# IMC Based Boundary Control of a Thermal Process with Parameter Uncertainty

Miklós Vajta, *Member, IEEE* and István Vajk

*Abstract*— **The aim of this paper is to present a control scheme for controlling the temperature in a slab which is not directly measurable. The process can only be controlled by manipulating the ambient temperature (boundary control). We describe the physical process and derive its (distributed) transfer function (DTF). The sensitivity of the model for one of the parameters is analyzed. We describe a model reduction technique which provides an accurate approximation of the distributed transfer function. Based on the reduced order model we present an Internal Model Control (IMC) scheme applied to the heat process and analyze the closed-loop.**

## I. Introduction

Infinite dimensional systems have gained considerable attention in recent years [5,7,9]. This is not surprising for all physical processes are distributed in nature due to their dimensions. Such processes are for example heat processes, flexible robot arms, vibrating systems etc. Distributed parameter (or infinite dimensional) systems are described by partial differential equations and the solutions also depend on the boundary conditions. We consider a heat process which describes the temperature distribution within a slab. The aim is to control the temperature at some point within the slab by acting on its boundary (boundary control). We assume the process to be stable but the system parameters may be uncertain.

## II. Problem Statement

Suppose we want to control the temperature within a slab which is heated (or cooled) on both sides. For sake of simplicity we assume the process to be one-dimensional. It is a typical industrial problem to keep the temperature profile within slab at a prescribed value or to follow a pre-specified reference signal. It is usually difficult (if not impossible) to measure the temperature which must be controlled. We can only manipulate the ambient temperature. However, an important parameter of the process, namely the heat transfer coefficient is usually not constant. It is difficult (or expensive) to measure and its value depends on several factors. The aim of our research is to devise an IMC based control scheme, which guaranties good control performance in spite of the largely uncertain parameter.

This work has been done while M.Vajta was on sabbatical at the Department of Automation and Applied Informatics, Budapest University of Technology and Economics, Budapest, Hungary.

M.Vajta is with the Dept. of Applied Mathematics, University of Twente, 7500 AE Enschede, The Netherlands (email: m.vajta@math.utwente.nl)

I.Vajk is with the Dept. of Automation and Applied Informatics, Budapest University of Technology and Economics, H-1111, Budapest, Goldmann Gyorgy ter-3, Hungary, (email: vajk@aut.bme.hu)

The temperature distribution in a slab can be described by the Kirchhoff-Fourier heat equation that is a 2nd order parabolic partial differential equation (PDE). We consider the nondimensionalized heat equation with normalized space-dimension and symmetric geometry. As heat transfer takes place on both surfaces, we consider Robin boundary conditions. Due to symmetry, the heat process can thus be described by the following equations [4]:

$$\begin{cases} T_\tau(\tau,z) = T_{zz}(\tau,z) + B\,T(\tau,z) & \text{in } (-1,1)\times(0,\infty) \\ T_z(\tau,0) = 0 & \text{in } (0,\infty) \\ T_z(\tau,1) = -\alpha^*\,(T(\tau,1) - T_a(\tau)) & \text{in } (0,\infty) \end{cases} \tag{1}$$

where $T(\tau, z)$ denotes the temperature distribution, $B$ is the gain of a linear internal source, $\alpha^*$ is the dimensionless heat transfer coefficient (Biot-number) on both sides, $T_a(\tau)$ is the ambient temperature, and subscripts $\tau$ and $z$ denotes derivative with respect to time and to space-dimension $z$, respectively.

Without the internal source term, the physical process is always stable: all poles are negative, real numbers. If $B = 0$ the smallest eigenvalue of the operator $\partial^2/\partial z^2$ is $-(\pi^2/4)$ with Dirichlet boundary condition. In case of Robin boundary condition the smallest eigenvalue can be expressed as $p_1 = -\beta_1^2$, where $\beta_1$ is the smallest positive solution of $\beta * \tan(\beta) = \alpha^*$. The effect of the internal source appears in shifting the poles to the right along the real axis, i.e. $p_k = -\beta_k^2 + B$ $(k = 1, 2, ..., \infty)$. From this condition, we can determine the maximum value of $B$ at which the system is still stable. $B$ is usually small (or zero) for solid slabs, and therefore we shall assume that the heat process is stable. For large values of $B$ (unstable process), a stabilizing controller was proposed in [3].

