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Brief Paper

Stability analysis of learning feed-forward control☆

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Abstract

In this paper, a learning control system is considered for motion systems that are subject to two types of disturbances; *reproducible* disturbances, that re-occur each run in the same way, and *random* disturbances. In motion systems, a large part of the disturbances appear to be reproducible. In the control system considered, the reproducible disturbances are compensated by a learning component consisting of a B-spline neural network that is operated in feed-forward. The paper presents an analysis of stability properties of the configuration in case of a linear process and second-order B-splines. The outcomes of the analysis are quantitative criteria for selection of the width of the B-splines, and of the learning rate, for which the system is guaranteed to be stable. These criteria facilitate the design of a learning feed-forward controller. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Learning feed-forward control; Adaptation; Neural networks; Iterative learning control; Stability analysis

1. Introduction

High-performance motion systems such as component mounters require both accurate and robust control. To design a model-based controller that satisfies these requirements, an accurate model of the process is needed. However, due to factors like process uncertainties, process non-linearities or time-varying parameters, the identification and modelling that is needed might be difficult, expensive and sometimes even impossible. To overcome this, several learning control methods have been proposed (Ng, 1997). In learning control, the controller is not designed on the basis of a process model. The controller is either trained on the basis of previously gathered data or is trained during control.

In this paper, a learning control system is considered for the processes that are subjected to two types of disturbances; *reproducible* disturbances, that depend on the state of the process and reoccur each time a motion is performed, and *random* disturbances. The learning control system has separate means for compensating both types of disturbances (Fig. 1) (Kawato, Uno, Isobe & Suzuki, 1988). The reproducible disturbances are compensated by a neural network (F). As these disturbances depend on the state of the process, they can be compensated in feed-forward. Besides the reproducible disturbances, F also compensates the process dynamics. The output of the feedback controller is chosen as a training signal for the neural network. The random disturbances are compensated by a model-based feedback controller (C). When random disturbances are small compared to the reproducible disturbances, this controller does not determine the tracking performance of the controlled system. Therefore, this controller can be designed for robustness mainly.

The type of neural network that is used is a B-spline network (BSN) (Brown & Harris, 1994). A BSN utilises piece-wise polynomial basis functions, known as B-splines, to store the feed-forward signal. This type of learning controller was introduced as the learning feed forward control scheme (LFFC) (Starrenburg, Luenen, Oelen & Amerongen, 1996). B-spline basis functions of

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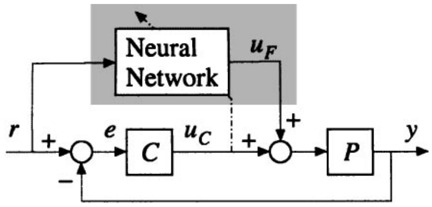

Fig. 1. Learning control system.

order n consist of piecewise polynomial functions of order $n-1$. Only second-order B-splines will be considered. The evaluation of the B-splines is generally called the *membership* and is denoted as μ. To create an I/O mapping, B-splines are placed on the domain of the input of the BSN, in such a way that at each input value the sum of all memberships equals 1 (Fig. 2). That part of the input space for which μ is not equal to 0 for a particular basis function is called its *support*. Note that a BSN can also be regarded as a fuzzy logic controller that has the B-spline functions as fuzzy premise sets and fuzzy singletons as consequence sets (Lee, 1990).

The variable x is the input of the BSN. The output of the BSN is a weighted sum of the B-spline evaluations:

$$u_{\mathrm{F}}(x) = \sum_{i=1}^{N} \mu_i(x) w_i, \tag{1}$$

where w_i is the weight associated to the ith B-spline and N is the number of B-splines. Training the network, in other words adapting the I/O mapping in such way that it comes closer to the desired I/O mapping, is done by adjusting the weights of the network using a so-called learning mechanism (to be presented later). This mechanism incorporates an adaptation gain referred to as the learning rate γ, and an approximation error. LFFC utilises the output of the feedback controller as a measure for the approximation error (Fig. 1). This choice is based on the intuitive reasoning that this signal is the feedback controller's best guess on how to decrease the tracking error. In the design of the LFFC, the following parameters have to be chosen:

The inputs of the BSN. The inputs are chosen on the basis of the plant and the type of disturbances that have to be compensated (Otten, Vries, Amerongen, Rankers & Gaal, 1997).

