

Fig. 3. Plot of the ratio of expressions for $\text{var}(\hat{\rho}_{III})$ to $\text{var}(\hat{\rho}_I)$ versus the population correlation for N equal to 4096.

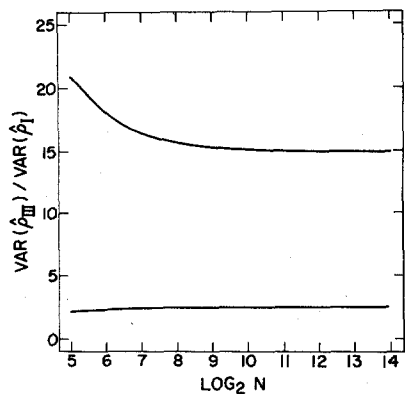


Fig. 4. Plot of the ratio of expressions for $\text{var}(\hat{\rho}_{III})$ to $\text{var}(\hat{\rho}_I)$ versus the sample number N for $\rho = 0.98$ (upper curve) and $\rho = 0.0$ (lower curve).

tion of N at $\rho = 0.98$ and $\rho = 0.0$. In all cases, the variances tend to zero as ρ approaches unity or as N increases. Moreover, for all the estimators, the variances are proportional to $1/N$.

In addition, Fig. 3 plots the functional ratio of $\text{var}(\hat{\rho}_{III})$ to $\text{var}(\hat{\rho}_I)$ versus the true population correlation (or equivalently, the ratio of samples required with estimator III to the samples required with estimator I for equal variance). Note that the closer the population correlation is to unity, the larger the ratio; this indicates that $\text{var}(\hat{\rho}_I)$ converges to zero faster than $\text{var}(\hat{\rho}_{III})$. Fig. 4 plots the functional ratio of $\text{var}(\hat{\rho}_{III})$ to $\text{var}(\hat{\rho}_I)$ versus N at $\rho = 0.98$ and 0.0 . At both population correlations, the ratio is constant indicating equal convergence rates as N gets large.

DISCUSSION

The number of samples required by the two indirect methods are within an order of magnitude of the samples required by the direct estimator for population correlations up to approximately 0.95. This is particularly significant for the polarity coincidence estimator which requires only comparators, a counter, and a table lookup scheme to estimate correlation. The other two methods require analog-to-digital conversion and either addition alone or both multiplication and addition to compute correlation. Furthermore, if the samples were to be stored, the polarity coincidence estimator need only retain

the sign of the samples, whereas the other two methods require storage of all or half the samples.

Thus, if one is willing to take twice or three times the number of samples, the computations involved in estimating correlation can be significantly simplified. However, it should be noted that if the polarity coincidence estimator is to be used for estimation of the cross-correlation function $R_{xy}(\tau)$, it can only estimate the *shape* of the function to within a constant factor $R_{xy}(0)$.

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Relative Linear Power Contribution with Estimator Statistics

PETER LÖHNBERG

Abstract—The relative contribution by a noiselessly observed input signal to the power of a possibly disturbed observed stationary output signal from a linear system is expressed into signal spectral densities. Approximations of estimator statistics and derived confidence limits agree fairly well with simulation results for white signals.

I. INTRODUCTION

An observed stationary signal y may often be considered as the output v of a linear system with frequency response function $H(f)$ driven by a noiselessly observed input signal u , with additive uncorrelated disturbance. Then it may be of interest to know the *relative contribution*

$$R = P_v/P_y \tag{1}$$

by u to the power P_y of y . The power of v

$$P_v = 2 \int_0^\infty G_{vv}(f) df \tag{2}$$

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with power spectral density of v

$$G_{vv}(f) = |H(f)|^2 G_{uu}(f) = \frac{G_{uy}(f) G_{yu}(f)}{G_{uu}^2(f)} G_{uu}(f) \\ = \frac{G_{uy}(f) G_{yu}(f)}{G_{uu}(f)} \quad (3)$$

Cross-spectral density $G_{uy}(f)$ with complex conjugate $G_{yu}(f)$, and power spectral density $G_{uu}(f)$ of u , may be estimated from recordings of u and y . Expression of P_y into power spectral density $G_{yy}(f)$ of y , similarly to (2), allows a simple total procedure with identical filtering of u and y . Substitution into (1) of this P_y and of P_v according to (2) with (3) yields

$$R = \int_0^\infty \frac{G_{uy}(f) G_{yu}(f)}{G_{uu}(f)} df / \int_0^\infty G_{yy}(f) df. \quad (4)$$

It follows that R is real, $0 \leq R \leq 1$, $R = 0$ for $H(f) = 0$, and $R = 1$ for no disturbance.

