotropic case, as the separation $\vec{r}-\vec{r}\,'$ between excitation and observation positions defines a particular direction in space.

Even within non-local media, equation (2) may be further simplified if the system is translationally invariant, with a response that depends on the separation $\vec{r}-\vec{r}\,'$, instead of being a function of both $\vec{r}$ and $\vec{r}\,'$. In this case, equation (2) is also a convolution and may be rewritten as a simple algebraic relation,

$$\vec{D}(\vec{k},\omega) = \epsilon(\vec{k},\omega) \cdot \vec{E}(\vec{k},\omega), \tag{3}$$

by taking spatial Fourier transforms with wave vector $\vec{k}$. Thus, spatial dispersion is frequently identified with a wave-vector-dependent dielectric response. Examples of spatially dispersive systems are excitonic semiconductors, where the spatial dispersion arises from the momentum dependence of the excitonic energy. Another well-known non-local system is the conduction electron gas of a conductor. Here the non-locality arises from the correlation hole that surrounds every electron, so that excitation of one electron at some position affects the response of the system within a neighbourhood on the order of the Thomas–Fermi screening distance. Furthermore, an electron excited at one position may contribute to the response at a distance of a mean free path away.

The identification of spatial dispersion with a wave-vector-dependent response has led to some subtle but pervading confusion in the literature when discussing the optical properties of non-local systems, as $\boldsymbol{\epsilon}(\vec{k}, \omega)$ *is not even a well-defined quantity* close to a surface. As translational invariance is necessarily lost, the response of the system has to be written fully as in equation (2), or be simplified at most to

$$\vec{D}(z, \vec{Q}, \omega) = \int \mathrm{d}z' \boldsymbol{\epsilon}(z, z', \vec{Q}, \omega) \cdot \vec{E}(z', \vec{Q}, \omega), \tag{4}$$

where we assumed that the surface is normal to the z axis and we took spatial Fourier transforms with wave vector $\vec{Q}$ along the x–y plane along which we may assume a 2D translational invariance. Nevertheless, specific meaning can sometimes be given to $\boldsymbol{\epsilon}(\vec{k}, \omega)$ close to a surface, but only under some additional simplifying assumptions about the nature of the response, some of which will be discussed below. These assumptions and their approximate nature should not be ignored.

3. Homogeneous systems

Within a homogeneous isotropic material, the only preferred direction is determined by $\vec{k}$, so that the dielectric tensor may be written as

$$\boldsymbol{\epsilon}(\vec{k}, \omega) = \epsilon_{\mathrm{l}}(\vec{k}, \omega)\mathbf{P}_{\mathrm{l}}(\vec{k}) + \epsilon_{\mathrm{t}}(\vec{k}, \omega)\mathbf{P}_{\mathrm{t}}(\vec{k}) = \epsilon_{\mathrm{l}}(\vec{k}, \omega)\frac{\vec{k}\vec{k}}{k^2} + \epsilon_{\mathrm{t}}(\vec{k}, \omega)\left(\mathbf{1} - \frac{\vec{k}\vec{k}}{k^2}\right), \tag{5}$$

where $\mathbf{P}_{\mathrm{l}}(\vec{k})$ and $\mathbf{P}_{\mathrm{t}}(\vec{k})$ are the longitudinal and the transverse projectors. Thus, for longitudinal and for transverse exciting fields, the response may be taken as scalar. However, the longitudinal dielectric function $\epsilon_{\mathrm{l}}(\vec{k}, \omega)$ is in general different from the transverse dielectric function $\epsilon_{\mathrm{t}}(\vec{k}, \omega)$, although they coincide among themselves and with the local dielectric function in the $k \to 0$ limit.

