Analysis of the Kirk Effect in Silicon-Based Bipolar Transistors With a Nonuniform Collector Profile

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***Abstract*—In this paper, the Kirk effect has been analyzed for silicon-based bipolar transistors (BJTs) with a nonuniform collector profile. We show that, for any arbitrary collector doping profile, the Kirk effect starts when the electron concentration equals the average doping concentration in the depletion region. We present a basic guideline for determining the collector current density at the onset of Kirk effect (J_K) for any collector doping profile and simple expressions for J_K and the electrical field in the collector drift region for the case of a linearly graded collector drift region. These analytical expressions are verified with device simulations. The Kirk effect for this kind of transistor is substantially different from that presented previously for transistors having a uniform collector drift region. For example, the possibility of the onset of the Kirk effect in a partially depleted collector occurs, while in a uniform collector profile the effect can only occur in a fully depleted collector. Our expressions can be used to do approximate analytical calculations for optimizing future BJTs.**

***Index Terms*—Heterojunction bipolar transistors (HBTs), high-frequency (HF) amplifiers, power semiconductor devices, silicon compounds, simulation.**

I. Introduction

BASE WIDENING into the collector drift region of a bipolar transistor (BJT) is well known to be the cause of the falloff of the cutoff frequency (f_T) at high current densities. For an npn silicon bipolar transistor with a uniformly doped drift region this so-called Kirk effect was found [1] to begin when the collector current density (J_c) reaches a critical value, i.e., the critical current density (J_K)

$$J_K = -q \cdot v_{\rm sat}\left(N_c + \frac{2V\varepsilon}{qW_c^2}\right) \tag{1}$$

where q is the elementary charge, ε is the dielectric constant, N_c is the doping concentration in the collector drift region, $v_{\rm sat}$ is the carrier saturation velocity, V is the applied voltage and W_c is the collector drift length.

For the derivation of (1) the abrupt depletion approximation was used, and (1) was found as the current density for which the electric field at the base-collector junction takes the value zero. An essential assumption was that the electron velocity saturates toward the value $v_{\rm sat} = 10^7$ cm/s, in which case, the electron concentration (n) at J_K can be described by [1]

$$n = \frac{J_K}{-q \cdot v_{\rm sat}} \tag{2}$$

and hence it was assumed that the electron concentration is constant in the collector drift region. This is applicable for silicon based bipolar transistors when the electric field is larger than 10^4 V/cm.

In [2] we have shown that by using the reduced surface field (RESURF) effect in a BJT, the Kirk effect is suppressed for a specific off-state breakdown voltage $(\mathrm{BV_{cbo}})$. For obtaining this Resurf effect, we used a field plate in a trench located along a linearly graded collector profile, as was originally proposed for lateral power devices [3]. Since record $f_T \cdot \mathrm{BV_{cbo}}$ and $f_T \cdot \mathrm{BV_{ceo}}$ values can be obtained from this device concept it is of interest to examine the Kirk effect for BJTs containing linearly graded drift regions. Note that during the investigation of this concept [2], our simulation results indicated that the field plate along the drift region hardly affects the current flow for collector-base voltages $(V_{\rm cb})$ less than the $\mathrm{BV_{cbo}}$, because of its thick dielectric (typically around 0.1 μm). Therefore, in this device concept the current flow has a one-dimensional (1-D) behavior at low $V_{\rm cb}$.

However, in [2] we did not analyze the Kirk effect in detail. Therefore, in this paper the Kirk effect has been analyzed for silicon-based bipolar transistors (BJTs) with a nonuniform collector profile and in particular a linearly graded collector profile. We show that the Kirk effect starts when the electron concentration equals the average doping concentration in the depletion region of any arbitrary collector doping profile. To our best knowledge, in the literature the difference between the Kirk effect for bipolar transistors having a uniformly doped collector on the one hand, and transistors having e.g., a linearly graded collector on the other hand, has not yet been recognized. The published results for the uniformly doped case are not valid for the linearly doped case.

In Section II, we come up with and explain a new situation for bipolar transistors with a nonuniform collector profile: The partially depleted collector at the onset of Kirk effect.

In Section III, we propose a basic guideline from which an expression for J_K can be derived for a BJT with any collector profile. By using this guideline we derive two simple expressions for J_K applicable to the linearly graded profile, the first of which is valid when the collector drift region is fully depleted and the second of which applies to the partially depleted collector drift region.

Manuscript received February 4, 2005; revised July 5, 2005. The review of this paper was arranged by Editor M. A. Shibib.

