STABILITY OF RESET SYSTEMS

S. POLENKOVA[*], J.W. POLDERMAN[*] AND R. LANGERAK[†]

Abstract. We derive sufficient conditions for asymptotic stability of state reset systems in terms of a linear matrix inequality. The reset system is modeled as a hybrid automaton with one discrete state. The guard on the transition is a switching surface and the reset map is a projection onto a subspace of the state space. A discrete stability indicator is introduced: the projection gain. A modified version of the LMI provides a estimate of the projection gain.

Key words. Reset systems, hybrid automaton, stability analysis

AMS subject classifications. 34K20, 34K34, 34K45

1. Introduction. In this paper we focus on the stability of systems with state reset. The main motivation for studying state reset systems lies in reset control where the controller states are reset to zero whenever its input meets a threshold. The first reset controllers were introduced by Clegg in 1958, the so-called Clegg integrator, whose output reset to zero when its input meets zero. Furthermore, in a series of papers, [7, 8], reset control systems have been advanced by introducing the first-order reset element. One of the main disadvantages of reset controllers is that the reset action may destabilize the system. Recent work, [1, 3, 6], addressed the stability problem of this type of systems.

Reset control systems can also be considered as a special case of hybrid systems. Stability analysis for hybrid systems is a much harder problem than it is for smooth systems. The reason appears to be the interplay between continuous time driven dynamics and discrete event driven dynamics. See [2, 5, 4, 10] and the references therein.

Necessary and sufficient conditions for stability of a seemingly simple situation, a single linear planar system with a state reset, are derived in [11].

Motivated by reset control systems, the main goal of this paper is to study stability of systems with state reset in a somewhat more general context.

We consider systems modeled as a hybrid automaton, as depicted in Figure 1.1. See [11] for details on hybrid automata.

Here A is a Hurwitz matrix and $\mathcal{V}$ is a linear subspace of the state space. Whenever the state trajectory hits $\mathcal{V}$, part of the state is put to zero by the projection operator Π.

The dynamics in the location is described by a system of differential equations:

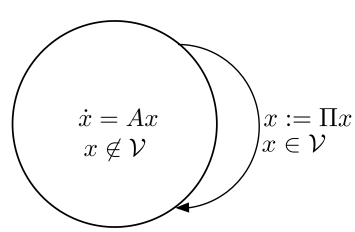

FIG. 1.1. *Linear system with state reset*

[*]Department of Applied Mathematics, University of Twente, The Netherlands.
[†]Department of Computer Science, University of Twente, The Netherlands.

$$\dot{x} = Ax. \tag{1.1}$$

The guard on the transition is the linear subspace $\mathcal{V}$ of dimension m, the generalization of the subspace corresponding to error zero in reset control systems.

$$\mathcal{V} = \{x \in \mathbb{R}^n \Big| x = My, y \in \mathbb{R}^m\}, \tag{1.2}$$

for some matrix $M \in \mathbb{R}^{n\times m}$ of rank m and $\Pi = \begin{bmatrix} \mathrm{I} & 0 \\ 0 & 0 \end{bmatrix}$ with $\mathrm{I} \in \mathbb{R}^{k\times k}$ is the orthogonal projection onto k-dimensional subspace of $\mathbb{R}^n$. We define

$$\mathcal{W} = \{\Pi x \in \mathbb{R}^n \Big| x \in \mathbb{R}^n\}. \tag{1.3}$$

The subspaces $\mathcal{V}$ and $\mathcal{W}$ are called the switching plane and projection plane respectively.

The system depicted in Figure 1.1 can be written as follows:

$$\begin{cases} \dot{x} &= Ax, \qquad x \notin \mathcal{V}, \\ x^+ &= \Pi x, \qquad x \in \mathcal{V}, \end{cases} \tag{1.4}$$

where x^+ is the state of the system after reset.

To avoid ill-posedness we assume that $\mathcal{V} \cap \mathcal{W} = \{0\}$. This implies that $k + m \leq n$.

The state is reset by orthogonal projection Π whenever the state trajectory crosses the switching plane $\mathcal{V}$. Although system (1.1) has no unstable poles, the state reset may lead to instability. Examples that illustrate this can easily be constructed. See Figure 1.2 for an example of a system that is destabilized due to state reset.