Substituting equation (5) into (3) and the resulting displacement into Maxwell's equations for non-magnetic systems, we obtain

$$\vec{k} \times \vec{E}(\vec{k}, \omega) = \frac{\omega}{c}\vec{B}, \tag{6}$$

$$\vec{k} \times \vec{B}(\vec{k}, \omega) = -\frac{\omega}{c}(\epsilon_{\mathrm{l}}(\vec{k}, \omega)\vec{E}_{\mathrm{l}}(\vec{k}, \omega) + \epsilon_{\mathrm{t}}(\vec{k}, \omega)\vec{E}_{\mathrm{t}}(\vec{k}, \omega)), \tag{7}$$

where $\vec{E}_\alpha = \mathbf{P}_\alpha \cdot \vec{E}$ $(\alpha = \mathrm{l}, \mathrm{t})$. Taking as usual the vector product of equation (7) with $\vec{k}$ and substituting (6), we obtain the dispersion relation for transverse waves ($E_{\mathrm{t}} \neq 0$)

$$k^2 = \frac{\omega^2}{c^2}\epsilon_{\mathrm{t}}(\vec{k}, \omega), \tag{8}$$

which differs from the usual local result only by the explicit dependence of ϵ_{t} on $\vec{k}$. On the other hand, taking the longitudinal projection of equation (7) we obtain that there might also be free longitudinal fields ($E_\mathrm{l} \neq 0$) within the bulk, provided the wave vector and frequency satisfy the longitudinal dispersion relation, given implicitly by

$$\epsilon_\mathrm{l}(\vec{k}, \omega) = 0. \tag{9}$$

Both equations (8) and (9) can provide more solutions within non-local media than the usual two independent transverse modes that may be sustained by local materials.

The specific form of the dielectric function depends on the nature of the material, dielectric or metallic, and the model used. For example, a simple model for semiconductors close to an excitonic transition is that of a Lorentz oscillator [17],

$$\epsilon_\mathrm{t}(\vec{k}, \omega) = \epsilon_\mathrm{l}(\vec{k}, \omega) = \epsilon_\infty + \frac{\omega_\mathrm{p}^2}{\omega_T^2(k) - \omega^2 - \mathrm{i}\gamma\omega}, \tag{10}$$

with weight ω_p^2, dissipation constant γ and with a wave-vector-dependent resonance energy $\hbar\omega_T(k) = E_\mathrm{g} - E_\mathrm{b} + K$ that incorporates the energy required to create an electron–hole pair given by the energy gap E_g of the semiconductor, the binding energy E_b of the exciton and its kinetic energy $K = \hbar^2k^2/2M$, where M is the excitonic mass. Here ϵ_∞ is the contribution from the other non-resonant transitions. An analogous simple model for metals is the hydrodynamic model, with a local transverse response $\epsilon_\mathrm{t}(\omega) = 1 - \omega_\mathrm{p}^2/(\omega^2 + \mathrm{i}\gamma\omega)$ given by the Drude model and a non-local longitudinal response

$$\epsilon_\mathrm{l}(\vec{k}, \omega) = 1 - \frac{\omega_\mathrm{p}^2}{\omega^2 + \mathrm{i}\omega\gamma - \beta^2k^2}. \tag{11}$$

Here, the spatial dispersion arises from the fact that electrons are fermions and so Pauli's principle implies that it takes energy to increase their density. Therefore, for longitudinal waves there is a restoring force proportional to $\beta^2 = 3v_\mathrm{F}^2/5$ [18] in addition to the electrical coupling to the electromagnetic field [19]. Here, v_F is the Fermi velocity and β is related to the compressibility of the metal.

More elaborate expressions may be obtained through a purely quantum-mechanical approach using linear response theory. One of such approaches is the random phase approximation (RPA), in which the response of the electron gas to the self-consistent oscillating electric field is identified with the response of a gas of independent fermions to an external perturbing field. The response may then be found from Kubo's formulae through the density–density and the current–current equilibrium correlation functions of the fermion gas, resulting in the Lindhard longitudinal dielectric function [20–22]

$$\epsilon_\mathrm{l}(\vec{k}, \omega) = 1 + \frac{3\omega_\mathrm{p}^2}{k^2v_\mathrm{F}^2} f_\mathrm{l}, \tag{12}$$

where

$$f_\mathrm{l} = \frac{1}{2} + \frac{1}{8w}\left[[1 - (w - u)^2]\ln\left(\frac{w - u + 1}{w - u - 1}\right) + [1 - (w + u)^2]\ln\left(\frac{w + u + 1}{w + u - 1}\right)\right], \tag{13}$$

$w = k/2k_\mathrm{F}$, $u = \omega/kv_\mathrm{F}$ and k_F is Fermi's wave vector. Similar expressions have been obtained for the non-local transverse dielectric function [11], including additional dissipation channels [23].

