TASTE: A TOOL FOR ANALOG SYSTEM TESTABILITY EVALUATION

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Abstract

In this paper a new method is presented to analyze the testability of both linear and nonlinear analog systems. It combines a rank-test algorithm with statistical methods. The algorithm will find sets of inseparable parameters and determine whether it is possible to calculate a certain parameter with sufficient accuracy. It also determines a subset of appropriate measurements if redundant measurements are present.

1. Introduction

With the increasing complexity of analog integrated circuits, the testability of those circuits becomes more difficult. It is of course of great importance to design circuits which are testable within a reasonable time. This can reduce the costs of testing. The presented method can be used to evaluate the testability of analog integrated circuits but its use is not restricted to this type of circuits. It can also be used to evaluate the testability of analog circuits (printed circuit boards) or other (elektromechanical, mechanical,...) analog systems.

An approach to investigate the correctness of a system is to determine all the parameters necessary for its function. This does not imply that all parameters have to be determined on the lowest possible level (for instance transistor level); it is sufficient to determine the high-level functional parameters. These high-level parameters can be for instance the gain of an opamp, the cut-off frequency or the Q-factor of a filter. It is clear that this approach is closely related to functional testing, which is applied to most analog circuits. With the method presented in this paper it is possible to carry out a functional testability evaluation in an early design stage since only a high-level functional description of the system is required. This is important in order to avoid expensive redesigns in a later stage. In general, not all functional parameters of a system can be determined independently from another, especially when a reduced set of testvectors is used. These parameters are called inseparable parameters. Separable parameters on the other hand are parameters which can be determined without knowledge of the other parameters. Parameters which are a member of a set of inseparable parameters can only be computed if some of the parameters of the set are known. The presented method will detect the sets of inseparable parameters and calculates the accuracy with which all parameters can be determined. This accuracy will be called the determination accuracy in the rest of this paper. Another feature of the method is that it determines the best subset of measurements required to compute the parameters.

In the following section some previous work will be considered. This section states the advantages and disadvantages of some testability evaluation methods as used in the past. Section 3 describes the relations between parameters and measurements. In that section the influence of measurement errors on the determination accuracies will be explained. In section 4 the algorithm is described. This algorithm is based on relations derived in section 3. An example of a simple circuit is included to illustrate the advantages of the proposed algorithm. The paper is completed with a discussion and conclusions. An appendix is included which describes the Gaussian elimination procedure with full pivoting. Knowledge of the Gaussian elimination process is necessary to understand the algorithm since the algorithm is mainly based on this elimination method.

2. Previous work

The testability of digital circuits can be described with measures like controllability and observability [1,2]. Unfortunately this approach is not very suitable for analog circuits. This is because many faults appearing in analog circuits are soft faults, being the result of a parameter deviating too much from its nominal value. In general, such soft faults are harder to detect than hard faults, as they do not cause a full absence of a function but merely result in deviating specifications. Hard faults can be considered as extreme large parameter deviations. In the succeeding part of this paper the assumption is made that a fault in an analog system is the result of a parameter deviating too much from its nominal value (soft fault).

Most testability evaluation methods which have been presented are based on a rank-test algorithm [3..6]. These methods determine the solvability of a set of equations describing the relations between measurements and parameters. The measure of solvability δ equals zero if all parameters can be determined independently from another; the equations then have a unique solution. δ=1 implies that one parameter must be known to determine the values of the other parameters. So with increasing values of δ the solvability of the set of equations is decreasing. A disadvantage of these algorithms is that the effects

of measuring errors on the determination accuracies are not taken into account. The set of equations describing the relations between parameters and measurements can be ill-conditioned due to 'almost' inseparable parameters. Solving such a set of equations inevitably leads to large faults. Even a very small measurement error can cause a large deviation of the computed parameter value from its real value. The method proposed here should overcome these disadvantages.

3. Relations between parameters and measurements

The algorithm described in this paper finds the sets of inseparable high-level parameters and computes the determination accuracy of the parameters. The latter is computed from a particular set of testvectors. If a parameter is a member of a set of inseparable parameters, a fault in that parameter may be detected. It is not certain however which parameter of the set caused the failure; in other words the fault can not be located exactly. The algorithm also determines an appropriate subset of measurements by removing measurements containing redundant data. The number of measurements can be less than the number of parameters in the system. This can result in extra inseparable parameters which can be removed by increasing the number of measurements. In general, a certain number of parameters will remain inseparable even when the number of measurements is increased. The only possibility to overcome this problem is to add test points in the circuit.

If a particular set of input signals is given then the relations between parameters and output signals can be written as follows:

$$\underline{u} = f(\underline{p}) \qquad (1)$$

with: $\underline{u}$ vector of output signals
$\underline{p}$ vector of parameters
f function describing the relations between parameters and output signals

The vector $\underline{u}$ contains the values of a number of output signals. These output signals are not necessarily represented in the time domain, they can be in the frequency domain as well. Since output signals can of course be measured, equation (1) can be used to describe the relation between the parameters and the measurements. These measurements can not be carried out with infinite accuracy and therefore an error vector $\underline{e}$ is introduced which depends on the accuracy of the measurement methods used. The following equation then describes the relation between parameters and measurements.

$$\underline{x} + \underline{e} = f(\underline{p}) \qquad (2)$$

with: $\underline{x}$ vector of measurements
$\underline{e}$ vector of measurement errors, resulting from the measurement method used