Checking stability of reset systems is non-trivial. In this paper we derive sufficient conditions for stability by constructing an appropriate quadratic Lyapunov function. Moreover by optimizing the Lyapunov function we provide an estimate of the so called *projection gain*, a measure of how much resetting the state contributes to stability.

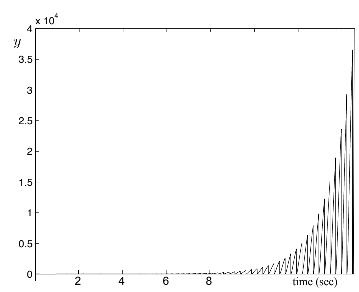

FIG. 1.2. *Unstable reset system*

2. LMI-based stability criterion. In this section we formulate sufficient conditions for stability of the reset system depicted in Figure 1.1. We assume that the dynamics in the location A, the switching plane $\mathcal{V}$ and projection matrix Π, hence also the projection plane $\mathcal{W}$, are given. Furthermore, a positive definite matrix P with $A^{\mathrm{T}}P + PA < 0$ is given. Our objective is to find criteria that guarantee stability of the system for the given triple $(A, \mathcal{V}, \Pi)$ and matrix P. First we shall give a geometric criterion. Using this we translate the geometric stability criterion into a linear matrix inequality.

We partition the state as $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, where $x_1 \in \mathbb{R}^k$. We partition P and M accordingly, $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}$, with $P_{11} \in \mathbb{R}^{k\times k}$, $M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}$, with $M_1 \in \mathbb{R}^{k\times m}$.

We define ellipsoidal set:

$$E = \{x \in \mathbb{R}^n \Big| x^{\mathrm{T}} P x = 1\}. \tag{2.1}$$

The intersection of E with $\mathcal{V}$:

$$E_1 = \{x \in E \Big| \exists y \in \mathbb{R}^m : x = My\}. \tag{2.2}$$

Finally we shall use the intersection of E and its interior with $\mathcal{W}$:

$$E_2 = \{x \in \mathcal{W} \mid x_1^{\mathrm{T}} P_{11} x_1 \leq 1\}. \tag{2.3}$$

Intuitively, the relevance of the sets defined above in relation to stability is as follows. Any state trajectory that starts from the boundary of E_2 will stay within the ellipsoid E, since P defines a quadratic Lyapunov function for the unswitched system. As soon as the state trajectory intersects with the switching plane $\mathcal{V}$ it is projected back onto $\mathcal{W}$. Since it hits $\mathcal{V}$ within E, this must be the case on or inside E_1. After projection onto $\mathcal{W}$ the state will therefore be on or inside ΠE_1. Now, stability is guaranteed if ΠE_1 is contained in E_2.

The following lemma relates this inclusion property to a linear matrix inequality.

Lemma 2.1. *$\Pi E_1 \subset E_2$ if and only if $M^{\mathrm{T}}\Pi P \Pi M - M^{\mathrm{T}} P M \leq 0$.*

Proof. ($\Rightarrow$) Assume that the $\Pi E_1 \subset E_2$. Choose $y \in \mathbb{R}^m$ with $y^{\mathrm{T}} M^{\mathrm{T}} P M y = 1$. It follows that $My \in E_1$ and therefore $\Pi M y \in \Pi E_1$. Since $\Pi E_1 \subset E_2$ it follows that $\Pi M y \in E_2$ and hence $y^{\mathrm{T}} M^{\mathrm{T}} \Pi P \Pi M y \leq 1$. This means that $y^{\mathrm{T}} M^{\mathrm{T}} P M y = 1$ implies $y^{\mathrm{T}} M^{\mathrm{T}} \Pi P \Pi M y \leq 1$ and hence $M^{\mathrm{T}}\Pi P \Pi M - M^{\mathrm{T}} P M \leq 0$.

($\Leftarrow$) Assume that $M^{\mathrm{T}}\Pi P \Pi M - M^{\mathrm{T}} P M \leq 0$. Then, for all $y \in \mathbb{R}^m$

$$y^{\mathrm{T}} M^{\mathrm{T}} \Pi P \Pi M y - y^{\mathrm{T}} M^{\mathrm{T}} P M y \leq 0. \tag{2.4}$$

Choose $x \in \Pi E_1$, then there exists $y \in \mathbb{R}^m$ with $x = \Pi M y$ and $y^{\mathrm{T}} M^{\mathrm{T}} P M y = 1$. Therefore we have that $y^{\mathrm{T}} M^{\mathrm{T}} \Pi P \Pi M y \leq 1$. Hence $x \in E_2$. □

The intuitive geometric criterion for stability has now been translated into a linear matrix inequality. Existence of global solutions and stability can now readily be proven by invoking Proposition 3.1 in [12].

