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**An  $n^2$ -bound for the ultimate equivalence problem of certain D0L systems over an  $n$ -letter alphabet. (English summary)**

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A D0L system  $(\Sigma, g, w)$  consists of an alphabet  $\Sigma$ , a morphism  $g: \Sigma^* \rightarrow \Sigma^*$  and a word  $w$  over  $\Sigma$ . It is called “growing” if for each  $a$  in  $\Sigma$ , we have  $\lim_{i \rightarrow \infty} |g^i(a)| = \infty$ , where  $|x|$  denotes the length of  $x$ . Given two D0L systems  $(\Sigma, g_k, w_k)$  ( $k = 1, 2$ ), the ultimate equivalence problem consists in deciding whether  $g_1^i(w_1) = g_2^i(w_2)$  holds for all but finitely many  $i \geq 0$ .

The author shows that for growing D0L systems satisfying some technical restrictions, the ultimate equivalence problem is equivalent to deciding whether  $g_1^{B+i}(w_1) = g_2^{B+i}(w_2)$  holds for  $i = 0, 1, \dots, n^2 - 1$ , where  $n$  is the number of symbols in  $\Sigma$  and  $B$  is a number bounded from above by a polynomial function in  $n$  and in some structural entities of the D0L systems such as  $\max\{|g_k(a)|: a \in \Sigma, k = 1, 2\}$ .

Reviewed by *Peter R. J. Asveld*

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