It can be very difficult to derive the function $f(\underline{p})$ especially if nonlinear systems have to be analyzed. However, simulation can be used to obtain a set of sensitivity vectors which describe the dependencies of parameters on output signals. These sensitivity vectors can be used to approximate equation (2) by a first order description. The function $f(\underline{p})$ is nonlinear even if a linear system is analyzed. The first order description can therefore only be used if we assume that the parameter deviations are sufficiently small. The first-order description is then given by:

$$\underline{x} + \underline{e} = \underline{x}_{nom} + \sum_{j=1}^{J} \underline{S}_j \cdot \Delta p_j \qquad (3)$$

with: $\underline{x}_{nom}$ vector of measurements with nominal parameters
$\underline{S}_j$ vector of sensitivities of the measurements for parameter j
J number of parameters

If a matrix notation is used then equation (3) can be rewritten as:

$$S\underline{\Delta p} = \underline{\Delta x} + \underline{e} \qquad (4)$$

with: S the sensitivity matrix of dimension I*J
I number of measurements
$\underline{\Delta p}$ vector of parameter deviations
$\underline{\Delta x}$ vector $\underline{x} - \underline{x}_{nom}$

If the rank of matrix S is smaller than J then this equation can not be solved and sets of inseparable parameters have to be found in order to reduce the number of variables. In this case the variables are the parameters which have to be determined. It is possible that the number of measurements is larger then the number of parameters (I>J). In that case there are at least (I-J) redundant measurements, which means that the number of measurements can be reduced. This procedure will be explained later.

Now the influences of inseparable and almost inseparable parameters are considered. First a small example will be given and then a general description of the problem is presented.

Consider a system with at least 3 parameters p_1, p_2, p_3 and assume that the corresponding sensitivity vectors are 'nearly' dependent. This means that one of the vectors can be written as a linear combination of the other vectors, with just a small error. $\underline{S}_3$ can be written as:

$$\underline{S}_3 \approx \alpha.\underline{S}_1 + \beta.\underline{S}_2 \qquad (5)$$

with: $\underline{S}_1, \underline{S}_2, \underline{S}_3$ sensitivity vectors and also column vectors of matrix S
α, β real constants

Writing equation (4) in a different form results in:

$$\underline{\Delta x} + \underline{e} = \underline{S_1}\Delta p_1 + \underline{S_2}\Delta p_2 + \underline{S_3}\Delta p_3 + \ldots \qquad (6)$$

Now equation (5) can be substituted in equation (6) to obtain equation (7).

$$\underline{\Delta x} + \underline{e} = \underline{S_1}(\Delta p_1 + \alpha\Delta p_3) + \underline{S_2}(\Delta p_2 + \beta\Delta p_3) + (\underline{S_3} - \alpha\underline{S_1} - \beta\underline{S_2})\Delta p_3 + \ldots \qquad (7)$$

The third right-hand term of equation (7) can be considered as an extra error vector resulting from the approximation made in equation 5. If one of the vectors $\underline{S_1}$, $\underline{S_2}$, $\underline{S_3}$ can be written as an exact linear combination of the others then this error vector will equal $\underline{0}$. The 'nearly' dependent column vector of the matrix S can now be removed. Note that also a suitable row vector (measurement) must be removed to construct a square matrix again. The selection of the redundant rows and the 'nearly' dependent parameters will be explained in the description of the algorithm in section 4. Resulting from this mechanism the extra error vector turns out to have no influence on the computations and can therefore be ignored. The resulting set of equations is better conditioned because one of the 'nearly' dependent vectors is removed. Note that also the number of variables is reduced. The variables are not equal to the parameter deviations anymore but are now equal to a linear combination of the parameter deviations ($y_1 = \Delta p_1 + \alpha\Delta p_3$ and $y_2 = \Delta p_2 + \beta\Delta p_3$). The variables can now be determined with a higher accuracy than before the reduction. Note that in this case p_1,p_2 and p_3 are a set of inseparable parameters. This means that one of the values Δp_1, Δp_2 or Δp_3 must be known to determine the two other parameter deviations. Consequently a deviation in one or more of these parameters may be detected but can not be located anymore.

In general, if there are D column vectors dependent or 'nearly' dependent on other column vectors then these dependent (or 'nearly' dependent) vectors must be removed. The procedure described above has to be repeated D times in that case. This results in a set of independent column vectors and the old matrix S is reduced to a matrix of dimension (J-D)*(J-D). The number of variables Δy is also reduced from J to (J-D). Applying this to the general equation (4), we thus obtain:

$$\underline{\Delta y} = S^{-1}(\underline{\Delta x} - \underline{e}) \qquad (8)$$

with: S^{-1} the inverted sensitivity matrix of dimension (J-D)*(J-D)
$\underline{\Delta y}$ variables of the reduced set of equations (vector with dimension (J-D))

In equation (8), $\underline{\Delta x}$ is a known (measured) vector but the error vector $\underline{e}$ is unknown. The elements of $\underline{e}$ are assumed to be normally distributed with a zero mean value and a standard deviation dependent on the measurement method and statistically independent of another. The assumption that the mean values of the elements of $\underline{e}$ are zero is not necessary but it simplifies the computations. Therefore the determination accuracies (ey_j) of the variables can be computed with the following equation:

$$ey_j^2 = \sum_{i=1}^{I} (S^{-1}{}_{ji} \cdot \epsilon_i)^2 \qquad (9)$$

with: ϵ_i standard deviation of the error in measurement i
ey_j determination accuracy of variable j