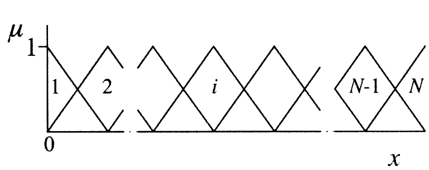

Fig. 2. One-dimensional second-order B-spline distribution.

The width of B-splines on each input axis. The accuracy of the LFFC depends on the width of the B-splines. The smaller the width of the B-splines, the more accurate the LFFC. However, a too small width may result in unstable behaviour. In the following, the effect of the width of B-splines on the robustness of the system will be further dealt with.

The learning rate. The learning rate γ determines how fast the weights of the BSN are adapted.

In Section 2, we discuss the type of LFFC for which the stability will be analysed. The influence of the width of the B-splines and the size of the learning rate on the stability of an LFFC-controlled system is derived quantitatively in Section 3. Simulation results that validate the stability analysis are presented in Section 4. We end with conclusions in Section 5.

2. LFFC for repetitive motions

In the standard configuration (Fig. 1), the input of the feed-forward controller consists of the reference path. Since the reproducible disturbances depend on the state of the process and the feed-forward signal is stored as a function of the reference trajectories of the states, this controller configuration is able to learn to track arbitrary reference paths. However, when repetitive motions are only considered, a fixed temporal sequence of combined positions, velocities and accelerations is present, and the control signal needed to compensate reproducible disturbances becomes a function of the periodic motion time. In that case, it is beneficial to choose the periodic motion time as the only input of the feed-forward part of the LFFC.

Consider a periodic motion with period T (s), and a BSN with uniformly distributed second-order B-splines with support width d (s). The membership of the ith B-spline is then defined as

$$\mu_i(t) = \begin{Bmatrix} \frac{2t-d(i-2)}{d} & \text{for } \frac{d}{2}(i-2) \le t \le \frac{d}{2}(i-1), \\ \frac{di-2t}{d} & \text{for } \frac{d}{2}(i-1) \le t \le \frac{d}{2}i, \\ 0 & \text{elsewhere}. \end{Bmatrix}. \tag{2}$$

The learning mechanism according to which the weights of the BSN are adapted is given by

$$\Delta w_i = \gamma \sum_{k=0}^{T/h} \mu_i(kh) u_{\mathrm{C}}(kh), \tag{3}$$

where Δw_i is the adaptation of weight i, and h is the sample time.

An LFFC that has the periodic motion time as input closely resembles another learning control system intended for processes that perform repetitive tasks, namely the iterative learning control scheme (ILC) (Moore, 1992) (Fig. 3).

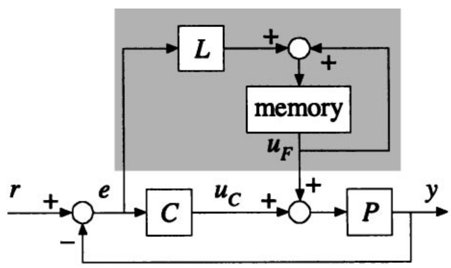

Fig. 3. Iterative learning control.

In an ILC the feed-forward signal is stored in a memory instead of a neural network. The feed-forward signal is adapted on the basis of the output of the learning filter, L, which filters the tracking error. In LFFC the feedback controller fulfils the role of the learning filter.