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Digital Object Identifier 10.1109/TED.2005.857176

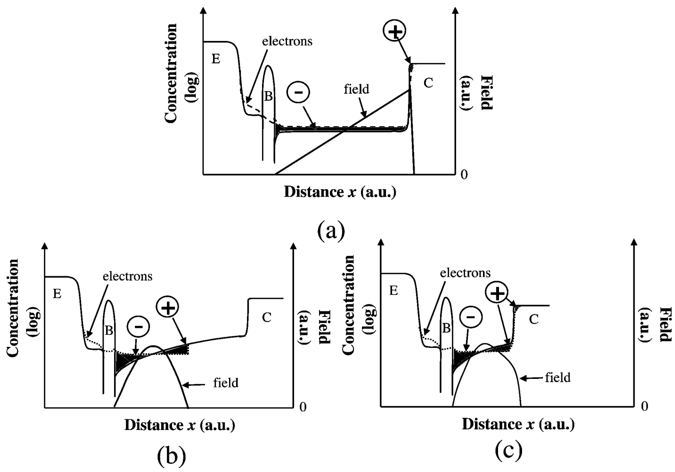

Fig. 1. Schematic drawings of the electron and electric field distribution at the onset of the Kirk effect for the npn BJT containing a collector drift region with a (a) uniform doping profile, (b) partially depleted linearly graded profile and, (c) fully depleted linearly graded profile. The shaded areas indicate the different space charge ($\rho = N_c^+ - n$) contributions. The areas indicated by a minus ("−") sign are overcompensated by electrons.

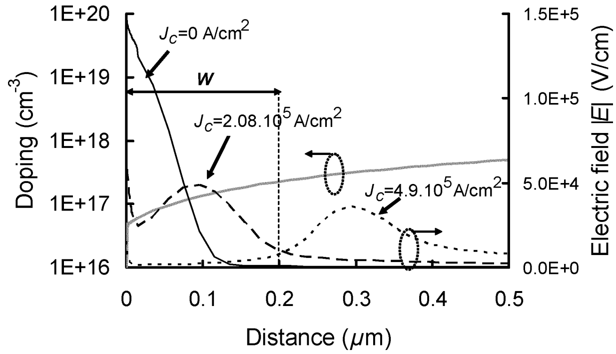

Fig. 2. Simulated electric field distributions in a one-sided linearly graded pn junction as proposed in [2] for three different collector current densities (J_c): $0, 2.08 \cdot 10^5$ and $4.90 \cdot 10^5$ A/cm^2 ($V_{cb} = 0$ V, $T = 300$ K). For convenience sake, the magnitude of the electric field is shown. The onset of the Kirk effect is when $J_c = J_K \approx 2.08 \cdot 10^5$ A/cm^2. The linearly graded doping profile is shown in the same figure. The drift length $W_c = 1.2$ μm, the depletion layer width at the onset of Kirk effect $W = 0.2$ μm, the slope of the graded profile $\alpha = 9 \cdot 10^{21}$ cm^{-4} and the doping at the pn junction $N_0 = 4 \cdot 10^{16}$ cm^{-3}. For clarification only a part of the doping profile is shown. The horizontal axis is with respect to the emitter–base junction.

Interestingly, our derivations show that analytical expressions for the depletion layer width are complicated, but our final results for J_K are rather simple. The analytical results for the linearly graded profile are compared with simulation data and are discussed in Section IV. Finally, in Section V we come up with conclusions.

II. New State: Partially Depleted Collector

In advanced SiGe:C heterojunction bipolar transistors (HBTs) the base typically has a doping concentration of 10^{19} cm^{-3} or higher [4], [5] and hence these HBTs contain one-sided (asymmetrical) p$^+$nn$^+$- junctions in which the penetration of the depletion layer into the heavily doped base region is negligible. Fig. 1(a) schematically shows the electron and electric field distributions for an npn BJT with a uniform collector profile at the onset of the Kirk effect. For this case, the slope of the electric field through the collector drift region is constant and hence, the collector drift region is fully depleted.

From (1) and (2) it can be seen that when $J_c = J_K$, n equals N_c plus a voltage dependent term $(2V\varepsilon/qW_c^2)$. As we shall show later, this voltage dependent term is actually the (positive) charge contribution from the n$^+$ collector depletion region divided by q, as indicated in Fig. 1(a).

For the npn BJT containing a linearly graded collector profile, schematic drawings of the electron and electric field distribution at the onset of Kirk effect $(J_c = J_K)$ are shown for two different conditions: 1) the condition as shown in Fig. 1(b), a so-called partially depleted case and 2) the condition as shown in Fig. 1(c), a fully depleted case. For all profiles, as for the uniform collector profile, the electron concentration in the depleted collector is about constant. In linearly graded doped collectors however, the field distribution has a parabolic shape; this remains true for J_c beyond the current density at which the maximum cutoff frequency (f_T) is obtained. The peak electric field is located at the position where the uniform electron distribution crosses the doping profile, i.e., where the electrons precisely compensate the ionized donors. In linearly graded doped collectors therefore, when $J_c = J_K$ the collector may be partially depleted, a situation that is excluded in uniformly doped collectors.

In Fig. 2 numerically calculated electric field distributions are plotted for a partially depleted linearly graded collector doping profile as proposed in [4] for three different collector current densities $J_c = 0, 2.08{\cdot}10^5\ (= J_K)$ and $4.90{\cdot}10^5$ A/cm^2, respectively. The simulations indeed show at $J_c = J_K$ and beyond, electric fields of more than 10^4 V/cm, yielding almost constant electron distributions across the depletion layer. Hence, (2) is applicable for the analysis which is discussed in the following sections.