Equation (4) carries some resemblance to the squared coherence function, defined as

$$C(f) = \frac{G_{uy}(f) G_{yu}(f)}{G_{uu}(f) G_{yy}(f)} \quad (5)$$

$= G_{vv}(f)/G_{yy}(f)$ in this case. The relation of R with $C(f)$ is illustrated by substituting the result of (5) $G_{uy}(f) G_{yu}(f)/G_{uu}(f) = C(f) G_{yy}(f)$ into (4), yielding $R = \int_0^\infty C(f) G_{yy}(f) df / \int_0^\infty G_{yy}(f) df$. This shows R to be a normalized average of $C(f)$ weighted by $G_{yy}(f)$. Note that for $C(f) = C$, $R = C$ and that for $H(f) = H$, R is the squared correlation coefficient.

Estimate \hat{R} of R according to (4) is obtained from estimates $\hat{G}_{uy}(f)$, $\hat{G}_{uu}(f)$, and $\hat{G}_{yy}(f)$ of the respective spectral densities. These may be obtained from recordings of u and y during time of length T by successive sampling, Fourier transformation, complex conjugate multiplication, and convolution by a spectral window $S(f)$ with properties

$$\int_{-\infty}^{\infty} S(f) df = 1, \quad (6)$$

$$\int_{-\infty}^{\infty} S^2(f) df = 1/W \quad (7)$$

with W the window bandwidth [1, (6.4.23)]. Then from (1)-(3) we find

$$\hat{G}_{vv}(f) = \hat{G}_{uy}(f) \hat{G}_{yu}(f) / \hat{G}_{uu}(f), \quad (8)$$

$$\frac{1}{2} T \hat{P}_v = \frac{1}{2} \hat{G}_{vv}(F_1) + \sum_{i=F_1+1/T}^{F_2-1/T} \hat{G}_{vv}(i/T) \\ + \frac{1}{2} \hat{G}_{vv}(F_2) \simeq T \int_{F_1}^{F_2} \hat{G}_{vv}(f) df \quad (9)$$

for frequencies of interest from F_1 to $F_2 = F_1 + F$ with signal bandwidth F , with $\frac{1}{2} T \hat{P}_y$ similarly, and finally

$$\hat{R} = \frac{1}{2} T \hat{P}_v / \frac{1}{2} T \hat{P}_y. \quad (10)$$

II. APPROXIMATE STATISTICS OF \hat{R}

A. Variance

By first-order Taylor expansion of the estimate \hat{R} (10) about its actual value R (1), we obtain the relative variance

$$V[\hat{R}, \hat{R}] \simeq V[\hat{P}_v, \hat{P}_v] + V[\hat{P}_y, \hat{P}_y] - 2V[\hat{P}_v, \hat{P}_y], \quad (11)$$

defining $V[\hat{P}_v, \hat{P}_y] = \text{cov}[\hat{P}_v, \hat{P}_y]/P_v P_y$ as the relative covariance between \hat{P}_v and \hat{P}_y . The terms on the right-hand side of (11) follow from (9) by

$$\text{cov}[\hat{P}_v, \hat{P}_y] = 4 \iint_{F_1}^{F_2} \text{cov}[\hat{G}_{vv}(f), \hat{G}_{yy}(g)] df dg \quad (12)$$

with similar formulas for $\text{var}[\hat{P}_v]$ and $\text{var}[\hat{P}_y]$. By first-order Taylor expansion of $\hat{G}_{vv}(f)$ (8) about $G_{vv}(f)$ (3), follow the relative covariances