## III. Transfer Function of the Process

We first determine the transfer function of the process. Taking the Laplace transform of (1) we get [4]:

$$\frac{d^2T(s,z)}{dz^2} - q^2\,T(s,z) = 0 \tag{2}$$

where

$$q = \sqrt{(s-B)}. \tag{3}$$

Substituting the general solution into (2) and taking the boundary conditions into account we can express the distributed transfer function (DTF) between the ambient temperature $T_a(\tau)$ (as input) and the temperature distribution

978-3-9524173-9-3

$T(\tau, z)$ (as output) as follows [4,9]:

$$P(s,z) = \frac{T(s,z)}{T_a(s)} = \frac{\alpha^* \cosh(qz)}{\alpha^* \cosh(q) + q \sinh(q)} \qquad (4)$$

Suppose we would like to control the maximum temperature at some point $z = z_0 \in [0, 1)$. As we can only measure the surface temperature $T(\tau, 1)$ we partition $P(s)$ into two parts:

$$P(s) = P_1(s)\, P_2(s, z_0) \qquad (5)$$

where

$$P_1(s) = \frac{T(s,1)}{T_a(s)} = \frac{\alpha^* \cosh(q)}{\alpha^* \cosh(q) + q \sinh(q)} \qquad (6)$$

$$P_2(s, z_0) = \frac{T(s,z_0)}{T_a(s)} = \frac{\cosh(qz_0)}{\cosh(q)} \qquad (7)$$

Note that $P_1(s)$ describes the relation between the input (control) signal $u(\tau) \equiv T_a(\tau)$ and the surface variable $y_1(\tau) \equiv T(\tau, 1)$ and includes the uncertain parameter $\alpha^*$. $P_2(s, z_0)$ represents the relation between the surface temperature and the temperature to be controlled $y_2 = T(\tau, z_0)$. It is independent of the uncertain parameter $\alpha^*$ but depends on the location $z = z_0$. As a worst-case (slowest response) we shall assume $z_0 = 0$, i.e. we control the temperature at the symmetry axes. Figure 1 shows the partitioning of the transfer function $P(s)$.

We may feel intuitively that the process is sensitive for the value of parameter $\alpha^*$. Figure 2 shows the frequency diagram of $P_1(s)$ with different parameter value $\alpha^*$. $P_1(s)$ changes significantly as $\alpha^*$ varies between 0.50 and 4.0. Figure 3 shows the step-response of $P_1(s)$ and $P(s) = P_1(s)P_2(s, 0)$. As we can see, the surface temperature $y_1 \equiv T(\tau, 1)$ reacts very fast to a step-change in ambient temperature and the controlled temperature $y_2 \equiv T(\tau, 0)$ lags behind. However, in both cases, we can see the effect of parameter $\alpha^*$.

## IV. Simulation of the Process

Equation (1) has an exact analytic solution expressed by an infinite sum for arbitrary input (ambient) temperature [4]. It is difficult, however, to use it for simulation due to slow convergence. Therefore, to simulate the process we apply a numerical scheme assuming equidistant grid points:

$$T_i^k = T(k\Delta\tau, i\Delta z) \qquad (8)$$

where $\Delta\tau$ and $\Delta z$ denotes the discrete time-step and discrete space-step, respectively. There are several numerical techniques to solve a PDE. We applied Crank-Nicolson finite difference scheme for it is stable for any value of $(\Delta\tau, \Delta z)$ [14]. Discretizing (1) and collecting similar terms we can express the discrete scheme as:

$$a_i\, T_{i-1}^{n+1} + b_i\, T_i^{n+1} + c_i\, T_{i+1}^{n+1} = d_i^n; \quad \text{i} = 2, 3, ..., N-1. \qquad (9)$$

with

$$\begin{aligned} a_i &= c_i = r \\ b_i &= -2 - 2\,r + \Delta\tau\, B \\ d_i &= -r\, T_{i-1}^n + (-2 + 2\,r - \Delta\tau\, B)\, T_i^n - r\, T_{i+1}^n \\ r &= \Delta\tau/(\Delta z)^2 \end{aligned} \qquad (10)$$