4. Surface

It is important to emphasize that a dielectric response of the form $\epsilon(\vec{k}, \omega)$ may only be defined within the bulk of a translational invariant system. When a surface is present, this invariance is broken along its normal direction and the full response $\epsilon(z, z', \vec{Q}, \omega)$ has to be employed. This may be obtained from microscopic surface screening calculations that include the self-consistent confining potential [24]. Alternatively, simplified models may be obtained by writing the dielectric response close to the surface in terms of the bulk response, introducing simplifying assumptions about the interaction of electrons with the surface. The latter calculations have the added difficulty of having to account for the possible excitation of longitudinal and/or additional transverse waves given by all the real and complex solutions of equations (8) and (9), beyond the usual two transverse modes of local optics.

As an example, consider p-polarized light incident on the surface of a metal described by the hydrodynamic model (11). Within the bulk, there is one p-polarized transverse and one longitudinal transmitted mode. Assuming that these modes may be extrapolated up to the surface, which is taken as a sharp discontinuity in the dielectric properties, the optical problem is reduced to the calculation of the amplitude of the reflected wave, the transmitted transverse wave and the longitudinal wave. Thus, three boundary conditions are required. Maxwell's equations provide only two independent conditions, so that the problem seems to be under-determined and additional boundary conditions (ABCs) of non-electromagnetic origin are called for. The problem arises of course from the assumption that the response is bulk-like up to the surface. Nevertheless, it is reasonable to assume that at the surface of a non-local metal *all of the components of all the fields ought to be continuous* [25], not only the usual field components $E_\parallel$, $H_\parallel$, $D_\perp$ and $B_\perp$, where $\parallel$ and $\perp$ denote parallel and perpendicular. For instance, a discontinuity in the normal component of the electric field $E_\perp$ would imply an infinite charge density at the surface, which would unrealistically require an infinite amount of energy according to Pauli's principle. Using this ABC, one can obtain the reflection amplitude

$$r_\mathrm{p} = \frac{\epsilon_\mathrm{t} k_\mathrm{v} - k_\mathrm{t} + Q^2(\epsilon_\mathrm{t} - 1)/k_\mathrm{l}}{\epsilon_\mathrm{t} k_\mathrm{v} + k_\mathrm{t} - Q^2(\epsilon_\mathrm{t} - 1)/k_\mathrm{l}}, \tag{14}$$

which differs from the local result $r_\mathrm{p} = (\epsilon_\mathrm{t} k_\mathrm{v} - k_\mathrm{t})/(\epsilon_\mathrm{t} k_\mathrm{v} + k_\mathrm{t})$ due to the excitation of longitudinal waves at the surface. Here, $k_\mathrm{v}, k_\mathrm{t}$ and k_l are the normal components of the wave vector of the incident wave in vacuum, of the transverse wave in the metal and of the longitudinal wave, respectively, for given values of $\vec{Q}$ and ω.

The above hydrodynamic result may be obtained as a particular case of the semi-classical infinite barrier model (SCIB) [26]. In this model, it is assumed that there are two ways in which an electron excited at $\vec{r}'$ within a semi-infinite conductor occupying the half-space $z > 0$ may propagate and contribute to the response at $\vec{r}$: either it travels directly from $\vec{r}'$ to $\vec{r}$ or else it first propagates to some point on the surface $z = 0$ where it is specularly reflected back into $\vec{r}$. Assuming no electrons are inelastically nor diffusively scattered at the surface, and ignoring the quantum interference between incident and reflected electronic wavefunctions, the polarization at $\vec{r}$ within the *real* semi-infinite conductor would be indistinguishable from the polarization in a *fictitious* infinite system, provided in the latter we impose the specular $z \leftrightarrow -z$ symmetry. For each electron at $z > 0$ moving with speed $-v_z$ towards the surface, there would be another electron in the fictitious system at $z < 0$ moving with speed $v_z > 0$; when the former reaches $z = 0$ and leaves the $z > 0$ half-space, the latter enters the half-space, as if the former were specularly reflected. In the fictitious system, $\vec{E}_\parallel$ must be an even function of z while $E_\perp$ must be an odd function. All other vectorial quantities must have the same behaviour, while pseudo-vectors such as $\vec{B}$ and $\vec{H}$ must have the opposite parity. In particular, the fields $\vec{H}_\parallel$ and $D_\perp$ are odd functions of z and are therefore discontinuous at the surface. This may seem surprising until we note that *this discontinuity applies only to the fields of the fictitious, not of the real system*, and that fictitious and real fields coincide only in the half-space $z > 0$. According to Maxwell's equations, the fictitious fields have a singular source given by a fictitious external surface current flowing at $z = 0$. By calculating the fields produced within a homogeneous metal by such a singular current, we may obtain the surface impedances [27]