$$V[\hat{G}_{vv}(f), \hat{G}_{yy}(g)] \simeq V[\hat{G}_{uy}(f), \hat{G}_{yy}(g)] \\ + V[\hat{G}_{yu}(f), \hat{G}_{yy}(g)] \\ - V[\hat{G}_{uu}(f), \hat{G}_{yy}(g)], \\ V[\hat{G}_{vv}(f), \hat{G}_{vv}(g)] \simeq 2V[\hat{G}_{uy}(f), \hat{G}_{yu}(g)] \\ + V[\hat{G}_{uu}(f), \hat{G}_{uu}(g)] \\ + 2V[\hat{G}_{uy}(f), \hat{G}_{uy}(g)] \\ - 2V[\hat{G}_{uy}(f), \hat{G}_{uu}(g)] \\ - 2V[\hat{G}_{yu}(f), \hat{G}_{uu}(g)]. \quad (13)$$

For arbitrary distribution of u and y , it follows from [1, (A9.1.25)] that if the spectral densities are smooth over the window bandwidth W , a similar derivation as for [1, (A9.1.28)] yields

$$V[\hat{G}_{ab}(f), \hat{G}_{cd}(g)] \simeq D(f-g) \frac{G_{ac}(k) G_{db}(k)}{G_{ab}(k) G_{cd}(k)} \quad (14)$$

with $k = (f+g)/2$ and defining

$$D(f-g) = \frac{1}{T} \int_{-\infty}^{\infty} S(f-h) S(g-h) dh, \quad (15)$$

which because of (6) has the property

$$\int_{-\infty}^{\infty} D(f) df = 1/T. \quad (16)$$

Substitution of the reverse of (5) into (14) and of the results into (13) yields

$$V[\hat{G}_{vv}(f), \hat{G}_{yy}(g)] \simeq D(f-g) \{2 - C(k)\}; \\ V[\hat{G}_{vv}(f), \hat{G}_{vv}(g)] \simeq D(f-g) \left\{ \frac{2}{C(k)} - 1 \right\}. \quad (17)$$

Substitution of (17) with (5) into (12) yields by use of (16)

$$\text{var}[\hat{P}_y] \simeq \frac{4}{T} \int_{F_1}^{F_2} G_{yy}^2(f) df, \\ \text{var}[\hat{P}_v] \simeq \frac{8}{T} \int_{F_1}^{F_2} G_{vv}(f) G_{yy}(f) df - \frac{4}{T} \int_{F_1}^{F_2} G_{vv}^2(f) df \\ \simeq \text{cov}[\hat{P}_v, \hat{P}_y]. \quad (18)$$

The spectral effects in (18) can be expressed by *cocoloration factors*, defined by

$$K_{vy} = 4F \int_{F_1}^{F_2} G_{vv}(f) G_{yy}(f) df / P_v P_y \quad (19)$$

with properties: $K_{vy} \geq 1$ for $G_{vv}(f) = G_{yy}(f)$; $K_{vy} = 1$ for v and/or y white from $f = F_1$ to F_2 . Similar formulas define K_{vv} and K_{yy} . Substitution of (19) into (18) yields, defining the relative power variance $Q = 1/FT$, $\text{var}[\hat{P}_y] \approx QK_{yy}P_y$ and $\text{var}[\hat{P}_v] \approx Q(2K_{vy}P_vP_y - K_{vv}P_v^2) \approx \text{cov}[\hat{P}_v, \hat{P}_y]$, so that

$$\begin{aligned} V[\hat{P}_y, \hat{P}_y] &\approx QK_{yy}; & V[\hat{P}_v, \hat{P}_v] &\approx Q(2K_{vy}/R - K_{vv}); \\ V[\hat{P}_v, \hat{P}_y] &\approx Q(2K_{vy} - K_{vv}R). \end{aligned} \quad (20)$$

Finally, substitution of (20) into (11) yields $V[\hat{R}, \hat{R}] \approx Q(2K_{vy}/R - K_{vv} + K_{yy} - 4K_{vy} + 2K_{vv}R)$, so that

$$\text{var}[\hat{R}] \approx QR(2K_{vy} - RK_{vv} + RK_{yy} - 4RK_{vy} + 2R^2K_{vv}). \quad (21)$$

If all K 's are 1, $R = C =$ the squared correlation coefficient and $\text{var}[\hat{R}] \approx 2QR(1 - R)^2$. Then $\text{var}[\hat{R}]$ is a factor F/W smaller than $\text{var}[\hat{C}(f)] = 2C\{1 - C\}^2/WT$ [1, (9.2.19)].