The coefficients $(a_1, b_1, c_1, d_1)$ and $(a_N, b_N, c_N, d_N)$ can be determined from the boundary conditions:

$$a_1 = 0; \quad b_1 = -1; \quad c_1 = 1; \quad d_1 = 0. \qquad (11)$$

and

$$\begin{aligned} a_N &= r \\ b_N &= -1 - r - r\,\Delta z\,\alpha^* + \Delta\tau\, B/2 \\ c_N &= 0 \\ d_N &= -r\, T_{N-1}^n + \gamma\, T_N^n + r\,\Delta z\,\alpha^* \left(T_a^n + T_a^{n+1}\right) \\ \gamma &= -1 + r + r\,\Delta z\,\alpha^* - \Delta\tau\, B/2 \end{aligned} \qquad (12)$$

The control signal appears in the expression of $d_N$. We can also express (9) in matrix form:

$$\mathbf{E}\,\mathbf{T}^{n+1} = \mathbf{d}^n \qquad (13)$$

where matrix $\mathbf{E}$ is tridiagonal. Taking advantage of this fact, the linear equation can very efficiently be solved by the Thomas algorithm [15]. Recognize, that the discrete descriptor state-space model follows immediately:

$$\begin{aligned} \mathbf{E}\,\mathbf{T}[n+1] &= \mathbf{A}\,\mathbf{T}[n] + \mathbf{B}\,u[n] \\ \mathbf{y}[n+1] &= \mathbf{C}\,\mathbf{T}[n] \end{aligned} \qquad (14)$$

where matrix $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ can directly be determined from (10) - (13). Note, that matrix $\mathbf{A}$ is also tridiagonal.

## V. Model Reduction

The transfer function of the process $P_1(s)$ is of infinite dimension and can be expressed by the residue theorem as:

$$\begin{aligned} P_1(s) &= \sum_{n=1}^{\infty} \frac{R_k}{s - p_k} \\ R_k &= \frac{2\alpha^* \beta_k^2}{\beta_k^2 + \alpha^*(1+\alpha^*)} \\ p_k &= -\beta_k^2 + B. \end{aligned} \qquad (15)$$

where $R_k$ is the $k$-th residue of $P_1(s)$ and $p_k$ is the $k$-th pole. $\beta_k$ $(k = 1, 2, ..., \infty)$ denotes the $k$-th positive solution of $\beta \tan(\beta) = \alpha^*$ which are periodic $(k-1)\pi < \beta_k < k\pi$, and converges to the value for large $k$:

$$\lim_{k\to\infty} \beta_k = (k-1)\pi \qquad (16)$$

There are many different ways to approximate the non-rational transfer function $P_1(s)$ by a reduced order model. It would seem natural to take for example the first $N$ terms of the infinite series of (15). Or one can express $P_1(s)$ by an infinite product or apply Padé-approximation [1]. Unfortunately, all these approaches usually suffer from slow convergence, thus requiring many terms. That, in turn, would lead to high-order lumped (or reduced order) model. It can also be shown that models based on the truncated series of (15) always underestimates $P_1(s)$, i.e. $|\tilde{P_1}(i\omega)| < |P_1(i\omega)|$, $\forall\omega \in \mathbb{R}$. To develop a low-order, accurate lumped model, we choose, therefore a different approach. Recall, that if $P_1(s)$ is stable it has the following important properties:

1) The poles $p_k$ of $P_1$ are real, $p_k \in \mathcal{R}$ for all $k \geqslant 1$,
2) $p_{k+1} < p_k$ for all $k \geqslant 1$,
3) The residues $R_k$ are positive and summable.