$$Z_s \equiv -\frac{E_y(0^+)}{H_x(0^+)} = \frac{\mathrm{i}}{\pi}\frac{\omega}{c}\int_{-\infty}^{\infty}\frac{\mathrm{d}k_z}{(\omega^2/c^2)\epsilon_t - k^2}, \tag{15}$$

and

$$Z_p \equiv \frac{E_x(0^+)}{H_y(0^+)} = \frac{\mathrm{i}}{\pi}\frac{\omega}{c}\int_{-\infty}^{\infty}\frac{\mathrm{d}k_z}{k^2}\left[\frac{k_x^2}{(\omega^2/c^2)\epsilon_l} + \frac{k_z^2}{(\omega^2/c^2)\epsilon_t - k^2}\right], \tag{16}$$

which are independent of the unspecified magnitude and phase of the external current. *As in the real system there are no singularities and the fields are continuous across the boundary*, we can write $Z_s = -E_y(0^-)/H_x(0^-)$ and $Z_p = E_x(0^-)/H_y(0^-)$ in terms of the fields $\vec{E}(0^-)$ and $\vec{H}(0^-)$ in vacuum, which may be written in terms of the incident and reflected waves, so that we may solve for the reflection amplitudes

$$r_s = \frac{Z_s - Z_{vs}}{Z_s + Z_{vs}}, \qquad r_p = \frac{Z_{vp} - Z_p}{Z_{vp} + Z_p}, \tag{17}$$

where $Z_{vs} = \omega/(k_v c)$ and $Z_{vp} = (k_v c)/\omega$ are the vacuum surface impedances.

It can easily be shown that Fresnel's reflection amplitudes may be obtained by substituting in equations (15)–(17) the local wave-vector-independent dielectric function. The excitation of collective modes may be accounted for by substituting the dielectric function (11), yielding the hydrodynamic result (14). A full quantum-mechanical bulk response such as Lindhard's formulae (12) may also be employed, accounting therefore also for electron–hole pair creation and for Landau damping. Nevertheless, the above SCIB results are still not exact, as they do not account for the microscopic nature of the surface, the shape of the confining surface potential, the quantum oscillations of the equilibrium and the induced density close to the surface.

Besides applying simplified models that approximate the surface response in terms of the bulk response of the system, it is possible to obtain the microscopic response of the surface $\epsilon(z, z', \vec{Q}, \omega)$ through the use of linear response theory. The most simple microscopic model for metallic surfaces is the jellium model, in which electrons are added to a homogeneous semi-infinite positive background. Using density functional theory in the local density approximation (LDA), the self-consistent confining potential, electronic wavefunctions and equilibrium density profile may be obtained through a solution of the Kohn–Sham equations [28]. Through a generalization of the RPA known as the time-dependent LDA (TDLDA), the surface susceptibility and dielectric response may be obtained [24]. Equation (4) is an integral relation between $\vec{D}$ and $\vec{E}$. As the width of the selvedge region is usually small [16] with respect to the relevant optical wavelength, the integro-differential Maxwell's equations may be solved using a long-wavelength approximation [29]. For example, the reflection amplitude for *p*-polarized light may be simply expressed as [30]

$$r_p = r_p^0\left[1 + \frac{2\mathrm{i}k_v\epsilon_t}{1 + \epsilon_t k_v^2/Q^2}d_\perp\right], \tag{18}$$

where r_p^0 is the non-perturbed reflection amplitude given by Fresnel's relations and

$$d_\perp \equiv \frac{\int \mathrm{d}z\, z\delta\rho(z)}{\int \mathrm{d}z\, \delta\rho(z)} \tag{19}$$

is the position of the centroid of the distribution of charge $\delta\rho$ induced at the surface of a metal in order to screen the normal component of the electric field. As an illustrative example, $d_\perp = -\mathrm{i}/k_l$ within the hydrodynamic model. It is then simply shown that equation (18) is consistent with (14) in the long-wavelength limit. Nevertheless, $d_\perp$ has been calculated within the TDLDA for many metals and its low frequency value has been tabulated [24].