The next step is the computation of the determination accuracy of the parameter deviations ($\underline{ep}$) from the determination accuracies of the variables ($\underline{ey}$). As seen previously, the elements of the vector $\underline{\Delta y}$ are linear combinations of the parameter deviations. So the relation between $\underline{\Delta p}$ and $\underline{\Delta y}$ can be written in a matrix form as shown in equation (10).

$$\underline{\Delta y} = T\underline{\Delta p} \qquad (10)$$

with: T transformation matrix of dimension (J-D)*J

The rank of matrix T equals J-D, which means that D determination accuracies must be known to compute all the remaining determination accuracies. If a row vector of T contains only one non-zero element then the associated parameter deviation and its determination accuracy can be computed independently of the other parameters. The remaining parameters form sets of inseparable parameters. At least one parameter has to be assumed fault free in order to be able to compute the parameter deviation and the associated determination accuracy for the other parameters in a particular set of inseparable parameters. The value of a fault free parameter is normally distributed with a mean value which equals the nominal value and a standard deviation which is related to the deviation of the parameter due to production variations. This decreases the determination accuracy of the other parameters of the set. The calculation of the best case determination accuracy of the inseparable parameters is explained in the description of the algorithm in the next section.

4. The algorithm

The input required for the algorithm is a set of sensitivity vectors. These vectors contain the sensitivity of the output signals of the system for a variation of the high-level parameters. These vectors can be determined by means of simulation. In our case we use high-level (circuit) models, for instance, operational amplifiers, multipliers, vco's, different kinds of filters and so on. A relevant set of high-level parameters for an amplifier can be for instance: differential-gain, common-mode gain, offset voltage and gain-bandwidth product.

The algorithm used to find inseparabilities between parameters and to compute the determination accuracy of the parameters is based on Gaussian elimination with full pivoting (see appendix). Together with data about the desired determination accuracy of the parameters it is also possible to find the nearly inseparable parameters.

Equation (4) can be rewritten as:

$$\underset{IA}{\begin{vmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{vmatrix}} \begin{vmatrix} \Delta x_1 + e_1 \\ \Delta x_2 + e_2 \\ \vdots \\ \Delta x_I + e_I \end{vmatrix} =$$

$$\underset{A}{\begin{vmatrix} \sigma_1 . S_{11} & \sigma_2 . S_{12} & \dots & \sigma_J . S_{1J} \\ \sigma_1 . S_{21} & \sigma_2 . S_{22} & \dots & \sigma_J . S_{2J} \\ \vdots & \vdots & & \vdots \\ \sigma_1 . S_{I1} & \sigma_2 . S_{I2} & \dots & \sigma_J . S_{IJ} \end{vmatrix}} \begin{vmatrix} \Delta p_1/\sigma_1 \\ \Delta p_2/\sigma_2 \\ \vdots \\ \Delta p_J/\sigma_J \end{vmatrix} \quad (11)$$

with: I number of measurements
J number of parameters
σ_i standard deviation of parameter i due to production variations
$\Delta p_i/\sigma_i$ normalised parameter deviation of parameter i

The matrix in the most left hand term of this equation is an I*I unity matrix (called IA) and the matrix in the right hand term is the normalized I*J sensitivity matrix (called A). Different from the description given in equation (4), the parameters in equation (11) are normalized with respect to their standard deviations as they result from production variations during the manufacturing process. This is done in order to simplify the pivot-finding routine as will become clear in the following part.

To compute the influence of measurement errors on the determination accuracies of the elements of vector $\underline{\Delta p/\sigma}$, the assumption $\underline{\Delta x} = \underline{0}$ can be made. If the measurement vector $\underline{\Delta x}$ equals $\underline{0}$ then the normalized parameter vector $\underline{\Delta p/\sigma}$ generally does not equal $\underline{0}$ due to the error vector $\underline{e}$. In this case equation (11) reduces to (in matrix notation):

$$IA.\underline{e} = A.\underline{\Delta p/\sigma} \quad (12)$$

with: $\underline{\Delta p/\sigma}$ normalized parameter deviation vector

Equation (12) is used as input for the algorithm shown in fig. 1.

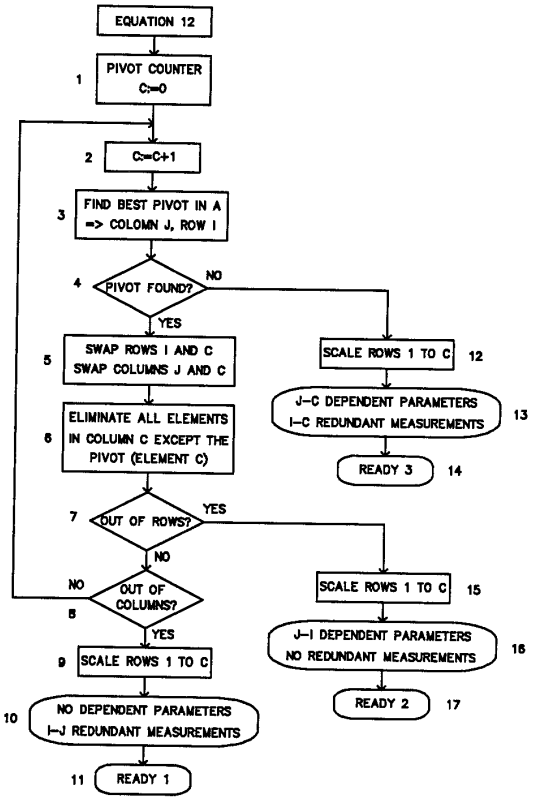


Figure 1 The algorith

In order to find the best pivot element in matrix A (see appendix) we look for the best determination accuracy of a parameter based on only one measurement. Suppose the system is described as in (12) and the first cycle (C=1) of the algorithm is in progress. Consider the first row of equation (12):