B. Bias

By second-order Taylor expansion of \hat{R} (10) about R (1) the relative bias, defined as bias $[\hat{R}]/R$, equals

$$\begin{aligned} B[\hat{R}] &\approx B[\hat{P}_v] - B[\hat{P}_y] - B[\hat{P}_v]B[\hat{P}_y] - V[\hat{P}_v, \hat{P}_y] \\ &\quad + B^2[\hat{P}_y] + V[\hat{P}_y, \hat{P}_y]. \end{aligned} \quad (22)$$

It follows from (2) that

$$\text{bias}[\hat{P}_v] = 2 \int_{F_1}^{F_2} \text{bias}[\hat{G}_{vv}(f)] df \quad (23)$$

and similarly, $\text{bias}[\hat{P}_y]$ follows from $\text{bias}[\hat{G}_{yy}(f)]$. By second-order Taylor expansion of $\hat{G}_{vv}(f)$ (8) about $G_{vv}(f)$ (3) follows for each frequency f

$$\begin{aligned} B[\hat{G}_{vv}] &\approx B[\hat{G}_{uy}] + B[\hat{G}_{yu}] - B[\hat{G}_{uu}] + B^2[\hat{G}_{uu}] \\ &\quad + V[\hat{G}_{uu}, \hat{G}_{uu}] + B[\hat{G}_{uy}]B[\hat{G}_{yu}] \\ &\quad + V[\hat{G}_{uy}, \hat{G}_{uy}] - B[\hat{G}_{uy}]B[\hat{G}_{uu}] \\ &\quad - V[\hat{G}_{uy}, \hat{G}_{uu}] - B[\hat{G}_{yu}]B[\hat{G}_{uu}] \\ &\quad - V[\hat{G}_{yu}, \hat{G}_{uu}]. \end{aligned} \quad (24)$$

If the spectra do not vary significantly over the window bandwidth W , it follows from [2, eq. (3)] that

$$B[\hat{G}_{uu}] = B[\hat{G}_{uy}] = B[\hat{G}_{yy}] = 0. \quad (25)$$

Substitution of the reverse of (5) into (14) and of the result and (25) into (24) yields

$$B[\hat{G}_{vv}(f)] \approx L \left\{ \frac{1}{C(f)} - 1 \right\}, \quad (26)$$

defining the relative spectral variance $L = D(0) = 1/WT$, the latter result following from substitution of (7) into (15) for $f = g$.

Substitution of (25), (26) into (23) yields with (5) bias $[\hat{P}_v] \approx LP_y - LP_v$ and $\text{bias}[\hat{P}_y] = 0$; therefore

$$B[\hat{P}_v] \approx L \left(\frac{1}{R} - 1 \right); \quad B[\hat{P}_y] = 0. \quad (27)$$

Finally, substitution of (20) and (27) into (22) yields $B[\hat{R}] \approx L(1/R - 1) - Q(2K_{vy} - RK_{vv} - K_{yy})$; therefore

$$\text{bias}[\hat{R}] \approx L(1 - R) - QR(2K_{vy} - RK_{vv} - K_{yy}). \quad (28)$$

For all K 's = 1, this is $(L - QR)(1 - R)$. This shows, then, that $\text{bias}[\hat{R}] > L(1 - R)^2 = \text{bias}[\hat{C}(f)]$ for $C(f) = R \neq 0$ or 1 [2, eq. (6)] because $Q = 1/FT < 1/WT = L$.