III. Kirk Effect Analysis

A. Basic Guideline

Following [1], we neglected the influence of hole diffusion into the collector drift region on the onset of Kirk effect; this approximation is further discussed in Appendix B. Furthermore, we shall now first discuss and then adopt the assumption that the drift velocity takes a constant saturated value $v_{\rm sat}$.

We first derive a general formula for the critical current density J_K for any arbitrary collector profile having a doping concentration $N_c(x)$, where x is the distance. This J_K is then given as a function of the collector depletion layer width (W).

At the onset of the Kirk effect $(J_c = J_K)$ n is not negligible with respect to the doping concentration. Applying Gauss's law to Poisson's equation $(\varepsilon(dE/dx) = q \cdot \{N_c(x) - n(x)\})$ while the electric field at both the base–collector junction and at the end of the depletion region equals zero we find

$$\int_0^W (N_c(x) - n(x))\,dx = \int_0^W \left(N_c(x) + \frac{J_K}{q \cdot v\,(E(x))}\right) dx = 0 \tag{3a}$$

where W is the depletion layer width for $J_c = J_K$ and the electron velocity v is a function of the electric field E which depends on the distance x.

We can derive from (3a) a general expression for J_K

$$J_K = -q \cdot \frac{\int_0^W N_c(x)dx}{\int_0^W \frac{1}{v(E(x))}dx} = -q \cdot \overline{N} \cdot \widetilde{v} \tag{3b}$$

where $\overline{N}$ is the average doping concentration in the depleted collector region and $\widetilde{v}$ is the harmonic average velocity in which the low velocity values are relatively more important and tend to reduce the average.

Equation (3b) is valid for all types of semiconductor. In view of the fact that $v(E)$ vanishes for $E = 0$, i.e., both at $x = 0$ and at $x = W$, the integrand $1/v$ in $\widetilde{v}$ is singular at the integral's boundaries. At low fields, however, it generally holds that $v = -\mu \cdot E$ and from Poisson's equation it may then be derived that $E(x) \propto \sqrt{x}$ for E going to zero. For such spatial dependence, the integral in $\widetilde{v}$ is convergent, so that (3b) is meaningful for all physical $v(E)$ models. For the often applied model $\widetilde{v} \approx v_{\mathrm{sat}}$ these subtleties disappear altogether.

When, as is usually done for silicon, v is modeled by $v = v_{\mathrm{sat}}$, it follows that $\widetilde{v} = v_{\mathrm{sat}}$ and hence (2) is applicable. From (2) and (3b) it then follows that at the onset of Kirk effect the (constant) electron concentration equals the average doping concentration in the depletion region of the collector profile

$$n = \frac{\int_0^W N_c(x)dx}{W} = \overline{N}. \tag{3c}$$

In Appendix A we use (3c) for showing that the voltage dependent term in (1) is actually the charge contribution in the depleted n+ collector divided by q.

However, for analyzing the Kirk effect, a formula for the electric field distribution $(E(x))$ is required.

From Poisson's equation $E(x)$ can be derived for silicon-based BJTs

$$E(x) = \frac{q}{\varepsilon}x \cdot \left\{ \frac{\int_0^x N_c(\xi)d\xi}{x} - n \right\} + c_1 \tag{4}$$

where c_1 is an integration constant and the integral term represents the average doping concentration in the depleted collector region.

The potential distribution $\psi(x)$ can be derived from the electric field as described by (4)

$$\psi(x) = -\frac{q}{2\varepsilon}x^2 \cdot \left\{ \frac{2 \cdot \int_0^x \int_0^\eta N_c(\xi)d\xi d\eta}{x^2} - n \right\} - c_1 \cdot x + c_2 \tag{5}$$

where c_2 is an integration constant.

The integration constants can be determined by using the correct boundary conditions.

When the Kirk effect starts, the electric field at the base–collector junction vanishes [1], and hence the boundary condition $E(0) = 0$ applies, then from (4) it follows that $c_1 = 0$. If we choose $\psi(0) = 0$ as a boundary condition, then from (5) it follows that $c_2 = 0$.

Finally, the total collector depletion region carries the total voltage drop and consequently $\psi(W) = V$. From (5) and this boundary condition an implicit equation for V, W and n [or, equivalently, J_K; see (2)] follows. Equation (3c) is also such an equation, so that we now have two equations with the two variables W and n. As an example we now apply this guideline to the linearly graded profile.

B. Linearly Graded Profile

A linearly graded drift doping concentration $N_c(x)$ can be described as

$$N_c(x) = N_0 + \alpha \cdot x \tag{6}$$

where N_0 is the drift doping concentration at the junction and α is the slope of the doping profile.

Analogous to the derivation of (4), from (6) we can derive that

$$E(x) = \frac{q}{\varepsilon}x \cdot \left\{ (N_0 - n) + \frac{1}{2} \cdot \alpha x \right\} + c_1. \tag{7}$$

Again applying the boundary condition $E(0) = 0$, yields $c_1 = 0$.

From (7) it can be derived that

$$\psi(x) = -\frac{q}{2\varepsilon}x^2 \cdot \left\{ (N_0 - n) + \frac{1}{3} \cdot \alpha x \right\} + c_2. \tag{8}$$

Using (8), if we choose $\psi(0) = 0$ as a boundary condition it follows that $c_2 = 0$.