$$e_1 = \sigma_1 . S_{11} . \Delta p_1/\sigma_1 + \sigma_2 . S_{12} . \Delta p_2/\sigma_2 + \dots + \sigma_J . S_{1J} . \Delta p_J/\sigma_J \quad (13)$$

This equation can be rewritten as:

$$\Delta p_1/\sigma_1 = \frac{e_1 - \sigma_2 . S_{12} . \Delta p_2/\sigma_2 - \dots - \sigma_j . S_{1j} . \Delta p_j/\sigma_j}{\sigma_1 . S_{11}} \quad (14)$$

In order to estimate the relative determination accuracy of $\Delta p_1/\sigma_1$, the other parameters are assumed to be fault free. This means a standard deviation of 1 for all $\Delta p_j/\sigma_j$ with $j \neq 1$. The measurement error e_1 is also assumed to be normally distributed with standard deviation ϵ_1 and statistically independent of

$\Delta p_j/\sigma_j$. In this case the estimated relative determination accuracy ep_1/σ_1 of $\Delta p_1/\sigma_1$ can be calculated with:

$$(ep_1/\sigma_1)^2 = \frac{\epsilon_1^2 + (\sigma_2.S_{12})^2 + ... + (\sigma_1.S_{11})^2}{(\sigma_1.S_{11})^2} \quad (15)$$

A similar equation can be derived for all parameters with respect to measurement 1. It is obvious from equation (15) that the parameter corresponding with the largest value of $\sigma_j S_{1j}$ has the smallest value of ep_j/σ_j. This is a result of the normalization of the parameter vector as mentioned previously and makes it superfluous to compute all the ep_j/σ_j of a row: looking for the largest value of $\sigma_j S_{ij}$ in row i of matrix A is sufficient to find the best element in that row. A comparison of the best values of ep_j/σ_j over all rows is carried out next and the row with the smallest value of ep_j/σ_j is chosen as the pivot row. The pivot column is the column with the largest absolute value of $\sigma_j S_{ij}$ in that row. This concludes the pivot-finding part of the algorithm (step 3).

Next, the pivot element is placed in the position of matrix element A_{11} by swapping the pivot row with row 1 and the pivot column with column 1 (step 5). The following step in the algorithm is to eliminate all elements in column 1 of matrix A except the pivot element (step 6). This is described in the conventional Gaussian elimination routine (see appendix). After the elimination of the first column a second column must be eliminated if possible. The same procedure will be used to find a new pivot element in the resulting rows (row 2,3,...).

For the general case it is now assumed that the elimination process is in the beginning of the C-th cycle. Hence, a pivot element must be found in the rows C up to I of matrix A. In these rows the first (C-1) elements are made zero by the previously executed elimination steps. Row i, with $i \geq C$, can now be described as:

$$\sum_{k=1}^{I} IA_{ik}.e_k = \sum_{m=C}^{J} A_{im}.(\Delta p'_m/\sigma'_m) \quad (16)$$

with: I number of measurements
J number of parameters
$\Delta p'_m/\sigma'_m$ swapped normalized parameter deviation

The swapped normalized parameter deviation vector is changing during the elimination process. If a column of matrix A is swapped, then the corresponding element of the normalized parameter deviation vector is swapped too. Equation (16) can be used to determine the remaining (J-C) parameter deviations. To compute the relative parameter deviation corresponding with the j-th column, with $j \geq C$, equation (16) can be rewritten as follows:

$$A_{ij}.(\Delta p'_j/\sigma'_j) = \sum_{k=1}^{I} IA_{ik}.e_k - \sum_{\substack{m=C, \\ m\neq j}}^{J} A_{im}.(\Delta p'_m/\sigma'_m) \quad (17)$$

As mentioned in section 3, the elements of the vector $\underline{e}$ are statistically independent and the mean values are zero. The relative determination accuracy (see equation 15) is then in the general case determined by:

$$(ep'_j/\sigma'_j)^2 = \sum_{k=1}^{I} \frac{(IA_{ik}.\epsilon_k)^2}{(A_{ij})^2} + \sum_{\substack{m=1, \\ m\neq i}}^{J} \frac{(A_{im})^2}{(A_{ij})^2} \quad (18)$$

with: i row number
I number of measurements
J number of parameters
ep'_j/σ'_j swapped normalized determination accuracy vector
ϵ_k standard deviation of measurement k

The smallest value of (ep'_j/σ'_j) over all columns C to J and measurements C to I provides the best expected determination accuracy and the associated element of matrix A is used as the pivot element. After this pivot-finding part row C and column C are respectively swapped with the pivot row and the pivot column. Now column C (except the pivot element A_{cc}) can be eliminated.