It can be shown that under these conditions the poles $p_k$ and zeros $z_k$ are *intertwined*, i.e. $p_{k+1} < z_k < p_k$ for all $k \geqslant 1$ [20]. Observe, furthermore, that the slope of the amplitude diagram of $P_1(i\omega)$ converges to -10 [dB/dec] and its phase converges to -45 [degree] for high-enough frequencies:

$$\lim_{s\to\infty} P_1(s) \Rightarrow \frac{\alpha^*}{\alpha^* + \sqrt{(s-B)}} \tag{17}$$

There is no lumped system, which has the same property. However, we can determine a lumped approximation, which in "average" approximate the amplitude diagram until a high-enough frequency. By appropriately chosen intertwined poles and zeros, one can achieve any slope between 0 and -20 [dB/dec]. We propose, therefore, the following approximation technique (in a way, we try to minimize the absolute error) [19]:

1) determine the first pole of $P_1(s)$ as $p_1 = -\beta_1^2 + B$, where $\beta_1$ is the smallest positive solution of $\beta * \tan(\beta) = \alpha^*$ (this is the exact pole of the process),
2) determine the other poles and zeros from the condition that the intersection frequencies $\omega_{k1}$ and $\omega_{k1}$ (of the asymptotes and $|P_1(i\omega)|$) are the geometric means of the poles and zeros $\omega_{k1} = \sqrt{p_k z_k}$ and $\omega_{k2} = \sqrt{p_{k+1} z_k}$ (equal distances on logarithmic scale).

Figure 4 demonstrates how the pole-zero configuration of the reduced order model is determined. The applied approximation always provide an accurate low-order (usually 3rd or 4th order) process model in a pre-defined frequency range (3-5 decades) and we kept the intertwined pole-zero property of the original system [19]. As an example, we give the 4th order approximation of $P_1(s)$ with $\alpha^* = 2$:

$$\tilde{P}_1(s) = \frac{78.36(s+2.5)(s+36)(s+325)}{(s+1.16)(s+16)(s+125)(s+998)} \tag{18}$$

Note, that the static gain is $P_1(0) = 1$. Figure 5 shows the frequency diagram of $P_1(s)$ and $\tilde{P}_1(s)$. As we can see, the approximation provides an accurate estimates of the process transfer function within 5 decades. We remark that fractional order systems have gained increased interest recently. In that regard we refer to the literature [2,12,13,17].

## VI. Internal Model Control Scheme

Internal model control has been widely applied and studied after the pioneering work of Morari [11]. The advantage of IMC is that the controller design for stable plants is based on the open-loop characteristics instead of the closed-loop behavior [8,11]. Figure 6 shows the IMC structure applied to our process. The transfer function of the IMC-loop is:

$$G(s) = \frac{Q(s)P_1(s)}{1 + Q(s)\left(P_1(s) - M(s)\right)} \tag{19}$$

In case of no model-mismatch (perfect model), the IMC is basically open-loop and the stability of $P_1(s)$ and $Q(s)$ implies open-loop stability. Remember, that the process transfer function $P(s)$ is partitioned into $P_1(s)$ and $P_2(s)$ because the controlled variable is not measurable.

In ideal case, the internal process model $M(s)$ is equal to the process model $P_1(s)$. Unfortunately, this is usually not the case. However, if we choose the steady-state gain of the controller $Q(0)$ to be $Q(0) = M^{-1}(0)$ then the gain between the set point $T_{r1}(s)$ and the surface temperature $T(s,1)$ is one, and the process output $y_1(t)$ has zero steady-state error.

A good controller would be the inverse of the realizable part of $M(s)$. However, our physical model is an infinite dimensional process, so it is not trivial to determine what actually the "realizable" part would be. Applying the IMC design scheme to distributed parameter systems, we have two options: i.) either approximate the distributed transfer function and design the controller based on the reduced order model or ii.) design the controller for the distributed process and approximate the (possibly infinite-dimensional) controller.