A stability analysis (Kavli, 1992; Moore, Dahleh & Bhattacharyya, 1992) shows that the control system is stable if

$$\left|1-\frac{LP}{1+CP}\right|_{\infty}<1. \tag{4}$$

To design an L such that (4) is satisfied for all frequencies, detailed knowledge of the process is required. For low-frequency dynamics, a competent model of the process often exists. However, for high frequencies this is usually not the case. This may result in an L that does not satisfy (4) and thus causes unstable behaviour. Several methods have been proposed to improve the stability robustness for unmodelled dynamics. This involves some alternation of the memory loop such that the high frequencies are not stored (Messner, Horowitz, Kao & Boals, 1991). In Hara, Yamamoto, Omata and Nakano (1988) this is realised by incorporating a low-pass filter in the memory loop, known as the Q-filter. The Q-filter is designed such that it suppresses the frequency components at which the process model was inaccurate. The lower frequencies, at which the model was accurate, are passed. This way, stability can be guaranteed. In time-indexed LFFC, a similar approach is pursued to cope with unstable behaviour. Here, the B-splines act as a low-pass filter. The approximation of a BSN consists of a linear interpolation between function evaluations at the centres of each two neighbouring B-splines. Therefore, the width of the B-splines determines the frequencies that can be approximated. To guarantee stability of time-indexed LFFC, the width of the B-splines should be chosen such that the BSN only stores the low-frequency signals for which (4) is satisfied. In the next section, rules are derived according to which the width and the learning rate can be chosen such that this is accomplished.

3. Stability analysis of LFFC

Firstly, to be able to analyse the stability of the LFFC, a number of (rather strong) assumptions were made.

(1) The process under control P is linear and time-invariant.

(2) The feedback controller C is linear, time-invariant and chosen such that the feedback loop is stable.

(3) A continuous version of the discrete learning rule (3) is used:

$$\Delta w_i=\gamma_c\int_0^T \mu_i(t)u_{\mathrm{C}}(t)\,\mathrm{d}t. \tag{5}$$

This implies that learning is linear in $u_{\mathrm{C}}(t)$, and hence the feed-forward adaptation loop is linear. Since the feedback loop is also linear, the reference path may be taken equal to zero in the analysis. This system is stable if an arbitrarily chosen initial feed-forward signal will not result in an unbounded output of the process. The (initial) feed-forward signal is determined by the (initial) values of the weights in the B-spline network. As the feedback controlled system is stable, the output can only become unbounded when the feed-forward signal $u_{\mathrm{F}}(t)$ becomes unbounded, which implies that at least one weight has become infinitely large. So, if the weights are adapted in such way that their value remains bounded, the system is stable; otherwise the system is unstable. The weights remain bounded if each weight adaptation satisfies the following condition:

$$\begin{aligned} &0\le \Delta w_i\le -2w_i \quad \text{for } w_i\le 0,\\ &-2w_i\le \Delta w_i\le 0 \quad \text{for } w_i>0. \end{aligned} \tag{6}$$

The problem is to select width d and learning rate γ_c in accordance with this. We will first consider the selection of d; after that, the learning rate γ_c is dealt with. In order to select d, we assume the following initial feed-forward signal.

(4a) The shape of the initial feed-forward signal, $u_{\mathrm{F}}(t)$ is triangular. This choice is motivated by the fact that experiments showed that when unstable behaviour occurs, the output of the neural network has a triangular shape (Velthuis, Vries & Amerongen, 1996). This I/O mapping can be realised by choosing the weights as $w_i=a$ for $i=1,3,5,\ldots$, and $w_i=-a$ for $i=2,4,6,\ldots$, where $a\in\mathbb{R}^+$.