This procedure continues as described above until one of the following conditions is satisfied:

A: the value of the pivot-counter equals the number of columns (after the elimination step). This means that all columns are eliminated; so there are no dependencies found between parameters and a number of redundant measurements are found (step 9,10 and 11).

B: the value of the pivot-counter equals the number of rows (after the elimination step). If also the number of columns is larger than the number of rows, then (J-I) dependent parameters are found. Some of the dependencies are presumably caused by a lack of measurements. The only way to check this is to increase the number of rows in equation (11) by adding measurement results and re-executing the algorithm (step 15,16 and 17).

C: no pivot is found. Resulting in a number of dependent parameters and redundant measurements. Probably the number of dependencies can be reduced by using other measurements (step 12,13 and 14).

In steps 9,12 and 15 the matrix A is scaled. This means that all diagonal elements of matrix A are made equal to 1 by multiplying the rows of matrix A and IA with a particular factor $(1/A_{ii})$.

The influence of dependent parameters is shown in the following part of this section. First an example will be given, followed by a description of the general case with D dependent parameters.

Assume that a system with 1 dependent parameter is given, then the algorithm will change equation (12) to:

$$\begin{vmatrix} IA_{11} & IA_{12} & \dots & IA_{1I} \\ IA_{21} & IA_{22} & \dots & IA_{2I} \\ . & . & & . \\ . & . & & . \\ IA_{R1} & IA_{R2} & \dots & IA_{RI} \end{vmatrix} \begin{Bmatrix} e_1 \\ e_2 \\ . \\ . \\ e_I \end{Bmatrix} =$$

$$\begin{vmatrix} 1 & 0 & \dots & A_{1J} \\ 0 & 1 & \dots & A_{2J} \\ . & . & . & . \\ . & . & . & . \\ 0 & 0 & \dots & 1 \; A_{RJ} \end{vmatrix} \begin{Bmatrix} \Delta p'_1/\sigma'_1 \\ \Delta p'_2/\sigma'_2 \\ . \\ . \\ \Delta p'_J/\sigma'_J \end{Bmatrix} \qquad (19)$$

with: R the number of dependent parameters subtracted from the number of parameters ((J-D); in this case (J-1))

Probably some rows in A will have only one non-zero element ($A_{ii}=1$ and $A_{ij}=0$). The column containing this element is related with a parameter deviation which can be determined independently of the other parameters; that parameter is a member of the set of separable parameters.

The determination accuracy of the parameters can in this case be computed by using the following equation:

$$(ep'_i/\sigma'_i)^2 = \sum_{j=1}^{I} (IA_{ij}.\epsilon_j)^2 + A_{iJ}^2 \qquad (20)$$

Note that the determination accuracy of the dependent parameter p'_J can not be computed with equation (20). It is also assumed that $\Delta p'_J/\sigma'_J$ is normally distributed with a zero mean value and a standard deviation equal to 1 (fault free parameter).

A parameter p_n with a determination accuracy not as small as desired ($ep_n > a_n$ with a_n the desired determination accuracy) can be a parameter which is a member of a set of nearly inseparable parameters. These inseparabilities can be determined with the described algorithm. The sensitivity vector corresponding with p_n must be placed in the last column of matrix A. The algorithm is executed again but it is not allowed to search for a pivot in the last column. Resulting from this it is also not allowed to eliminate the last column. After the scaling the matrix A has the same form as in equation (19).

In the general case there can be more parameters which are dependent (or nearly dependent) of the other parameters. Equation (19) can then be written in the following general form:

$$\begin{vmatrix} IA_{11} & IA_{12} & \dots & IA_{1I} \\ IA_{21} & IA_{22} & \dots & IA_{2I} \\ . & . & & . \\ . & . & & . \\ IA_{R1} & IA_{R2} & \dots & IA_{RI} \end{vmatrix} \begin{Bmatrix} e_1 \\ e_2 \\ . \\ . \\ e_I \end{Bmatrix} =$$

$$\begin{vmatrix} 1 & 0 & \dots & A_{1R} & \dots & A_{1J} \\ 0 & 1 & \dots & A_{2R} & \dots & A_{2J} \\ . & . & . & . & & . \\ . & . & . & . & & . \\ 0 & 0 & \dots & 1 \; A_{RR} & \dots & A_{RJ} \end{vmatrix} \begin{Bmatrix} \Delta p'_1/\sigma'_1 \\ \Delta p'_2/\sigma'_2 \\ . \\ . \\ \Delta p'_J/\sigma'_J \end{Bmatrix} \qquad (21)$$

with: R the number of dependent parameters subtracted from the number of parameters (J-D)

The coefficients of the D last columns of matrix A describe the dependencies between the dependent parameters and the other parameters. Note that one measurement is removed for each parameter which is assumed to be dependent of the others. The matrix IA contains (J-D) columns which not equal $\underline{0}$. The measurements corresponding with these columns are used to determine the parameter deviations. The measurement which is removed contains the information which distinguishes the nearly dependent parameter from the other parameters. After this the nearly dependent parameter is considered to be fully dependent of the other parameters. The determination accuracy of the other parameters can now be computed by assuming all the dependent parameters to be normally distributed with zero mean and standard deviation σ (fault free parameters). Equation (20) can now be rewritten in the following general form:

$$(ep'_i/\sigma'_i)^2 = \sum_{j=1}^{I} (IA_{ij}.\epsilon_j)^2 + \sum_{m=J-D}^{J} A_{im}^2 \qquad (22)$$