C. Confidence Intervals for White Signals

For simplicity, white signals were studied further. For these, $\text{var}[\hat{R}]$ according to (21) was similar to $\text{var}[\hat{C}(f)]$, and according to [1, sect. 9.2.3], $\text{var}[\sqrt{\hat{C}(f)}]$ is similar to the variance of a sample correlation coefficient [3, sect. 19.12]. Hence, similarly, confidence intervals for R may be found from \hat{R} easily via a transformation $\hat{Z} = Z(\hat{R})$ such that $\text{var}[\hat{Z}] \approx 1$ for any \hat{R} . From this requirement, if $Z(\cdot)$ would be approximately linear first-order Taylor expansion of \hat{Z} about R that for white signals $1 \approx \text{var}[\hat{Z}] \approx (dZ/dR)^2 \text{var}[\hat{R}]$. It follows by use of (21) that

$$\frac{dZ}{dR} \approx \frac{1}{\sqrt{\text{var}[\hat{R}]}} \approx \frac{1}{\sqrt{2QR(1 - R)}}. \quad (29)$$

Integration of (29) over R yields the required transformation

$$\begin{aligned} Z(\hat{R}) &= \int_0^{Z(\hat{R})} dZ \approx \int_0^{\hat{R}} \frac{dR}{\sqrt{2QR(1 - R)}} \\ &= \sqrt{\frac{2}{Q}} \text{arctanh}(\sqrt{\hat{R}}) \end{aligned} \quad (30)$$

which is the Fisher z -transformation of $\sqrt{\hat{R}}$ [3, (19.12.4)]. Because of the similarity of \hat{R} to $\hat{C}(f)$ and approximate normality of $Z\{\hat{C}(f)\}$ [1, sect. 9.2.3], near normality of \hat{Z} is expected. Further, according to [3, (5.8.5)] under the assumption made, the mean of \hat{Z} is $\bar{Z} \approx Z(\bar{R})$ with \bar{R} the mean of \hat{R} . Hence, the lower and upper 95 percent confidence limits for \bar{Z} are

$$\bar{Z}_L \approx \bar{Z} - 1.96; \quad \bar{Z}_U \approx \bar{Z} + 1.96. \quad (31)$$

From these limits follow lower and upper 95 percent confidence limits for \bar{R} as the inverse of (30)

$$\bar{R}_L \approx \left\{ \tanh \left(\sqrt{\frac{Q}{2}} \bar{Z}_L \right) \right\}^2 \quad (32)$$

and similarly \bar{R}_U from \bar{Z}_U . Lower and upper 95 percent confidence limits for R can be derived finally as $R_L = R(\bar{R}_L)$ and $R_U = R(\bar{R}_U)$ with the inverse of $\bar{R} = R + \text{bias}[\hat{R}]$ with $\text{bias}[\hat{R}]$ according to (28)

$$R(\bar{R}) \approx \frac{Q + L - 1 + \sqrt{(1 - L - Q)^2 - 4Q(L - \bar{R})}}{2Q}. \quad (33)$$

III. SIMULATION EXAMPLE FOR WHITE SIGNALS

In order to test the effect of the approximations, y was formed from two uncorrelated random generators u and w , uniformly distributed between 0 and 1, as

$$y = \sqrt{R}u + \sqrt{1 - R}w. \quad (34)$$

This implies $H(f) = \sqrt{R}$ and $P_y = P_u$. Substitution of (34) into (4) shows that this simulation indeed yields the correct R .

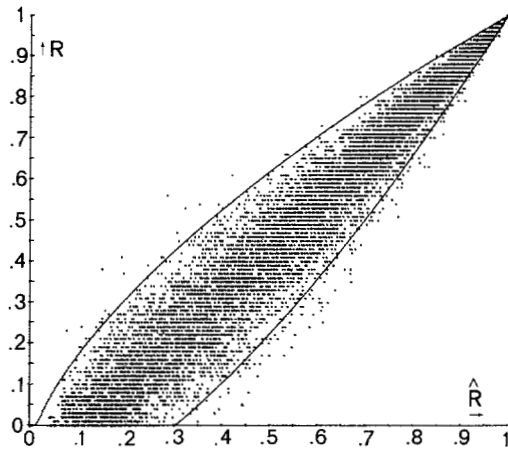


Fig. 1. Estimates \hat{R} with confidence intervals for relative linear power contribution R from 100 simulations each of 100 values of R .