From (8) and the boundary condition $\psi(W) = V$, it follows that

$$V = \frac{-qW^2 \cdot \{3(N_0 - n) + \alpha \cdot W\}}{6\varepsilon}. \tag{9}$$

This expression we use for the final determination of J_K for both the fully depleted and the partially depleted situation.

For the fully depleted case we assume that the depletion layer width equals the collector drift length $(W = W_c)$.

Then, from the substitution of (2) in (9) an analytical expression for J_K follows:

$$J_K = -q \cdot v_{\mathrm{sat}} \left(N_0 + \frac{W_c \cdot \alpha}{3} + \frac{2V\varepsilon}{q \cdot W_c^2} \right). \tag{10}$$

According to (3b) and (3c) the three terms in (10) between the brackets represent the electron charge that compensates the average doping concentration. The third term between the brackets in (10), like in (1), represents the charge contribution in the n^+ collector (see also Appendix A for the uniform doping profile). Note that when $\alpha = 0$ the expression (10) reduces to the traditional expression (1) for the uniformly doped drift region [1].

As mentioned earlier, in a linearly doped collector, the electric field is parabolic and therefore, in contrast with the case of a uniformly doped collector, the collector may be partially depleted at the onset of Kirk effect $(E(0) = 0)$. The collector will then be divided in a depleted zone of width $W < W_c$ and a (nondepleted) ohmic region, as indicated in Fig. 1(b).

In this partially depleted case, the electric field at the depletion layer edge vanishes, and hence, $E(W) = 0$. Again, with (7) and the boundary condition $E(W) = 0$ it follows that:

$$W = \frac{2(n - N_0)}{\alpha}. \quad (11)$$

From (9) and (11) it can be derived that

$$W = \left(\frac{12\varepsilon V}{q\alpha}\right)^{\frac{1}{3}} \quad (12)$$

this corresponds with the depletion layer width of a double-sided linearly graded pn junction [6]. The explanation for this correspondence is as follows. Since the uniform electron distribution partly overcompensates the doping in the drift region, as schematically shown in Fig. 1(b), the space charge distribution (ρ) in the collector of our one-sided junction is similar to the overall space charge distribution in the double-sided graded, current-less, junction. Because in both cases the voltage drop is across a similar space charge region $(\rho \neq 0)$, the full width of the space charge region in both cases obeys a similar formula, namely (12).

From (2), (9), (11), and (12), an analytical expression for J_K can be derived

$$J_K = -q \cdot v_{\text{sat}} \cdot \left\{N_0 + \frac{\alpha W}{2}\right\} = -q \cdot v_{\text{sat}} \cdot \left\{N_0 + \left(\frac{3\varepsilon V \alpha^2}{2q}\right)^{\frac{1}{3}}\right\}. \quad (13)$$

The term $N_0 + ((\alpha \cdot W)/2)$ in (13) between the brackets represents the electron charge that compensates the average doping concentration. When $\alpha = 0$, (13) reduces to $J_K = -q \cdot v_{\text{sat}} \cdot N_0$. According to (2) then $n = N_0$ and from (7) it then follows that $E(x) = 0$, because $c_1 = 0$. This state corresponds to a fully ohmic collector in the limit of zero resistance. Hence, in this limit (13) does not apply for a realistic situation.

Equation (13) holds when the length of the collector drift region W_c is larger than or equal to W. In the case of $W = W_c$ using (10) and (12) it can be verified that (10) and (13) are equivalent.

Equation (13) was derived under condition that the voltage drop across the ohmic region is negligible. If it is not, i.e., if a voltage drop $J_c \cdot R_c$ across the resistance R_c of the ohmic part of the collector has to be taken into account at the onset of the Kirk effect, it can be derived that

$$V - \psi^*(\alpha, W, J_K, W_c) = J_K \cdot R_c \quad (14)$$

where $\psi^*(\alpha, W, J_K, W_c)$ is a function depending on α, W, J_K and W_c; it is a third order function of W. Further, for R_c the following expression can be derived:

$$R_c = \int_W^{W_c} \rho dx = \int_W^{W_c} \frac{1}{q \cdot \mu_n \cdot (N_0 + \alpha \cdot x)} dx = \frac{1}{q \cdot \mu_n \cdot \alpha} \cdot \ln\left(\frac{N_0 + \alpha W_c}{N_0 + \alpha W}\right) \quad (15)$$

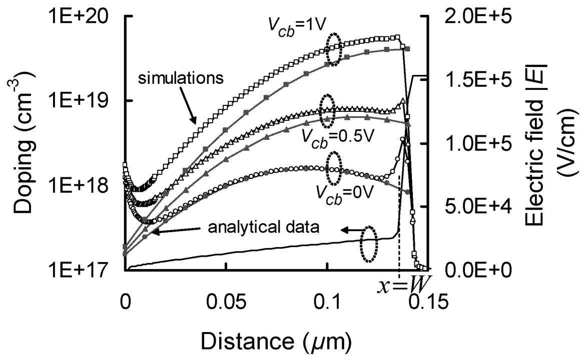

Fig. 3. Electric field distribution for a fully depleted one-sided linearly graded pn-junction for three different biases: $V_{\text{cb}} = 0, 0.5$ and 1 V corresponding with $J_K = 3.31 \cdot 10^5, 3.69 \cdot 10^5$, and $4.10 \cdot 10^5$ A/cm^2, respectively. The magnitude of the electric field is shown. The open symbols are simulation data and closed symbols represent the analytical data (7). The drift length $W_c = 0.14$ μm, the slope of the graded profile shown is $\alpha = 10^{22}$ cm^{-4} and the doping at the pn junction $N_0 = 10^{17}$ cm^{-3}. The horizontal axis is with respect to the emitter–base junction.

where μ_n is the electron (majority) mobility and is in (15) assumed to be independent of the doping.