Having determined a low-order lumped approximation of $P_1(s)$, it is natural to use it for the controller design. Depending on the requirements regarding the closed-loop (bandwidth, for example) we may choose a 2nd, 3rd or 4th order approximation of $P_1(s)$ leading to the following controllers:

$$\begin{aligned} Q_1(s) &= 2.155\frac{(s+1.16)}{(s+2.50)} = \frac{1+0.86s}{1+0.4s} \\ Q_2(s) &= 4.849\frac{(s+1.16)(s+16)}{(s+2.50)(s+36)} \\ Q_3(s) &= 38.328\frac{(s+1.16)(s+16)(s+125)}{(s+2.50)(s+36)(s+988)} \end{aligned} \tag{20}$$

Clearly, these are phase-lead (or PD) compensators. We have simulated the closed-loop process and assumed that the physical process $P_1(s)$ had parameter $\alpha^* = 1$ but the internal model $M(s)$ had parameter $\alpha^* = 2$. In Table I we also give the value of the dominant (first) pole of the process for different parameter value $\alpha^*$. To compensate the effect of $P_2(s)$ and to improve tracking, a pre-filter can be chosen as the inverse of the process model $P_2(s)$:

$$T(s) = \frac{1}{P_2(s)} = \cosh(q) = \prod_{k=1}^{\infty}\left(1 + \frac{4q^2}{(2k-1)^2\pi^2}\right) \tag{21}$$

Since $q = \sqrt{s-B}$, the desired pre-filter $T(s)$ has undesirable characteristics at high frequencies. This can simply be dealt with by considering only the first term of the infinite product which gives:

$$\tilde{T}(s) = \frac{1+sT_D}{1+(\epsilon\, T_D)s} \tag{22}$$

where $T_D = 4/\pi^2 \approx 0.4$ and $\epsilon$ is a design parameter. However, to avoid a large filter output, we may restrict the value of $\epsilon$. Usually $\epsilon = [0.4 - 1]$ provides a good compromise between accelerating tracking but limiting the reference signal.

Figure 7 shows the closed-loop performance and the control signal applied ($B = 0$) with pre-filter ($\epsilon = 0.8$). At time $0.25$, the process parameter $\alpha^*$ dropped from $3.0$ to $1.0$ and then jumped to $4.0$ and dropped again, meanwhile the model parameter remained unchanged. We can see how fast the controller reacts to the parameter changes and drives the controlled variable to its required value. The closed-loop with the given controller is stable and it retains property (17) for high-frequencies:

$$\lim_{s\to\infty} G(s) \Rightarrow \frac{\alpha^*(\alpha_M^* + q)}{\alpha^*\alpha_M^* + q^2} \qquad (23)$$

where $\alpha^*$ and $\alpha_M^*$ is the process parameter and model parameter, respectively. Figure 8 shows the closed-loop Nyquist diagram. Notice, how the phase approaches the origo.

We note, that it is also possible to express an IMC controller directly from the distributed process transfer function: $\tilde{Q}(s) = 1/M(s)$ which is also infinite dimensional [10]. Since $P_1(s)$ is strictly proper one must augment $\tilde{Q}(s)$ with a low-pass filter $F(s,\mu)$ which must satisfy certain conditions. A possible infinite-dimensional controller can thus be expressed in the form:

$$Q(s) = \tilde{Q}(s)F(s,\mu) = \frac{1 + \frac{q}{\alpha_M^*}\tanh(q)}{1 + \mu\frac{q}{\alpha_M^*}\tanh(q)} \qquad (24)$$

where $F(s,\mu)$ is the IMC filter with design parameter $\mu$. The optimal value of $\mu$ can then be determined from the robust performance criteria [6,10]. However, to implement the resulting controller, one has to approximate now $Q(s)$ instead of the process model $P_1(s)$.

## VII. Conclusion

We have considered the problem of controlling the temperature within a slab, which is not directly measurable. The temperature can only be controlled via boundary control by measuring the surface temperature. The process is described by a 2nd order parabolic partial differential equation. We give the distributed transfer function of the process, and showed how sensitive it is for parameter $\alpha^*$ (heat transfer coefficient). To design an IMC controller we devised a model reduction technique, which provides an accurate low-order process model. The proposed IMC controller compensates the dominant pole-zero of the model and provides a robust closed-loop behavior.

## Acknowledgment

Miklos Vajta greatly appreciates the financial support of the Department of Applied Mathematics, University of Twente, the Netherlands during his sabbatical leave.