We want to apply this proposition to quadratic Lyapunov function $V(x) = x^{\mathrm{T}} P x$, where P is a positive-definite symmetric matrix.

Before we can do that, we need to establish that the switching time instants are separated by a uniform positive constant δ. It is exactly this point where the assumption that the switching plane $\mathcal{V}$ and the projection plane $\mathcal{W}$ intersect trivially is used.

Lemma 2.2. *Let $\mathcal{V}, \mathcal{W} \subset \mathbb{R}^n$ be linear subspaces of $\mathbb{R}^n$ and $A \in \mathbb{R}^{n\times n}$. If $\mathcal{V}$ and $\mathcal{W}$ are such that $\mathcal{V} \cap \mathcal{W} = \{0\}$ then there exists $\delta > 0$ such that for all $x \in \mathcal{W}$, $x \neq 0$, we have that $\mathrm{e}^{At} x \notin \mathcal{V}$ for $0 \leq t < \delta$.*

Proof. Since the traveling time from a point in $\mathcal{W}$ to a point in $\mathcal{V}$ is scale invariant, we can restrict the attention to points on the unit sphere. Suppose that the statement is not true. Then for all $\delta > 0$ there exists $x \in \mathcal{W}$, and, $||x|| = 1$, and $0 \leq t < \delta$ such that $\mathrm{e}^{At}x \in \mathcal{V}$.

As a consequence there exist sequences $\{t_n\}$, $t_n > 0$ and $\{x_n\}$, $||x_n|| = 1$ with $\lim_{n\to\infty} t_n = 0$, such that $\mathrm{e}^{At_n}x_n \in \mathcal{V}$ for all n, and $\lim_{n\to\infty} x_n = x^*$. It follows that $\lim_{n\to\infty} \mathrm{e}^{At_n}x_n = x^* \in \mathcal{V}$ since $\mathcal{V}$ as a finite-dimensional linear subspace is closed.

Since $x_n \in \mathcal{W}$ for all n it follows that $x^* \in \mathcal{W}$ since also $\mathcal{W}$ as a finite-dimensional linear subspace is closed.

It follows that $x^* \in \mathcal{V} \cap \mathcal{W}$ and hence $x^* = 0$. Since $||x_n|| = ||x^*|| = 1$, we have a contradiction. This proves the statement. □

We are now ready to formally present the geometric condition for stability.

THEOREM 2.3. *If there exists $P = P^{\mathrm{T}} > 0$ such that $A^{\mathrm{T}}P + PA < 0$ and $\Pi E_1 \subset E_2$, then*

- *there exists a left-continuous function $x(t)$, satisfying (1.4) for all $t \geq 0$;*
- *the equilibrium point $x = 0$ is asymptotically stable.*

Proof. The existence of solutions of (1.4) follows from Proposition 3.1 in [12] and Lemma 2.2.

Define a quadratic Lyapunov function $V(x) = x^{\mathrm{T}}Px$.

We have

$$\dot{V}(x) < 0, \qquad x \neq 0. \tag{2.5}$$

It remains to show, [12][Proposition 3.1], that $\Pi E_1 \subset E_2$ is equivalent to

$$V(\Pi x) - V(x) \leq 0, \qquad x \in \mathcal{V}. \tag{2.6}$$

By Lemma 2.1, we have that $\Pi E_1 \subset E_2$ implies

$$y^{\mathrm{T}}M^{\mathrm{T}}\Pi P \Pi M y - y^{\mathrm{T}}M^{\mathrm{T}}PMy \leq 0, \qquad \text{for all } y \in \mathbb{R}^m. \tag{2.7}$$

This equivalent to

$$(\Pi x)^{\mathrm{T}}P(\Pi x) - x^{\mathrm{T}}Px \leq 0, \qquad x \in \mathcal{V}. \tag{2.8}$$

It follows that (2.8) implies (2.6). This completes the proof. □

COROLLARY 2.4. *If there exists $P = P^{\mathrm{T}} > 0$ such that $A^{\mathrm{T}}P + PA < 0$ and*

$$M^{\mathrm{T}}\Pi P \Pi M - M^{\mathrm{T}}PM \leq 0, \tag{2.9}$$

then the equilibrium point $x = 0$ is asymptotically stable.