The combination of (14) and (15) makes the derivation of a practical expression for J_K difficult, perhaps even impossible. Therefore, for the partially depleted drift region we only analyzed cases in which R_c is negligible, and compared predictions based on (7), (10), and (13) with simulation data in Section IV.

IV. Results and Discussion

A. Device Parameters

In order to verify the analytical expressions described in Section III, simulations with MEDICI [7] were performed for 1-D npn BJTs. All our device structures contain a highly doped base region with a doping concentration of $5 \cdot 10^{19}$ cm^{-3} and a total width of 15 nm [3]. The collector drift region contains a linearly graded doping profile in which α, N_0, and W_c are varied. Generally, the collector drift region is much lower doped than the base, and hence a one-sided pn junction is appropriate here. In this paper the doping close to the base–collector junction varies in between 10^{16} and $2 \cdot 10^{17}$ cm^{-3}, and hence the built-in voltage (V_{bi}) varies between 0.93 and 0.97 V. Since this parameter does not affect the results so much (up to 2%), we approximated in our analytical data V_{bi} with a constant value of 0.95 V. This value was used in our analytical data via $V = V_{\text{bi}} + V_{\text{cb}}$, where V_{cb} is the applied collector–base voltage. For various values of V_{cb}, α, N_0 and W_c, the theoretical electric field distributions at $J_c = J_K$ were compared with device simulations, for both a fully and partially depleted collector drift region.

B. Fully Depleted Case

In Fig. 3 open symbols show the simulated electric field distributions at $J_c = J_K$ for the fully depleted case for three different V_{cb}: 0, 0.5 and 1 V. Like Fig. 2, the plots show that the field has a parabolic distribution. For this device structure the device parameters were $N_0 = 10^{17}$ cm^{-3}, $\alpha = 10^{22}$ cm^{-4}, and $W_c = 0.14$ μm. Using (10) it could be calculated that $J_K = 3.31 \cdot 10^5$ $(V_{\text{cb}} = 0$ V), $3.69 \cdot 10^5$ $(V_{\text{cb}} = 0.5$ V) and

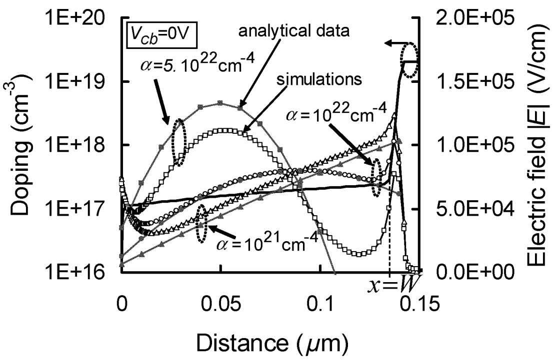

Fig. 4. Electric field distribution for a fully depleted one-sided linearly graded pn-junction for three different slopes: $\alpha = 10^{21}$, 10^{22}, and $5 \cdot 10^{22}$ cm^{-4}. The magnitude of the electric field is shown. The current densities J_K are $2.52 \cdot 10^5$, $3.31 \cdot 10^5$, and $6.42 \cdot 10^5$ A/cm^2 for $V_{\rm cb} = 0$ V, respectively. The open symbols are simulation data and the closed symbols represent the analytical data (7). The drift length $W_c = 0.14\,\mu$m and the doping at the pn junction $N_0 = 10^{17}$ cm^{-3}. The graded profile is shown for $\alpha = 10^{22}$ cm^{-4}. The horizontal axis is with respect to the emitter–base junction.

$4.1 \cdot 10^5$ A/cm^2 ($V_{\rm cb} = 1$ V). Using (2) and (11) we verified that the condition for full depletion $W > W_c$ was indeed satisfied. For comparison, the analytical field distribution data are shown by closed symbols as described by (7) in which we used (10) for determining the electron concentration. A good agreement with the simulation data is found. The discrepancies are due to the hole diffusion charge from the pn base–collector junction which was not taken into account in the analytical expressions (7) and (10). The hole diffusion is discussed in Appendix B. Next, in order to show the influence of α, in Fig. 4 the field distributions are plotted for several values of α, while $N_0 = 10^{17}$ cm^{-3}, $V_{cb} = 0$ V and $W = 0.14$ μm, were fixed. For α the values of 10^{21}, 10^{22}, and $5 \cdot 10^{22}$ cm^{-4} were taken resulting in $J_K = 2.52 \cdot 10^5$, $3.31 \cdot 10^5$ and $6.42 \cdot 10^5$ A/cm^2, respectively. As can be seen, for a small α the field distribution shows the well-known triangular shape [1] while for large α the field distribution is parabolic. The analytical data show the same trend, again with small differences. However, according to the simulations for $\alpha = 5 \cdot 10^{22}$ cm^{-4} we enter the regime of partial depletion and there is a larger difference between the analytical solution (which still assumes full depletion) and the simulated electric field.