TABLE I

Dominant poles of a heat process

| $\alpha^*$ | $\beta_1$ | $p_1$ |
|---|---|---|
| 0.01 | 0.09983 | −0.0100 |
| 0.05 | 0.22176 | −0.0492 |
| 0.10 | 0.31105 | −0.0968 |
| 0.25 | 0.48009 | −0.2305 |
| 0.50 | 0.65327 | −0.4268 |
| 1.00 | 0.86033 | −0.7402 |
| 1.50 | 0.98824 | −0.9766 |
| 2.00 | 1.07687 | −1.1596 |
| 5.00 | 1.31384 | −1.7262 |
| 10.0 | 1.42887 | −2.0417 |

## References

[1] Abramowitz, M. and I.A.Stegun: *Handbook of Mathematical Functions*, Dover Publications Inc., New York, 1972.

[2] Aoun,M., R.Malti, F.Levron and A.Oustaloup: Synthesis of fractional Laguerre basis for system approximation, *Automatica*, Vol.43, 2007, pp.1640-1648.

[3] Boškovic,D.M., M.Krstić and W.Liu: Boundary Control of an Unstable Heat Equation Via Measurement of Domain-Averaged Temperature, *IEEE Trans. on AC*, vol.46, No.12, 2001, pp.2022-2028.

[4] Carslaw,H.S. and J.C.Jaeger: *Conduction of Heat in Solids*, Clarendon Press, Oxford, 2nd edition, 2001.

[5] Curtain,R.F. and H.J. Zwart: *An Introduction to Infinite-Dimensional Systems Theory*, Text in Applied Mathematics Vol. 21, Springer Verlag, New-York, 1995.

[6] Doyle,J.C., B.A.Francis and A.Tannenbaum: *Feedback Control Theory*, Macmillan Publishing Company, New York, 1992.

[7] El-Farra,N.H., A.Armaou and P.D.Christofides: Analysis and control of parabolic PDE systems with input constraints, *Automatica*, Vol.39, pp.715-725, 2003.

[8] Farkas,I. and I.Vajk: Internal model-based controller for solar plant operation, *Computers and Electronics in Agriculture*, vol.49, 2005, pp.407-418.

[9] Hulkó,G., M.Antoniová, C.Belavý, J.Belanský, J.Szuda and P.Végh: *Modeling, Control and Design of Distributed Parameter Systems with Demonstrations in MATLAB*, Publishing House of STU, Bratislava, 1998.

[10] Kishida,M. and R.D.Braatz: Internal Model Control of Infinite Dimensional Systems, *47th IEEE Conf. on Decision and Control*, Cancun, Mexico, De.9-11, 2008, pp.1434-1441.

[11] Morari,M. and E.Zafiriou: Robust Process Control, Prentice-Hall,Inc., Englewoods Cliffs, N.J., 1989.

[12] Oustaloup,A. and B.Mathieu: *La commande CRONE: du Scalaire au Multivariable*, Hermes, Edition CNRS, Science Publications, Paris, 1999.

[13] Oustaloup,A., F.Levron, B.Mathieu and F.M.Nanot: Frequncy-band complex noninteger differentiator: characterization and synthesis, *EEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, Vol.47, January 2000, pp.25-39.

[14] Özisik,M.N.: *Finite Difference Methods in Heat Transfer*, CRC Press, Boca Raton, 1994.

[15] Press,W.H., B.P.Flannery, S.A.Teukolsky and W.T.Vetterlink: *Numerical Recipes*, Cambridge University Press, Cambridge, 1986.

[16] Spiegel,R.M.: *Laplace Transforms*, in Schaum's Outline Series, McGraw-Hill, New York, 1965.

[17] Tenreiro Machado,J. and I.S.Jesus: Fractional Order Dynamics in Some Distributed Parameter Systems, *Proc. of the 24th IASTED Int. Conf. on Modeling, Identification and Control*, Innsbruck, Austria, February 16-18, 2005, pp.29-34.

[18] Vajta,M.: A New Model Reduction Technique for a Class of Parabolic Partial Differential Equations, *IEEE International Conference on System Engineering*, pp.311-315, August 1-3, 1991, Dayton, Ohio, USA.