5. Casimir forces between non-local media

In the previous sections, we have discussed the calculation of the electromagnetic response and the optical properties of non-local systems. These may be immediately related to the calculation of the Casimir force between spatially dispersive media by noting that the Lifshitz formula, when written in terms of the reflection amplitudes,

$$F(L) = \mathcal{A}\frac{\hbar c}{2\pi^2}\mathrm{Re}\int_0^\infty \mathrm{d}Q\,Q\int_{q\geqslant 0}\mathrm{d}k_\mathrm{v}\,\frac{\tilde{k}_\mathrm{v}^2}{q}\left[\frac{r_\mathrm{s}^{(1)}r_\mathrm{s}^{(2)}\,\mathrm{e}^{2\mathrm{i}\tilde{k}_\mathrm{v}L}}{1-r_\mathrm{s}^{(1)}r_\mathrm{s}^{(2)}\,\mathrm{e}^{2\mathrm{i}\tilde{k}_\mathrm{v}L}}+\frac{r_\mathrm{p}^{(1)}r_\mathrm{p}^{(2)}\,\mathrm{e}^{2\mathrm{i}\tilde{k}_\mathrm{v}L}}{1-r_\mathrm{p}^{(1)}r_\mathrm{p}^{(2)}\,\mathrm{e}^{2\mathrm{i}\tilde{k}_\mathrm{v}L}}\right], \qquad (20)$$

is applicable to a wide class of systems, including homogeneous, inhomogeneous, semi-infinite or finite, insulator or metallic, dissipationless or absorptive, and local or spatially dispersive media [9–13, 31]. Here, $\tilde{k}_\mathrm{v} = k_\mathrm{v} + \mathrm{i}\eta$ where $\eta \to 0^+$ is a positive infinitesimal and the integral over k_v runs from $\mathrm{i}Q$ to 0 and then to ∞, so that $q = \omega/c$ remains real and positive, although the integration trajectory may be manipulated into a more convenient one over the imaginary axis, and we assumed the zero temperature case. The reason for the generality of equation (20) is that α-polarized photons ($\alpha = s, p$) that are not reflected coherently at the ath wall ($a = 1, 2$) of the cavity with amplitude $r_\alpha^{(a)}$ are lost from the cavity with probability $1 - \left|r_\alpha^{(a)}\right|^2$. However, detailed balance in thermodynamic equilibrium implies that those photons are replaced by similar photons through incoherent radiation from the walls or by being transmitted from the vacuum region beyond the system, at the same rate as they are lost. Thus, both the coherent and incoherent contributions to the field are determined by the same reflection amplitudes. By using an ancillary system with the same optical coefficients as the real cavity walls, the generality of equation (20) has been proved [9] for a wide class of isotropic systems, and it has recently been generalized to anisotropic [32] and to fermion mediated interactions [33].

Non-local effects in the Casimir force can therefore be obtained quantitatively simply by substituting the appropriate non-local reflection amplitudes in equation (20). For example, in figure 1 we show the non-local corrections $\delta F/F \equiv (|F_\mathrm{nl}| - |F_\mathrm{l}|)/|F_\mathrm{l}|$ to the Casimir force calculated with the hydrodynamic and the self-consistent jellium models [13], where the subscripts nl and l denote non-local and local, respectively. The exactly solvable hydrodynamic model predicts that non-locality decreases the force due to the excitation of additional waves in the media [9]. At the closest distances for which Casimir forces have been measured, $L \sim 50$ nm, the non-local correction $|\delta F/F|$ is about half a per cent. Similar results are also obtained [11] from models that employ a more sophisticated bulk dielectric response, such as a Lindhard-type dielectric function in the region of anomalous dispersion, including a correction to account for inter-band transitions, but which nevertheless truncate the system abruptly at the surface employing the SCIB or similar models. As in the hydrodynamic model, the force is smaller than in the local case. The left panel of figure 1 also illustrates the accuracy of the long-wavelength approximation (equations (18)). Contrariwise, the jellium model predicts a non-local correction of a similar size but of the opposite sign (note the labelling), that is, non-local effects increase the magnitude of the Casimir force [13]. The reason for this increase is that in realistic models of metallic surfaces, the electronic density is not truncated abruptly at the position of the nominal surface, but it decays smoothly to zero, extending beyond the metal and into vacuum. Actually, the negative electronic charge outside the nominal boundary and the compensating positive charge within the metal form the surface dipole that is the source of the self-consistent potential that actually confines the electrons within the metal.