3. Projection gain γ.

Theorem 2.3 provides an intuitive and appealing geometric condition for stability of the switched system. However, more information can be extracted from the underlying linear matrix inequality. Indeed, refer to Figure 3.1. Obviously, $\Pi E_1 \subset E_2$ indicates stability. Intuitively it is clear that the further apart the boundary of E_2 and ΠE_1 are, the more switching contributes to stability.

If the system is initialized in x_0 on the boundary of E_2, i.e on the level set of P, corresponding to level equal to one, then after the switch the state is projected into the set ΠE_1. Now, define γ as the worst case level of P taken over ΠE_1. So, if $\gamma < 1$, then by switching the state moves from level one to a lower level. Hence the switch adds a discrete factor γ to the continuous stability of the system. We call γ the projection gain. The formal definition is given below.

DEFINITION 3.1. (**Projection gain**.) *The projection gain γ of switched system is defined as*

$$\gamma = \max_{x_1 \in \Pi E_1} x_1^{\mathrm{T}} P_{11} x_1. \tag{3.1}$$

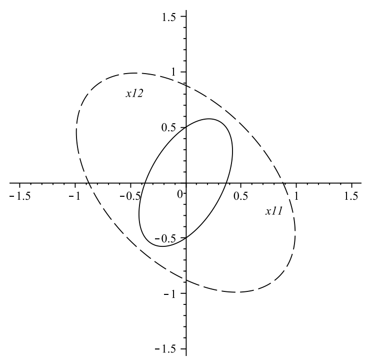

FIG. 3.1. *The ellipsoidal sets E_2 (dashed) and ΠE_1 (solid)*

The next result shows how γ can be computed.

THEOREM 3.2.

1. $\gamma = \max\limits_{y^{\mathrm{T}} M^{\mathrm{T}} P M y = 1} y^{\mathrm{T}} M^{\mathrm{T}} \Pi P \Pi M y.$
2. $\gamma = \lambda_{\max}((M^{\mathrm{T}} P M)^{-1/2} M^{\mathrm{T}} \Pi P \Pi M (M^{\mathrm{T}} P M)^{-1/2}).$

Here $\lambda_{\max}$ denotes the maximum eigenvalue. Note that since M has full column rank and $P > 0$ the matrix $M^{\mathrm{T}} P M$ is non-singular.

Proof. Part 1. This follows directly from the observation that for each $x_1 \in \Pi E_1$ there exists a $y \in \mathbb{R}^m$ with $y^{\mathrm{T}} M^{\mathrm{T}} P M y = 1$ such that $x_1^{\mathrm{T}} P_{11} x_1 = y^{\mathrm{T}} M^{\mathrm{T}} \Pi P \Pi M y$ and vice versa.

Part 2. Define $R = M^{\mathrm{T}} P M$ and $S = M^{\mathrm{T}} \Pi P \Pi M$, then:

$$\gamma = \max_{y^{\mathrm{T}} R y = 1} y^{\mathrm{T}} S y = \max_{z^{\mathrm{T}} z = 1} z^{\mathrm{T}} R^{-1/2} S R^{-1/2} z = \lambda_{\max}(R^{-1/2} S R^{-1/2}).$$

□

4. Optimal Lyapunov function. The projection gain γ is related to stability in that $\gamma \leq 1$ guarantees stability. Additionally, smaller γ implies that at each switch there is boost in stability. Indeed, if $\gamma < 1$, then due to switching the state moves from level one to a lower level and asymptotic stability of reset system is guaranteed. Disregarding the continuous time stability we see that at each switching time s_k we have

$$x(s_k)^{\mathrm{T}} x(s_k) \leq \gamma^k \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} x(0)^{\mathrm{T}} x(0). \tag{4.1}$$

From (4.1) it follows the smaller γ, the faster convergence of the state to zero. From Theorem 3.2 it follows that γ depends on P. It therefore makes sense to search for a P that minimizes γ. As the dependence of γ is highly non-linear, this appears not to be an easy task. On the other hand, inequality (2.9) allows the situation where, see Figure 3.1, E_2 and ΠE_1 intersect tangentially, thus leading to $\gamma = 1$. By replacing the right hand side of (2.9) by a negative definite matrix, this is avoided.