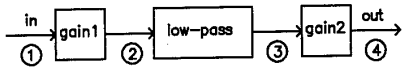
It is not possible to compute the determination accuracies of the dependent parameters p'_R to p'_J with equation (22). To compute the determination accuracy of these D dependent parameters, D other parameters must be assumed to be fault free. To determine the most suitable parameters the described algorithm can be used again in a slightly modified form. The sensitivity vectors corresponding with one of the D dependent parameters is placed in the first column of matrix A. Then the first pivot used for the elimination process must be found in this first column. After the first cycle the elimination process can be continued in the normal way. The elimination process must be stopped when D columns remain. Then (J-D) columns are eliminated and also (J-D) measurements are left. Care must be taken that these are the same measurements as in the original solution. Now again a set of equations like equation (21) is the result of the elimination process. The determination accuracy of the dependent parameter can now be determined with equation (22). This procedure is repeated until all determination accuracies are computed.

The assumption that a parameter is nearly dependent on other parameters usually results in a better determination accuracy of the set of inseparable parameters. If this is not the case, then the determination accuracy can only be improved by using other testvectors.

5. Example

In this section an example of a testability analysis is given. The analysis is carried out with the help of the computer program "TASTE". The algorithm described in this paper and a simulation program which is used to compute the necessary sensitivity vectors are both implemented in the program "TASTE". The models implemented in the simulation part of the program are high-level models, so it is possible to analyze the testability of a system in an early design stage.

The analyzed system is very straightforward and linear in order to facilitate the interpretation of the results of the analysis. Of course nonlinear systems can be analyzed too, but the results of such an analysis are not as easy to interpret as the results maintained with a simple linear circuit.



ⓘ: node number i

Figure 2 Analog system with one output node

In fig. 2 a linear analog system is given. The low-pass filter is a second order filter. Its transfer function can be described as:

$$H(j\omega) = \frac{1}{1 + a.j\omega + b.(j\omega)^2} \qquad (23)$$

with: ω frequency in rad/s
a,b filter coefficients

A deviation of this transfer function from the nominal transfer function can be represented as a deviation in the coefficients a and b. It is not necessary to use these parameters, it is also possible to use for instance the Q-factor and the cut-off frequency as the parameters which describe the behavior of the filter. The Q-factor and cut-off frequency can be written as a function of the coefficients a and b. The function of a gain block can be described with only one parameter: the input signal is multiplied with the corresponding gain to obtain the output signal.

The behavior of this system can be described with 4 parameters: a, b, gain1 and gain2. If we want to determine all parameters then 4 measurements are required. Consider that only the output signal of the system can be observed. Therefore it is obvious that the parameters gain1 and gain2 can not be determined independent of each other, only their product can be determined. Thus only three measurements are required.

The algorithm described in this paper can be used to select an opimal set of measurements. For this an input signal with an excessive number of frequency components can be used. A very suitable signal is the sum of a number of sine functions with random phase:

$$in(t) = (A/\sqrt{m}).\sum_{n=1}^{m} \sin(n.2\pi f_o.t + \varphi_n) \qquad (24)$$

with: m the number of frequency components
f_o the lowest frequency component in the input signal
A determines the amplitude of the signal
φ_n phase of n-th frequency component

Due to the random phase components φ_n this signal approximates a white noise signal. For m>3 the output signal will contain a number of redundant frequency components, that will be removed by the algorithm.

To use the algorithm the following data are needed: the nominal values, the standard deviations and the desired determination accuracies of the parameters, the standard deviations of the measurement errors and the definition of the input signal. These data are listed below:

nominal value	standard deviation	desired accuracy (% of nominal value)
$a = 1.4142/2\pi f_c$	10%	10%
$b = 1.0/(2\pi f_c)^2$	10%	10%
gain1 = 2.0	5%	5%
gain2 = 2.0	10%	10%

with: f_c = 1.0 (cut-off frequency in Hz.)

It is assumed that the standard deviation of the measurement errors equals 1E-3, if the output signal is a voltage this corresponds with a voltage of 1.0 mV.

The input signal consists of a number of frequency components with equal amplitude of 25 mV and random phase.

$$in(t) = 0.025.\sum_{n=1}^{20} \sin(n.2\pi f_o.t + \varphi_n) \qquad (25)$$

with: f_o = 0.1 Hz.

First a simulation of the system is carried out in order to obtain the sensitivity vectors. The absolute value of the relative sensitivities (sensitivity multiplied by the standard deviation of the corresponding parameter) is shown in a graphical form in figure 3.

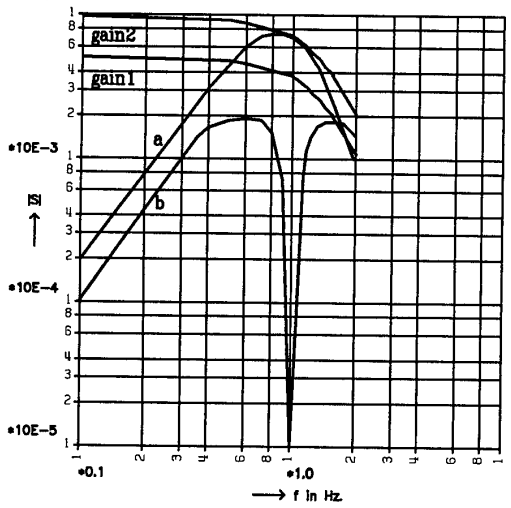


Figure 3 Absolute value of the relative sensitivities on node 4

The relative sensitivity vectors obtained by the simulation are used as input for the testability analysis algorithm. This algorithm is used to find dependencies between parameters, to compute the determination accuracies and to select an optimal set of measurements.