[19] Toure,Y. and L.Josserand: An extension of the IMC to boundary distributed parameter systems, *Proc. of the IEEE SMC-CCS*, 1997, pp.2426-2431.

[20] Zwart,H. and M.B.Hof: Zeros of Infinite-Dimensional Systems, *IMA Journal of Mathematical Control Information*, Vol.14, 1997, pp.85-94.

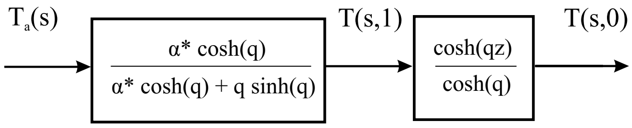


Fig. 1. Partitioning the process transfer function $P(s) = P_1(s)P_2(s, z_0)$.

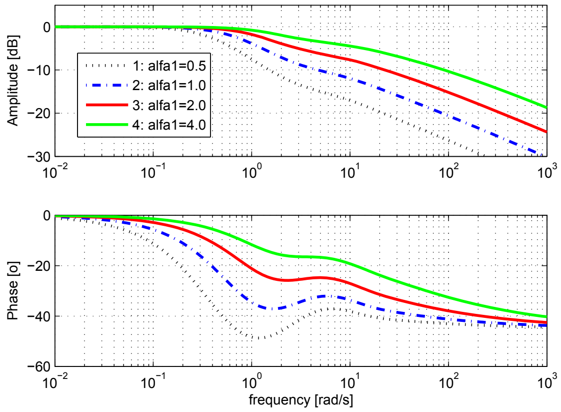


Fig. 2. Frequency diagram of transfer function $P_1(s)$ for different values of the parameter $\alpha^*$.

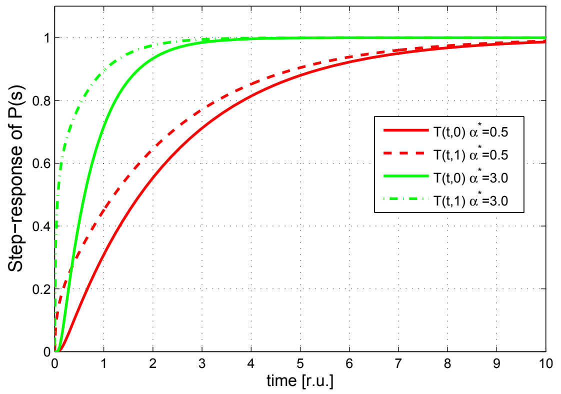


Fig. 3. Open-loop step-response of the process with different parameter $\alpha^*$. ($y_1(\tau)$ = continuous, $y_2(\tau)$ = dashed line)

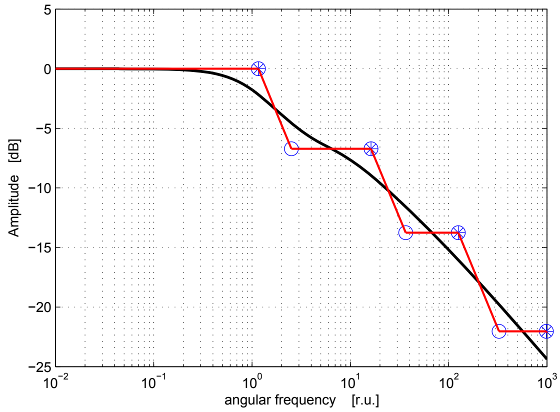


Fig. 4. Pole-zero configuration of the lumped approximation of $P_1(s)$ and their asymptotes.

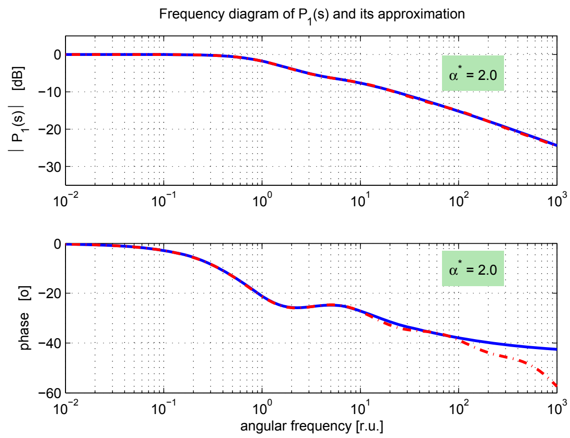


Fig. 5. Frequency diagram of $P_1(s)$ (continuous) and its 4-th order approximation (dashed) ($\alpha^* = 2.0$ $B = 0$).