So, consider

$$A^{\mathrm{T}}P + AP < 0, \quad M^{\mathrm{T}}\Pi P\Pi M - M^{\mathrm{T}}PM \leq -\varepsilon I, \quad \varepsilon > 0. \tag{4.2}$$

The joint problem of checking for stability and finding a $\gamma < 1$ now becomes:

Find maximal $\varepsilon \in \mathbb{R}$ such that (4.2) has a positive definite solution P.

Of course, since (4.2) is linear in P and ε we need to normalize P appropriately to guarantee the existence of a maximal ε. The following theorem ensures that this can be done. We choose a $x_0 \in \mathbb{R}^n$ of norm one, that is, $x_0^{\mathrm{T}}x_0 = 1$ and define the set

$$\Omega = \{P \in \mathbb{R}^{n\times n} | P = P^{\mathrm{T}} \geq 0, A^T P + PA \leq 0, x_0^{\mathrm{T}}Px_0 = 1\}. \tag{4.3}$$

THEOREM 4.1 ([9]). *Let $A \in \mathbb{R}^{n\times n}$ be a Hurwitz matrix and let $x_0 \in \mathbb{R}^n$ be a nonzero vector. If x_0 does not belong to a proper A-invariant subspace then Ω is compact.*

COROLLARY 4.2. *For a generic choice of x_0 on the unit sphere there exists $\varepsilon_{\max} \in \mathbb{R}$ such that for all $\varepsilon \geq \varepsilon_{\max}$ (4.2) does not have a positive definite solution.*

A Lyapunov function is called optimal if the projection gain γ is minimal over Ω. For given matrices A, Π, M an optimal Lyapunov function exists since from Theorem 4.1 follows that the set Ω is compact.

One could expect that somehow γ decreases monotonically with increasing ε. However, counter examples may be constructed that show that such a conjecture is wrong. But fortunately γ admits an upper bound that is monotonically decreasing as ε increases. This is the content of the following theorem.

THEOREM 4.3. *The projection gain γ admits a monotonically decreasing upper bound.*

Proof. Since Ω is compact we can define

$$C = \min_{P\in\Omega} \lambda_{\max}(M^{\mathrm{T}}PM).$$

From Corollary 2.4 and Theorem 3.2 it follows that:

$$\gamma = \max_{y^{\mathrm{T}}M^{\mathrm{T}}PMy=1} y^{\mathrm{T}}M^{\mathrm{T}}\Pi P\Pi My \leq -\varepsilon y^{\mathrm{T}}y + 1 \tag{4.4}$$

$$=1 - \frac{\varepsilon}{\lambda_{\max}(M^{\mathrm{T}}PM)} \leq 1 - \frac{\varepsilon}{C}. \tag{4.5}$$

□

EXAMPLE 4.4. *Consider the reset system (1.4) with Hurwitz matrix A and projection matrix Π given by*

$$A = \begin{bmatrix} -4.25 & -0.5 & 1.75 & -1 \\ -2 & -2 & 1 & 0 \\ -0.75 & -1.5 & -0.75 & -3 \\ 1.25 & -0.5 & -0.75 & -3 \end{bmatrix} \quad \Pi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The state $x \in \mathbb{R}^4$ is reset by orthogonal projection Πx whenever the state trajectory hits the switching plane $\mathcal{V}$. The switching plane is given by $\mathcal{V} = \left\{ x \in \mathbb{R}^4 \middle| x = My, y \in \mathbb{R}^2 \right\}$, where $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$. We check whether system (1.4) is asymptotically stable.

We choose the vector $x_0 = [1\ 0\ 0\ 0]^T$. By Theorem 4.1 it follows that Ω is compact. Using LMI toolbox, we find $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4.195 & -2.3715 & 2.3715 \\ 0 & -2.3715 & 5.2083 & -3.6768 \\ 0 & 2.3715 & -3.6768 & 5.0207 \end{bmatrix}$ and $\varepsilon_{\max} = 9.065$ satisfy (4.2). It follows that the reset system is asymptotically stable. We compute the projection gain $\gamma = \lambda_{\max}((M^{\mathrm{T}} PM)^{-1/2} M^{\mathrm{T}} \Pi P \Pi M (M^{\mathrm{T}} PM)^{-1/2}) = 0.6992$.