The results of the analysis are listed below:

parameter	relative determination accuracy	required non faulty parameters
gain2	0.510	gain1
a	0.180	none
b	0.549	none
gain1	2.010	gain2

The relative determination accuracy of a parameter equals the determination accuracy divided by the standard deviation (due to the production process) of the parameter.

The selected frequency components are: 0.1, 1.0 and 1.7 Hz.

To explain the results obtained by the algorithm the plot of the relative sensitivities (fig. 3) can be used. The sensitivity of the amplitude of the 1.0 Hz component on parameter a is much larger than the sensitivity on parameter b, at 1.7 Hz. the sensitivities on both parameters are almost the same, thus these measurements are very suitable to determine the parameters a and b independent from another.

The parameters gain1 and gain2 are dependent, so only one extra measurement is required to determine the product of these parameters. The frequency selected by the algorithm is 0.1 Hz., at this frequency the sensitivity on gain1 and gain2 is large compared to the sensitivity on the parameters a and b. To determine the determination accuracy of parameter gain1 the parameter gain2 must be assumed to be non faulty and vice versa. The relative determination accuracy of the parameter gain1 is not quite as good as the relative determination accuracy of parameter gain2, this is caused by the larger standard deviation of parameter gain2 compared to the standard deviation of gain1. The parameter gain1 can not be determined with sufficient accuracy (relative determination accuracy > 1.0).

6. Discussion

The algorithm described in this paper is implemented on an APOLLO workstation. The program is written in PASCAL and can be used to perform an analysis in the frequency domain. In order to obtain the required sensitivity vectors a simple high level simulation program is developed. First the response of the nominal circuit is simulated and after that further simulations are required to determine the sensitivity vectors. Because of the possible nonlinearities in the system a simulation must be carried out for each parameter, so it is clear that this simulation part of the algorithm will be time consuming. After the necessary simulations the testability analysis can be carried out

At this moment only the amplitude of the frequency components is used in the analysis. In most practical situations this will be sufficient but for example a delay-line will cause testability problems when no phase information is available. The program can easily be changed to overcome this problem.

As shown in the example the algorithm is useful to analyze the testability of an analog circuit. It is also shown that the algorithm can be used for test frequency selection. Therefore it might be possible to use this algorithm as part of an automatic test pattern generation program.

7. Conclusions

An algorithm is developed to compute the determination accuracies of the high-level parameters in an analog system based on user-defined measurements. The algorithm detects dependencies and near dependencies between parameters of the system and selects the best set of measurements from a given set. It can be used for both linear and nonlinear systems. The algorithm combines a rank-test method with statistical methods.

The algorithm has been implemented on a workstation and evaluations with small linear and nonlinear analog systems show the usefulness of the method.

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Appendix: Gaussian elimination procedure

The Gaussian elimination procedure with full pivoting is used to compute the solution of a set of n equations with n variables. Due to finite word-length effects in digital computers large faults can occur if the set of equations is ill-conditioned. A set of equations described with a matrix representation is ill-conditioned if two or more column vectors of the matrix are nearly dependent on other column vectors. In that case, a so called pivoting technique can be used to obtain the highest possible accuracy. First the Gaussian elimination with partial pivoting will be explained.

Assume that a vector $\underline{x}$ which represents the n variables x_1, $x_2,\ldots,x_n$ and a matrix A with dimension n*n are given:

$$\begin{vmatrix} A_{11} & A_{12} & \ldots & A_{1n} \\ A_{21} & A_{22} & \ldots & A_{2n} \\ A_{31} & A_{32} & \ldots & A_{3n} \\ . & . & & . \\ A_{n1} & A_{n2} & \ldots & A_{nn} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ x_n \end{vmatrix} = \begin{vmatrix} b_1 \\ b_2 \\ b_3 \\ . \\ b_n \end{vmatrix} \qquad (A.1)$$

Now a suitable element of the first column of matrix A, A_{i1}, must be found such that $|A_{i1}| \geq |A_{k1}|$ for all k=1,2,...,n. If for instance A_{31} satisfies this condition, then this element is chosen to be the pivot element. Rows 1 and 3 are swapped and thus A_{31} is moved to the position of A_{11}. The equations are now changed to:

$$\begin{vmatrix} A_{31} & A_{32} & \ldots & A_{3n} \\ A_{21} & A_{22} & \ldots & A_{2n} \\ A_{11} & A_{12} & \ldots & A_{1n} \\ . & . & & . \\ A_{n1} & A_{n2} & \ldots & A_{nn} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ x_n \end{vmatrix} = \begin{vmatrix} b_3 \\ b_2 \\ b_1 \\ . \\ b_n \end{vmatrix} \qquad (A.2)$$

This can be rewritten as:

$$\begin{vmatrix} A'_{11} & A'_{12} & \ldots & A'_{1n} \\ A'_{21} & A'_{22} & \ldots & A'_{2n} \\ A'_{31} & A'_{32} & \ldots & A'_{3n} \\ . & . & & . \\ A'_{n1} & A'_{n2} & \ldots & A'_{nn} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ x_n \end{vmatrix} = \begin{vmatrix} b'_1 \\ b'_2 \\ b'_3 \\ . \\ b'_n \end{vmatrix} \qquad (A.3)$$

With: $A'_{11}=A_{31}$, $A'_{12}=A_{32}$, ...
$b'_1=b_3$, $b'_2=b_2$, ...