Under this assumption the signal $u_{\mathrm{F}}(t)$ can be written as a Fourier series:

$$u_{\mathrm{F}}(t)=\frac{8a}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty}\frac{\cos(\omega_n t)}{n^2} \tag{7}$$

with

$$\omega_n = \frac{2\pi n}{d} \quad (\text{rad s}^{-1}). \tag{8}$$

The relation between the output of the feed-forward controller $U_F(j\omega)$ and the learning signal $U_C(j\omega)$ is given by the negative closed loop transfer function $-T(j\omega)$. $-T(j\omega)$ amplifies each frequency component ω_n of (7) by a factor a_n and introduces a phase shift φ_n, so $u_C(t)$ can be written as

$$u_C(t) = \frac{8a}{\pi^2} \sum_{n=1,3,5,\ldots}^{\infty} \frac{a_n \cos(\omega_n t + \varphi_n)}{n^2}. \tag{9}$$

Substitution of (2) and (9) to (5) and reformulation gives

$$\Delta w_i = \begin{cases} \dfrac{-16\gamma_c da}{\pi^4} \displaystyle\sum_{n=1,3,5,\ldots}^{\infty} a_n \frac{\cos(\varphi_n)}{n^4} \\ \quad \text{for } i = 2,4,6,\ldots, \\ \dfrac{16\gamma_c da}{\pi^4} \displaystyle\sum_{n=1,3,5,\ldots}^{\infty} a_n \frac{\cos(\varphi_n)}{n^4} \\ \quad \text{for } i = 1,3,5,\ldots . \end{cases} \tag{10}$$

It can be seen that all weights that have the same initial value are equally adapted. Therefore, learning does not change the shape of the feed-forward signal; the learning mechanism only adapts the amplitudes a and a_n. Hence, for each iteration the signal can again be expressed as (7) and the weight adaptation can be written as (10). In the following, the adaptation of weights that had a positive initial value, $w_i = a$, will be considered; for the other case, an analogous analysis can be made. Substituting (10) in (6) results in

$$\frac{-\pi^4}{8\gamma_c d} \leq \sum_{n=1,3,5,\ldots}^{\infty} a_n \frac{\cos(\varphi_n)}{n^4} \leq 0. \tag{11}$$

The width of the B-splines d should now be chosen such that (11) holds for the given process and controller.

We consider the right-hand side inequality of (11). Selection of a certain d determines the frequency of the triangular feed-forward signal, and therefore also the values of ω_n, a_n and φ_n. In case an exact model of the process and the controller is available, the values of a_n and φ_n can be calculated for all frequencies. This would allow the selection of the minimal d for which the right-hand side inequality of (11) is satisfied by means of a simple search. However, generally only the low-frequency dynamics of the process are known and the model is inaccurate at high frequencies. Therefore, we take an approach that seems somewhat conservative; we choose d such that the term for $n = 1$ in (11) is negative and has a larger amplitude than the maximum value of the sum of the rest of the terms, so that the right-hand side inequality of (11) will be satisfied for all possible values of φ_n, $n = 3,5,\ldots$

$$a_1 \cos(\varphi_1) \leq -\sum_{n=3,5,\ldots}^{\infty} \frac{a_n \cos(\varphi_n)}{n^4}. \tag{12}$$

Next, the maximum value of the rest of the terms is determined,

$$-\sum_{n=3,5,\ldots}^{\infty} \frac{a_n \cos(\varphi_n)}{n^4} \geq -\sum_{n=3,5,\ldots}^{\infty} \frac{a_n}{n^4}. \tag{13}$$

This implies that the right-hand side of (11) is always satisfied if

$$\cos(\varphi_1) \leq -\frac{1}{a_1} \sum_{n=3,5,\ldots}^{\infty} \frac{a_n}{n^4}. \tag{14}$$