Equation (34) was simulated for $R = 0, 0.01, \dots, 1$. For each value of R , 100 independent realizations of u and w of 64 samples during $T = 10$ s each were used. From each resulting pair of realizations of u and y , \hat{R} was estimated using a Papoulis correlation window [4] with maximal window lag of 15 samples, yielding a window bandwidth [1], [4] $W = 1.896 \times 64 / 15T = 0.80$ Hz for $F_1 = 0.1$ and $F_2 = 3.2$; therefore $F = 3.1$ Hz.

Fig. 1 shows $\hat{R}(R)$ with 95 percent confidence interval approximations calculated according to (30)–(33). For the 10 000 values of \hat{R} obtained, 34 of the actual R were over and 152 under the calculated 95 percent confidence limit approximations.

IV. CONCLUSIONS

Equation (21) shows that the variance of relative linear power contribution estimate \hat{R} is lower than that of squared coherence function estimate $\hat{C}(f)$. This is accomplished at the cost of a larger bias in \hat{R} compared to $\hat{C}(f)$, according to (28). Such bias might be corrected for.

Fig. 1 shows that for white signals, confidence intervals derived according to (30)–(33) are reasonable bounds on the simulated R for resulting estimate \hat{R} . The number of occurrences of R under the lower and over the upper 95 percent confidence limit did not reach the mathematical expectation 250 each, however. These deviations may be due to approximation errors like no linearity of \hat{Z} required for (29) over the wide range of \hat{R} for 64 samples or nonnormality of \hat{Z} . Similar analysis could be carried out for arbitrary spectra by using cocoloration factors according to (19).

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Noise Sensitivity of Band-Limited Signal Derivative Interpolation

ROBERT J. MARKS II

Abstract—The sensitivity of interpolation of the p th derivative of a band-limited signal directly from the signal's samples in the presence of additive stationary noise is considered. Oversampling and filtering generally decrease the interpolation noise level when the data noise is not band-limited. A lower bound on the interpolation noise level can be approached arbitrarily closely by increasing the sampling rate. The lower bound is equivalent to the noise level obtained by low-pass filtering and p th-order differentiation of the unsampled additive input noise.

INTRODUCTION

Given the sufficiently closely spaced samples of a band-limited signal, we can directly generate the p th derivative of the signal through appropriate interpolation functions [1].¹ Digital filters can generate good approximations of the samples of the p th derivative, given the signal samples as inputs [2]–[4]. The effects of filter design have been considered under the assumption of noiseless data [5]. Similarly, digital filters for sample interpolation ($p = 0$) have also been considered [6]–[7].

In this paper, an ideal p th-order differentiator is assumed and its operation in the presence of additive input data is investigated. We demonstrate that the noise level, in general, can be reduced by increasing the sampling rate. The reduction, however, is sometimes insignificant. A lower bound for the noise level is shown to be that resulting from passing the unsampled input noise through a cascaded low-pass filter and p th-order differentiator.

In the next section, preliminary concepts are introduced. General formulas for the interpolation noise level are then derived, followed by establishment of a corresponding lower bound. In the final two sections, the specific cases of Laplace and triangular autocorrelations are considered. When appropriately parameterized, both degenerate to the special case of white noise samples.

PRELIMINARIES

Let \mathfrak{B}_W denote the class of L_2 band-limited signals with bandwidth $2W$. That is, if $x(t) \in \mathfrak{B}_W$, then

$$x(t) = \int_{-W}^W X(f) \exp(j2\pi ft) df$$

where

$$\begin{aligned} X(f) &= \mathcal{F}x(t) \\ &= \int_{-\infty}^{\infty} x(t) \exp(-j2\pi ft) dt \end{aligned}$$

and \mathcal{F} denotes the Fourier transform operator. Then the p th derivative of $x(t)$ is

$$x^{(p)}(t) = (2B)^p \sum_{n=-\infty}^{\infty} x\left(\frac{n}{2B}\right) d_p[2Bt - n] \quad (1)$$

where the sampling rate $2B$ exceeds $2W$,

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¹The interested reader is referred to the introduction of [1] for a discussion of additional motivation for this work.