Fig. 5 shows simulated f_Ts as a function of J_c for the three BJTs of Fig. 4 and again the J_K values obtained from (10) are indicated. It can be seen that $J_c = J_K$ is reached beyond the J_c values at which the peak f_Ts are obtained. Besides, for uniformly doped collector profiles the peak f_Ts correspond often with the situation in which a uniform field is obtained. For linearly graded doping profiles however, a uniform field is never obtained as can be seen in e.g., Fig. 2. In the inset, simulated J_cs as a function of the base–emitter voltage $V_{\rm be}$ are shown. In the same plot the J_K values are indicated which indeed correspond with the point of transition from the diffusion regime (60 mV/dec) toward the base widening regime (120 mV/dec).

Fig. 6 shows the analytical data of J_K from (10) as a function of α for a drift length W_c varying between 0.05 and 0.2 μm. In the same plot the J_K values are indicated which were verified with simulations. As expected, the figure shows that indeed the J_K is more dominated by α when W_c increases and consequently more parabolic field distributions are obtained at the onset of the Kirk effect.

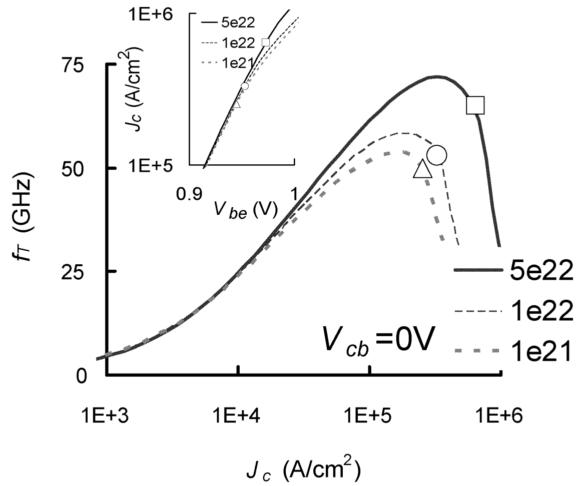

Fig. 5. Cutoff frequency f_T as a function of J_c for three device structures having each a different slope: $\alpha = 10^{21}, 10^{22}, 5 \cdot 10^{22}$ cm^{-4} (see Fig. 4). The J_K values calculated by (10) are indicated by open symbols, showing that they are beyond those J_c values at which peak f_Ts are obtained. The drift length $W_c = 0.14$ μm and the doping at the pn junction $N_0 = 10^{17}$ cm^{-3}. In the inset the collector current density J_c as a function of the base-emitter voltage $V_{\rm be}$ is shown for the three devices ($V_{\rm cb} = 0$ Vand $T = 300$ K).

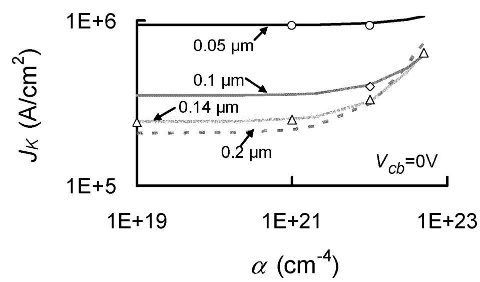

Fig. 6. Analytical current densities at the onset of Kirk effect J_K (10) as a function of the slope α for various drift lengths W_c (= 0.05, 0.1, 0.14 and 0.2 μm; fully depleted collector drift region). The open symbols indicate the J_K values for which the analytical and simulated field distributions were compared. Some typical results are shown in Fig. 4. The doping at the pn junction $N_0 = 10^{17}$ cm^{-3}. ($V_{cb} = 0$ V, $T = 300$ K). For α below 10^{21} cm^{-4} the slope is so small, that it has nearly no influence.

C. Partially Depleted Case

We also evaluated the simulations for the case when the collector region is partially depleted.

For this condition in Fig. 7 the simulated electric field distributions for a BJT are shown for three different applied biases V_{cb}:0, 0.5 and 1 V. For this device structure the device parameters were $N_0 = 4 \cdot 10^{16}$ cm^{-3}, $\alpha = 9 \cdot 10^{21}$ cm^{-4} and $W_c = 0.3$ μm. With (13) we calculated that $J_K = 2.08 \cdot 10^5$ ($V_{\rm cb} = 0$ V), $2.26 \cdot 10^5$ ($V_{cb} = 0.5$ V) and $2.43 \cdot 10^5$ A/cm^2 ($V_{cb} = 1$ V). From (2) and (11) we find that $W < W_c$ for all cases at hand, so that the collector is partially depleted but such that the collector resistance (R_c) is not important, as assumed. For comparison the analytical field distribution data are shown as described by (7) in which we used (13) for determining the electron concentration. A good agreement with the simulation data is observed.