To guarantee stability we have to choose ω_1 in such way that (14) is satisfied. The B-spline width d is directly related to ω_1 by (8) for $n = 1$. The minimum stable value of d corresponds to the maximum stable value of ω_1. When detailed process knowledge is available, this value can be found by means of a simple iterative search. However, a_n is often not known for high frequencies, which requires some sort of worst-case approximation. In this paper, we pursue the following approach. Typically, the phase shift of $-T(j\omega)$ is $-\pi$ (rad) for small values of ω and changes thereafter. This means that if we choose ω_1 small, $\cos(\varphi_1) \approx -1$ and $\cos(\varphi_1)$ increases when we increase ω_1. How far we can increase ω_1 and $\cos(\varphi_1)$ before the system becomes unstable is determined by the value of the right-hand side of (14). Instead of using the exact value of the right-hand side of (14), we will determine ω_1 on the basis of a lower bound of its minimum. We first calculate the minimum value of a_1, which we denote as N_l. This is done over the largest range of stable values of ω_1, which is obtained for the maximum value of right-hand side of (14). Since $a_1, a_n > 0$ the maximum value is 0. Thus,

$$N_l = \min_{\{\omega_1 \in \mathbb{R}^+ | \cos(\varphi_1) \leq 0\}} |-T(j\omega_1)|. \tag{15}$$

The angular velocity at which $|-T(j\omega)| = N_l$ will be denoted as ω_{N_l}. As an upper bound of a_n we take the maximum value of $|-T(j\omega)|$ for $\omega > \omega_{N_l}$:

$$a_n \leq \max_{\forall \omega > \omega_{N_l}} |-T(j\omega)| = N_h. \tag{16}$$

Using (15) and (16) we can derive

$$-\frac{1}{a_1} \sum_{n=3,5,\ldots}^{\infty} \frac{a_n}{n^4} \geq -\frac{1}{N_l} \sum_{n=3,5,\ldots}^{\infty} \frac{N_h}{n^4} \tag{17}$$

which gives

$$\cos(\varphi_1) \leq -\frac{N_h}{N_l} \sum_{n=3,5,\ldots}^{\infty} \frac{1}{n^4} = -0.0147 \frac{N_h}{N_l}. \tag{18}$$

From the Bode plot of $-T(j\omega)$ we can calculate the minimum value of d, denoted as $d_{\min}$, by searching the largest ω_1 for which (18) holds.

The value of $d_{\min}$ has been determined using the right-hand side inequality of (11), and hence of (6) for $w_i > 0$. For the maximum value of the learning rate, γ_c, the left-hand side inequality of (6) for $w_i > 0$ will be used, however, not for a triangular feed-forward signal. Consider an arbitrary chosen feed-forward signal. As before, the feedback signal can be written as a Fourier series. The low-frequency terms of the Fourier series cause a relatively stronger weight adaptation than the high-frequency terms. Namely, when for a low-frequency term, $u_C(t) = c\cos(\omega_l t + \varphi_l)$, $2\pi/\omega_l \gg d$, we may assume $u_C(t)$ constant over the support of one B-spline. Using (2) and (5) it can be calculated that the minimum adaptation of a sine-wave is obtained for $u_C(t) = -c$:

$$\Delta w_i = -\gamma_c \frac{c\,d}{2}. \tag{19}$$

For a high-frequency term, $u_C(t) = c\cos(\omega_h t + \varphi_h)$, $\omega_h \gg \omega_l$, $u_C(t) \geq -c$. We may thus conclude that the weight adaptation is larger than (19). Hence, to calculate the maximum learning rate we assume that

(4b) $u_F(t)$ is a constant signal, i.e. the initial weights are chosen $w_i = a$ for all i.

To obtain the feedback signal with the largest amplitude, we assume maximum amplification of $u_F(t)$:

$$u_C(t) = -a|-T(j\omega)|_\infty. \tag{20}$$

With (20) and (19), the left-hand side inequality of (6) for $w_i > 0$ results in

$$-2a \leq \gamma_c \frac{-a\,d|-T(j\omega)|_\infty}{2}. \tag{21}$$

Therefore, the learning rate should satisfy

$$\gamma_c \leq \frac{4}{|-T(j\omega)|_\infty d}. \tag{22}$$

Herewith, a maximum value of the learning rate for a continuous learning mechanism has been obtained. However, when implemented, the LFFC utilises the discrete learning mechanism (3). To calculate the maximum value of γ, a discrete approximation of the continuous learning mechanism is made. Assume that $h \ll d$, then $\mu_i(t)$ and $u_C(t)$ are almost constant over one sample interval. The continuous adaptation of the weights can then be written as