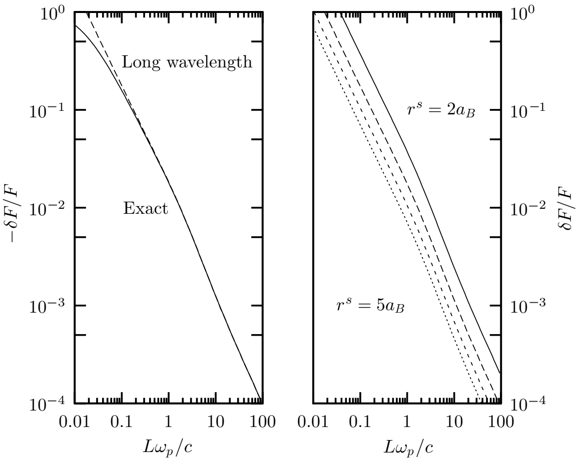

Figure 1. Normalized non-local corrections $\delta F/F$ to the Casimir force as a function of distance L calculated for Au using the hydrodynamic model (left panel) and for free electron metals of different densities corresponding to $r^s/a_\mathrm{B} = 2$, 3, 4 and 5 using the self-consistent semi-infinite jellium model (right panel), where a_B is Bohr's radius and r_s is related to the electronic number density $n = 3/(4\pi r_\mathrm{s}^3)$.

It turns out that the region just outside of the metal is much more polarizable than within the metal, so that most of the screening takes place outside of the nominal surface. Thus, from the electromagnetic point of view, the effective distance between two conductors is smaller than the nominal distance, and therefore, the Casimir force is larger. Although the non-local correction $|\delta F/F|$ is relatively small at currently accessible distances, it grows roughly as L^{-1} and thus will become very important as smaller distances are explored. Figure 1 also shows that the non-local correction increases with the electronic density.

The role of thin metallic coatings in the calculation of Casimir forces has also been studied taking into account spatial dispersion [12] within the Kliewer and Fuchs formalism. It was found that the main non-local contributions come from the coupling of the longitudinal guided collective modes of the thin films with p-polarized light. Although it could have been expected that non-local effects would be more important for thin films than for semi-infinite media, as the width introduces an additional small length scale besides the relatively large wavelength, they were found not to exceed about 7% at small separations. For current experimental setups and separations, non-local corrections are on the order of 0.4%. The effect of thin films within a local approximation has been explored by Lissanti *et al* [34].

6. Conclusions

We have reviewed the calculation of the optical properties of non-local systems emphasizing some of the concepts that have frequently been a source of confusion. We have discussed the meaning of a wave-vector-dependent dielectric response when surfaces are present, the problem of additional boundary conditions and the continuity conditions for the fields. We have shown some expressions which may be simply plugged into the Lifshitz formula in order to calculate the Casimir force including non-local corrections. The effects of non-locality are small at currently accessible distances but they might become very important in future experiments that explore much smaller distances. Self-consistent theories produce a non-local correction which has the opposite sign as that predicted by other models, such as the hydrodynamic model and, more generally, the SCIB model, in which the surface is unrealistically assumed to be truncated abruptly. Thus, to obtain the correct sign, the microscopic electronic density profile must not be disregarded. In this paper, we have concentrated on the non-local corrections at zero temperature and we have not touched upon the important and controversial issue of the thermodynamics of the Casimir force at large separations, where it is believed that non-locality also plays an important role [14].

Acknowledgments

This work was partially supported by DGAPA-UNAM under grants IN117402, IN118605 and IN101605, and CONACyT grant 44306.