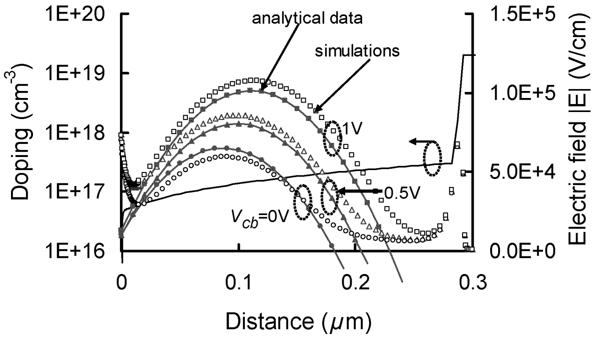

Fig. 7. Electric field distribution for a partially depleted one-sided linearly graded pn-junction for three different biases: $V_{cb} = 0, 0.5$, and 1 V corresponding with $J_K = 2.09 \cdot 10^5, 2.26 \cdot 10^5$, and $2.43 \cdot 10^5$ A/cm^2, respectively. For these current densities the depletion layer width W varies between 0.2 and 0.25 μm. For convenience sake, the magnitude of the electric field is shown. The open symbols are simulation data and closed symbols represent the analytical data (7). The drift length $W_c = 0.3$ μm, the slope of the graded profile shown is $\alpha = 9 \cdot 10^{21}$ cm^{-4} and the doping at the pn junction $N_0 = 4 \cdot 10^{16}$ cm^{-3}. The horizontal axis is with respect to the emitter–base junction.

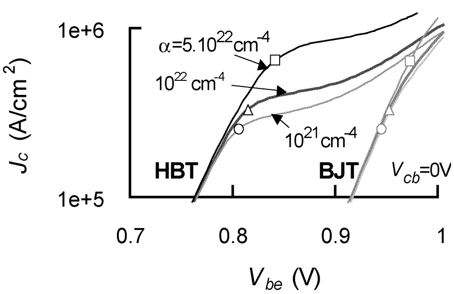

Fig. 8. Collector current density J_c as a function of the base–emitter voltage $V_{\rm be}$ for the SiGe HBT and Si BJT (Fig. 5) each having three different collector profiles: $\alpha = 10^{21}, 10^{22}$, and $5 \cdot 10^{22}$ cm^{-4}. The J_K values calculated by (10) are indicated by open symbols. The drift length $W_c = 0.15$ μm and the doping at the pn junction $N_0 = 10^{17}$ cm^{-3} ($V_{\rm cb} = 0$ V, $T = 300$ K).

While in the cases discussed above, R_c does not have significant influence, in other cases it could. The chance that it does increases if W_c becomes longer, because if W_c increases, the voltage drop across the neutral collector drift region would increase. In Fig. 2 for instance, the simulations show that the electric field has a constant value of $5 \cdot 10^3$ V/cm at $x = 0.3$ μm for the situation when $J_c = J_K \approx 2.08.10^5$ A/cm^2. Using (7) for $J_c = J_K$ we obtained a 20% lower peak field compared with data shown in Fig. 2. The fact that we have an ohmic collector region is an essential difference with a uniform collector profile since the latter is fully depleted for the situation when the Kirk effect occurs.

D. Application to SiGe Devices

We also compared simulation results of three fully depleted device structures, with graded collector similar to the devices as discussed for Fig. 4, but which contain a 20% uniform Ge profile across the base extending 20 nm into the collector drift region. The peak electrical fields obtained with (7) showed differences of at most 20% with these simulation data, caused by the heterojunction barrier induced field in the collector drift region. Fig. 8 shows J_c for the SiGe heterojunction bipolar transistor (HBT) having the same doping profiles as discussed for the BJT (Fig. 4). For comparison the simulation data of the Si BJT considered earlier (Fig. 5) are repeated in Fig. 8. As discussed in [8], the onset of Kirk effect in the SiGe HBT is more pronounced than in the Si BJT. The analytically calculated J_K values are indicated in the same plot, showing that (10) can still be used as a rule of thumb for determining the onset of the Kirk effect. Note that the J_K is more than doubled when α increases from 10^{21} to $5 \cdot 10^{22}$ cm^{-4}.

V. Conclusion

In this paper, we have shown that the onset of Kirk effect for silicon-based bipolar transistors with any arbitrary collector profile starts when the electron concentration equals the average doping concentration in the collector depletion region. The Kirk effect for bipolar transistors containing a one-sided pn junction with a linearly graded doping profile has been analyzed using simulations and analytical calculations. We have shown that at the onset of the Kirk effect for these transistors two conditions could occur in which the collector drift region is either 1) fully depleted or 2) partially depleted. We proposed a basic guideline for determining the collector current density at the onset of Kirk effect for any collector doping profile and, as a result, we found two simple expressions applicable to the linearly graded collector drift region which can be used as a rule of thumb for optimising future bipolar transistors.