$$\Delta w_i = \gamma_c h \sum_{k=0}^{T/h} \mu_i(kh) u_C(kh). \tag{23}$$

The adaptation of the weights by the discrete approximation of the continuous learning rule, (23), is equal to the adaptation by the discrete learning rule, (3), when

$$\gamma = \gamma_c h \leq \frac{4h}{|-T(j\omega)|_\infty d}. \tag{24}$$

4. Simulations

In this section, the stability conditions that have been derived will be validated by means of simulations. Furthermore, it will be examined how conservative these values are. Namely, in the stability analysis a number of worst-case assumptions was made which might result in conservative values of the minimum B-spline width and of the maximum learning rate.

The plant that is simulated is a linear motion motor system (LiMMS) (Otten et al., 1997). The motor consists of a base plate on which permanent magnets are placed, and a translator that contains iron-core coils (a moving mass with $m = 37$ kg). The thrust force is generated by applying a three-phase current to the coils. The iron cores in the coils have magnetic interaction with the permanent magnets in the base plate. This phenomenon is generally known as cogging. The cogging force can be modelled as a sinusoidal shaped input disturbance force that depends on the motor position, which makes this a non-linear system. The amplitude of the cogging force is 10 N, and its pitch is $1.6e^{-2}$ m. Furthermore, the translator is subject to friction, which is considered to be viscous (friction coefficient 10 N m s^{-1}). The position of the translator is controlled by means of a PD-type feedback controller:

$$C(j\omega) = 275280\left(\frac{0.02j\omega + 1}{0.002j\omega + 1}\right). \tag{25}$$

The Bode plot of $-T(j\omega)$ is shown in Fig. 4.

From Fig. 4 it can be derived that $N_l = 0.7395$ (for $\omega = 220$ rad s^{-1}). Since the amplitude of $-T(j\omega)$ drops

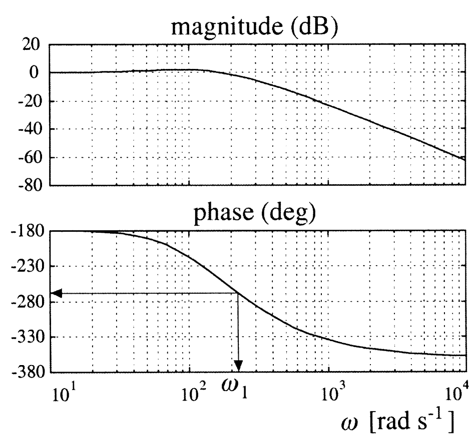

Fig. 4. Bode plot of the LiMMS.

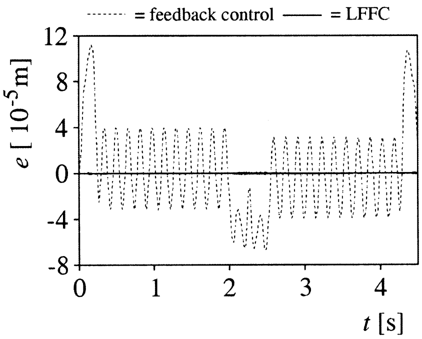

Fig. 5. Tracking error, d, conforms to the stability criterion.

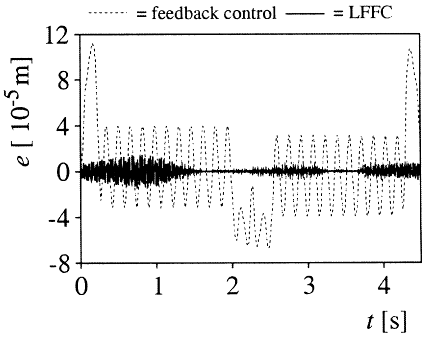

Fig. 6. Tracking error, d, overpassing the value given by the stability criterion.