References

[1] Lamoreaux S K 1997 *Phys. Rev. Lett.* **78** 5

[2] Mohideen U and Roy A 1998 *Phys. Rev. Lett.* **81** 4549
Chen F, Mohideen U, Klimchitskaya G L and Mostepanenko V M 2005 *Phys. Rev.* A **72** 020101
Chen F, Mohideen U, Klimchitskaya G L and Mostepanenko V M 2002 *Phys. Rev. Lett.* **66** 032113

[3] Chan H B, Aksyuk V A, Kliman R N, Bishop D J and Capasso F 2001 *Science* **291** 1942

[4] Iannuzzi D, Lisanti M and Capasso F 2004 *Proc. Natl Acad. Sci. USA* **101** 4019
Lissanti M, Iannuzzi D and Capasso F 2005 *Proc. Natl Acad. Sci. USA* **102** 11989

[5] Decca R S, López D, Fischbach E and Krause D E 2003 *Phys. Rev. Lett.* **91** 050402

[6] Kats E I 1977 *Sov. Phys.—JETP* **46** 109

[7] Heinrichs J 1975 *Phys. Rev.* B **11** 3625

[8] Buhl W 1976 *Z. Phys.* B **23** 221

[9] Esquivel-Sirvent R, Villarreal C and Mochán W L 2003 *Phys. Rev.* A **68** 052103
Esquivel-Sirvent R, Villarreal C and Mochán W L 2005 *Phys. Rev.* A **71** 029904

[10] Esquivel-Sirvent R and Mochán W L 2004 *Quantum Field Theory Under the Influence of External Conditions* ed K Milton (New Jersey: Rinton) p 90

[11] Esquivel R and Svetovoy V B 2004 *Phys. Rev.* A **69** 062102

[12] Esquivel-Sirvent R and Svetovoy V B 2005 *Phys. Rev.* B **72** 045443

[13] Contreras-Reyes A M and Mochán W L 2005 *Phys. Rev.* A **72** 034102

[14] Sernelius B E 2005 *Phys. Rev.* B **71** 235114

[15] Svetovoy V B and Esquivel R 2005 *Phys. Rev.* E **72** 036113

[16] Halevi P 1992 *Spatial Dispersion in Solids and Plasmas* (Amsterdam: North-Holland/American Elsevier)

[17] Cocoletzi G H and Mochán W L 2005 *Surf. Sci. Rep.* **57** 1–58

[18] Cottam M G and Tilley T R 1989 *Introduction to Surface and Superlattice Excitations* (Cambridge: Cambridge University Press)

[19] Barton G 1979 *Rep. Prog. Phys.* **42** 65

[20] Lindhard J 1954 *K. Dan. Vidensk. Selsk. Mat.-Fys. Medd.* **28** 8

[21] Ziman J M 1972 *Principles of the Theory of Solids* (Cambridge: Cambridge University Press)

[22] Kittel C 1987 *Quantum Theory of Solids* (New York: Wiley)

[23] Ford G W and Weber W H 1984 *Phys. Rep.* **113** 195

[24] Liebsch A 1997 *Electronic Excitations at Metal Surfaces* (New York: Plenum)

[25] Forstmann F and Gerhardts R R 1986 *Metal Optics Near the Plasma Frequency* (Berlin: Springer)

[26] Kliewer K L and Fuchs R 1968 *Phys. Rev.* **172** 607

[27] Stratton J A 1941 *Electromagnetic Theory* (New York: McGraw-Hill)

[28] Kohn W and Sham L J 1965 *Phys. Rev.* A **1133**

[29] Mochán W L, Fuchs R and Barrera R G 1983 *Phys. Rev.* B **27** 771–80

[30] Feibelman P J 1982 *Prog. Surf. Sci.* **12** 287

[31] Jaekel M T and Reynaud S 1991 *J. Physique* I **1** 1395
Genet C, Lambrecht A and Reynaud S 2003 *Phys. Rev.* A **67** 043811

[32] Torres-Guzmán J C and Mochán W L 2006 *J. Phys. A: Math. Gen.* **39** 6791

[33] Procopio L M, Villarreal C and Mochán W L 2006 *J. Phys. A: Math. Gen.* **39** 6679

[34] Lissanti M, Iannuzzi D and Capasso F 2005 *Preprint* quant-ph/0502123v1