Now (n-1) factors can be defined:

$$m_i = A'_{i1}/A'_{11} \qquad i=2,3,\ldots,n$$

After this operation m_i times the first equation is subtracted from the i-th equation (for i=2 to n). This results in a new equation:

$$\begin{vmatrix} A_{11} & A_{12} & \ldots & A_{1n} \\ 0 & A_{22} & \ldots & A_{2n} \\ 0 & A_{32} & \ldots & A_{3n} \\ . & . & & . \\ 0 & A_{n2} & \ldots & A_{nn} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ x_n \end{vmatrix} = \begin{vmatrix} b_1 \\ b_2 \\ b_3 \\ . \\ b_n \end{vmatrix} \qquad (A.4)$$

The first column of the matrix is now eliminated except for the pivot element A_{11}. Note that the coefficients A'_{ij} and b'_i are renamed to A_{ij} and b_i and also that these coefficients do not equal the coefficients appearing in equation (A.1).

The next step is finding a suitable pivot element in the second column of matrix A, A_{i2} (i=2,3,...,n). Again (n-1) factors can be defined:

$$m_i = A'_{i1}/A'_{11} \qquad i=1,3,4,\ldots,n$$

The second column can now be eliminated and the equation changes to:

$$\begin{vmatrix} A_{11} & 0 & \ldots & A_{1n} \\ 0 & A_{22} & \ldots & A_{2n} \\ 0 & 0 & \ldots & A_{3n} \\ . & . & & . \\ 0 & 0 & \ldots & A_{nn} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ x_n \end{vmatrix} = \begin{vmatrix} b_1 \\ b_2 \\ b_3 \\ . \\ b_n \end{vmatrix} \qquad (A.5)$$

This process is continued until all n columns are eliminated. The following form is then obtained:

$$\begin{vmatrix} A_{11} & 0 & \ldots & 0 \\ 0 & A_{22} & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ . & . & & . \\ 0 & 0 & \ldots & A_{nn} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ x_n \end{vmatrix} = \begin{vmatrix} b_1 \\ b_2 \\ b_3 \\ . \\ b_n \end{vmatrix} \qquad (A.6)$$

The solution of the set of equations can be easily computed ($x_i = b_i / A_{ii}$ for i=1 to n).

If Gaussian elimination with full pivoting is used, then a pivot element is not just searched in one column but over all columns. In this case the columns of matrix A must be swapped and also the corresponding elements of vector $\underline{x}$. Suppose that in (A.1), element A_{32} is found to be the best pivot element, then after the swapping of columns and rows (A.1) changes to:

$$\begin{vmatrix} A_{32} & A_{31} & \dots & A_{3n} \\ A_{22} & A_{21} & \dots & A_{2n} \\ A_{12} & A_{11} & \dots & A_{1n} \\ . & . & & . \\ A_{n2} & A_{n1} & \dots & A_{nn} \end{vmatrix} \begin{bmatrix} x_2 \\ x_1 \\ x_3 \\ . \\ x_n \end{bmatrix} = \begin{bmatrix} b_3 \\ b_2 \\ b_1 \\ . \\ b_n \end{bmatrix} \qquad (A.7)$$

Note that the elements 1 and 2 of vector $\underline{x}$ are swapped because the columns 1 and 2 are swapped. The elimination process can now be continued as previously, only the searching of the pivot and the swapping of the columns and the elements of vector $\underline{x}$ is changed.

The above described Gaussian elimination procedure can also be used to compute the inverse of matrix A, therefore only minor modifications are required. The procedure can only be used with a set of equations consisting of n equations and n unknown variables. It is also necessary that the rank of matrix A equals n (this means that the determinant of A not equals zero) to guarantee that a valid solution of the equations can be found. A problem which occurs in the algorithm (section 4) is the fact that in general the number of equations not equals the number of variables and also the rank of the matrix A not equals the number of variables. To overcome this problem the procedure is modified. Also the criterion used to find a pivot element is changed, in order to select the best subset of measurements.

Nomenclature

J	number of parameters
I	number of measurements
p_j	nominal value of parameter j
Δp_j	deviation of p_j
σ_j	standard deviation of p_j
a_j	required accuracy of parameter j
ep_j	inaccuracy of the computed value of parameter j
$\underline{x}$	measurement vector
$\underline{x}_{nom}$	measurement vector of nominal system
$\Delta\underline{x}$	$x-x_{nom}$
$\underline{e}$	measurement error vector
ϵ_i	standard deviation of measurement i
S_{ij}	sensitivity of measurement i for deviation of parameter j
S	sensitivity matrix
S^{-1}	inverse of matrix S
D	number of dependent column vectors
T	transformation matrix
$\Delta\underline{y}$	solution of reduced set of equations

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