for $\omega > 220\ \text{rad s}^{-1}$, $N_h = 0.7395$. Hence,

$$\cos(\varphi_1) \leq -0.0147 \Rightarrow 1.5854 \leq \varphi_1 \leq 4.6976. \tag{26}$$

This yields that for a stable system we should choose $\omega_1 \leq 220\ \text{rad s}^{-1}$ and $d_{\min} = 2.855 \times 10^{-2}$ s. Furthermore,

$$|-T(\mathrm{j}\omega)|_\infty = 1.2937.$$

Now, simulations will be done in which the LiMMS is to track a smooth reference path over 0.2 m back and forth. The simulation step size that is used is $h = 1 \times 10^{-4}$ s. The tracking error that is obtained, when the LiMMS is controlled by the feedback controller only, is given in Fig. 5 (dotted line). It can be seen clearly that the feedback controller is not able to fully compensate for the cogging.

In the first series of simulations with the LFFC, the width of the B-splines is larger than its minimum value, $d = 2.91 \times 10^{-2}$ s. The learning rate should now be chosen as

$$\gamma \leq \frac{4h}{|-T(\mathrm{j}\omega)|_\infty d} = \frac{4 \times 10^{-4}}{1.2937 \times 2.91 \times 10^{-2}} = 1.08 \times 10^{-2} \tag{27}$$

We choose $\gamma = 4 \times 10^{-3}$. In Fig. 5 the tracking error after 100.000 s is shown (solid line). The LFFC is able to learn to compensate for the cogging force and obtains a considerably smaller tracking error than the feedback controller. Furthermore, when learning is continued the system remains stable.

In the following series of simulations the width of the B-splines is chosen $d = 2.78 \times 10^{-2}$ s. For this width, the system becomes unstable as can be seen in Fig. 6, where the tracking error after 2000 s is depicted. We may conclude that even though the LiMMS is a non-linear system, the value of $d_{\min}$ is still valid and not conservative. Next, the maximum value of the learning rate will be examined for which the system is stable. In these simulations we take $d = 2.91 \times 10^{-2}$ s. It showed that the system becomes unstable when $\gamma > 1.37 \times 10^{-2}$. So, the maximum of the learning rate as determined in (27) gives a conservative value.

5. Conclusions

Learning feed-forward control is a learning control scheme which compensates for reproducible disturbances in a way similar as iterative learning control, and hence has similar stability properties. The feed-forward signal is stored in a B-spline network. In case of periodic motions, the width of the B-splines determines the frequency components that can be stored. If the width is chosen too small, unmodelled dynamics may cause unstable LFFC. Secondly, a too large learning rate can cause instability. Stability analysis of an LFFC that utilises second-order B-splines gives, in terms of the low-frequency behaviour of the closed-loop system,

(1) a quantitative criterion for the minimum width of the B-splines (Eqs. (8) and (18)), and
(2) a quantitative criterion for the maximum learning rate (Eq. (24)).

To verify the criteria, simulations were performed, in which LFFC was used to control a linear motor set-up. In spite of the fact that the plant is non-linear, simulations showed that the minimum width of the B-splines is valid and not conservative. However, the value of the maximum learning rate as given by (24) proved to be conservative. The validity of the criteria has also been verified using the actual linear motor set-up (Velthuis, Vries & Gaal, 1998). These experiments showed the same results as the simulations presented here.

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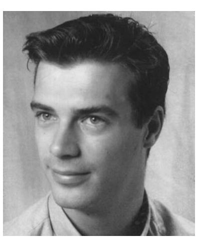

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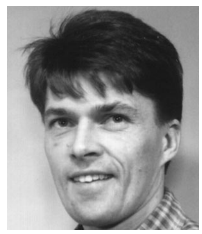

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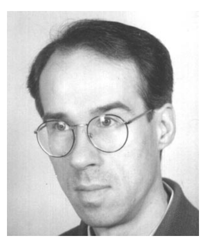

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