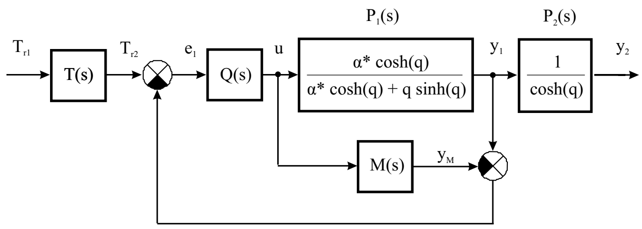


Fig. 6. Internal Model Control (IMC) structure for the heat process. $y_1(\tau)$ is the measured temperature $T(s, 1)$ and $y_2(\tau)$ is the controlled temperature $T(s, 0)$.

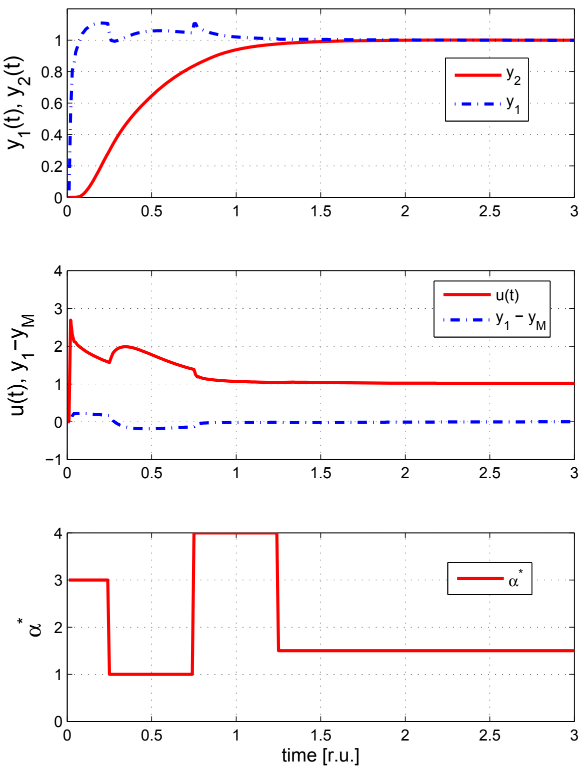


Fig. 7. Closed-loop performance of the IMC scheme with pre-filter ($\epsilon = 0.8$). a.) Measured temperature $y_1(\tau)$ and controlled temperature $y_2(\tau)$, b.) control signal $u(\tau)$ and model error $y_1 - y_M$, c.) parameter $\alpha^*$.

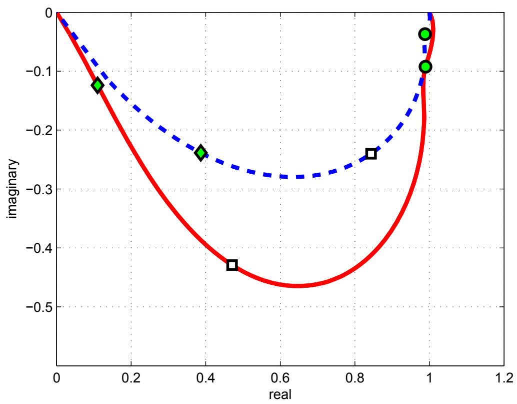


Fig. 8. Closed-loop Nyquist diagram with IMC controller $Q_2(s)$ for $\alpha^*_{min} = 1.0$ (continuous) and $\alpha^*_{max} = 4.0$ (dashed). ($\circ$ : $\omega = 10$, $\Box$ : $\omega = 100$ and ♦ : $\omega = 1000$).