Appendix A

In order to show that for the uniformly doped collector drift region [see, e.g., Fig. 1(a)] the term $2V\varepsilon/qW_c^2$ in (1) corresponds to the contribution of the depleted n+ collector charge, we start with a description for the collector profile $N_c(x)$

$$N_c(x) = N_c + (N_s - N_c) \cdot U(x - W_c) \tag{A1}$$

where $U(x)$ is a step function that equals unity if $x \geq 0$, and equals zero otherwise, and N_s is the doping concentration of the n+ collector.

From (3c) and (A1) it can be derived that for the electron concentration it holds that

$$\begin{aligned} n &= \frac{\int_0^W (N_c + (N_s - N_c) \cdot U(x - W_c))\, dx}{W} \\ &= \frac{N_c \cdot W_c + N_s \cdot (W - W_c)}{W} \\ &\approx N_c + \overline{N_s}, \end{aligned} \tag{A2}$$

where $\overline{N_s} = N_s \cdot (W - W_c)/W$ and is in fact the contribution of the depleted n+ collector to the average doping concentration of the total depletion region W. The approximation applied in (A2) is only valid when $W \approx W_c$ (or $N_s \gg N_c$).

The potential distribution at $J_c = J_K$ can be derived by substituting (A1) in (5). Using the boundary condition $\psi(W) = V$ this results in [see also (8) and (9) for $\alpha = 0$]

$$\frac{q}{2\varepsilon} \cdot \left\{(n - N_c) \cdot W_c^2 + (n - N_s) \cdot (W - W_c)^2\right\} = V. \tag{A3}$$

Substituting (A2) in (A3) and assuming $W \approx W_c$ (or $N_s \gg N_c$) results in

$$\overline{N_s} = \frac{2V\varepsilon}{q \cdot W_c \cdot (2 \cdot W_c - W)} \approx \frac{2V\varepsilon}{qW_c^2}. \tag{A4}$$

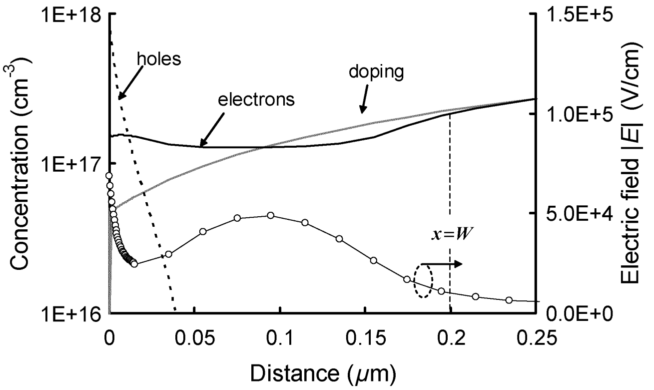

Fig. 9. Electric field and charge carrier concentrations at the onset of Kirk effect in a one-sided linearly graded pn junction as proposed in [2] for $J_c = J_K \approx 2.08 \cdot 10^5$ A/cm^2 (see also Fig. 2). The linearly graded doping profile is shown in the same figure. The drift length $W_c = 1.2$ μm, the depletion layer width at the onset of Kirk effect $W = 0.2$ μm, the slope of the graded profile $\alpha = 9 \cdot 10^{21}$ cm^{-4} and the doping at the pn junction $N_0 = 4 \cdot 10^{16}$ cm^{-3}.

Next, using (A2) and (A4) in (2) results in the critical current density J_K according to (1).

Hence, according to (A4) the term $2V\varepsilon/qW_c^2$ in (1) is actually the (positive) charge contribution from the n $^+$ collector depletion region divided by q.

Appendix B

In this Appendix, we briefly discuss the neglect of the hole contribution in for instance (7) and (8). As is well-known, at the onset of the Kirk effect the positive space charge in the drift region is overcompensated by electrons and the electric field at the base-collector junction becomes low. As a result, the hole distribution in the base is expanding or in fact diffusing into the collector. In Fig. 9 the charge carrier concentrations and the electric field are shown for the device structure proposed in [2] when $J_c = J_K \approx 2.08 \cdot 10^5$ A/cm^2 (see also Fig. 2). The peak field at the junction is $6.9 \cdot 10^4$ V/cm, rather than 0 V/cm as assumed in our analysis and also in [1], which corresponds to an integrated carrier concentration of $4.3 \cdot 10^{11}$ cm^{-2}. This peak field at the junction is due the hole diffusion. In the same figure we see that the diffused holes expand in the collector drift region. If we focus on the part of the electric field distribution that we compared with our analytical data, and locate the minimum field at $x = 0.02$ μm, we see that the hole concentration is about 20% with respect to the electron concentration. Hence, this value cannot be neglected and as a result (7) and (8) do not represent the complete physics and discrepancies occur between the presented theory and simulation data. This hole diffusion is always present irrespective of the base and collector profile. So far a maximum error of about 10% has been obtained in our results because of the neglect of the hole diffusion.

Acknowledgment

The authors would like to thank Dr. G.A.M. Hurkx and A. Heringa, Philips Research Leuven, Belgium, for critically reading this paper and the useful discussions.

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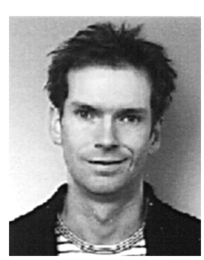

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