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**Ceccherini-Silberstein, Tullio** (I-SAN-EN)

**On the growth of linear languages. (English summary)**

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First the author replaces the condition $f(n) \leq c \cdot g(n)$ in the definition of "$f \in O(g)$" by $f(n) \leq g(c \cdot n)$, resulting in a relation $f \preccurlyeq g$. Consequently, all polynomial functions of the same degree are still equivalent (i.e., $f \preccurlyeq g$ and $g \preccurlyeq f$) but now all exponential functions—like $\exp(n)$, $2^n$ and $a^n$ $(a > 1)$—are equivalent as well.

The growth function of a language $L$ is the function $\gamma_L(n) = \#\{w \in L \mid |w| \leq n\}$ which counts the words of $L$ of length at most $n$. It is known that the growth $\gamma_L$ of a context-free language $L$ is either polynomial or exponential. This paper presents an algorithm to decide this alternative for the subclass of unambiguous linear languages. This algorithm associates an oriented graph to the linear grammar, it applies a criterion for the growth of such a graph established in [V. A. Ufnarovskiĭ, Mat. Zametki **31** (1982), no. 3, 465–472, 476; MR0652851 (83f:05026)], and it determines the degree of the polynomial.

In a note added in proof the author remarks that such an algorithm already follows from Theorem 5.2 in [S. Ginsburg and E. H. Spanier, Trans. Amer. Math. Soc. **113** (1964), 333–368; MR0181500 (31 #5729)]—without, however, producing the degree.

Reviewed by *Peter R. J. Asveld*

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*