One could hope that stability conditions, obtained in Corollary 2.4, are necessary and sufficient. We construct an example, that demonstrates, however, that the conditions are only sufficient, i.e., asymptotic stability of reset systems does not imply existence of P, satisfying all the conditions of Corollary 2.4.

EXAMPLE 4.5. *Consider the reset system (1.4) with $A = \begin{bmatrix} 1.775 & -2.125 \\ 2.125 & -1.975 \end{bmatrix}$, A is Hurwitz, the state $x = [x_1\ x_2]^T \in \mathbb{R}^2$ is reset by orthogonal projection $\Pi = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, whenever the state trajectory hits the switching plane $\mathcal{V}$. The switching plane is given by $\mathcal{V} = \left\{ x \in \mathbb{R}^2 \middle| x = My, y \in \mathbb{R}, \right\}$, with $M = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$.*

Using necessary and sufficient conditions for stability, given in [11], we can prove that the reset system is asymptotically stable. For more details see Example 6.3 of [11].

We show that there does not exist a symmetric matrix $P \in \mathbb{R}^2$ such that:

$$\begin{aligned} & P > 0, \\ & A^T P + PA < 0, \\ & M^T \Pi P \Pi M - M^T PM \leq 0. \end{aligned} \tag{4.6}$$

Without loss of generality, we normalize the set of Lyapunov functions as follows:

$$\bar{P} = \left\{ \begin{bmatrix} 1 & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \middle| p_{22} - p_{12}^2 > 0, p_{22} > 0 \right\}. \tag{4.7}$$

Substitute A, Π, M into (4.6) we derive the following LMI system:

$$\left\{ \begin{array}{rl} p_{22} - p_{12}^2 & > 0 \\ \begin{bmatrix} 3.55 + 4.25p_{12} & -0.2p_{12} + 2.125p_{22} - 2.125 \\ -0.2p_{12} + 2.125p_{22} - 2.125 & -4.25p_{12} - 3.92p_{22} \end{bmatrix} & < 0 \\ -1.25p_{12} - p_{22} & \leq 0 \end{array} \right. \tag{4.8}$$

In Figure 4.1 we have depicted the solutions of the system (4.8) in the $p_{12}p_{22}$- plane: $\bar{\Omega}$ denotes the solutions of the first and second inequalities in (4.8). The shaded region forms the solution set of the third inequality in (4.8). As these sets do not intersect it follows that there does not exists $P \in \bar{P}$, satisfying (4.6).

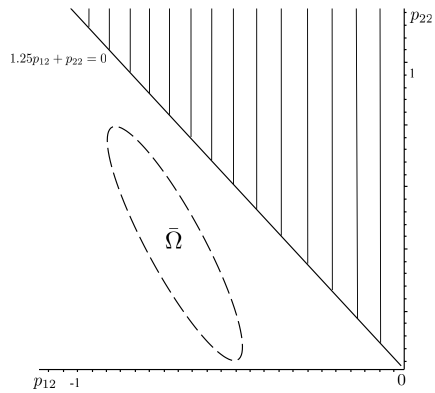

FIG. 4.1. *Solutions of (4.8) in* $p_{12}p_{22}$*- plane*

5. Conclusions. Motivated by the stability problem of reset control systems we have studied stability of more general state reset systems. Through a quadratic Lyapunov function of the unswitched system we gave a geometric interpretation of stability of the reset system. Subsequently, the geometric condition was translated into a linear matrix inequality. The projection gain was introduced as a discrete stability indicator. Finally we showed how to optimize the estimate of the projection gain by an appropriate modification of the linear matrix inequality.

REFERENCES

[1] W.H.T.M. Aangenent, G. Witvoet, W.P.M.H. Heemels, and M.J.G. van de Molengraft. An lmi-based l_2 gain performance analysis for reset control and systems. *Proceedings of the 2008 American Control Conference*, 2008.
[2] Andrei A. Agrachev and Daniel Liberzon. Lie-algebraic stability criteria for switched systems. *SIAM Jour. Ctr. Opt.*, 40:253–269, 2001.
[3] O. Beker, C.V. Hollot, Y. Chait, and H. Han. Fundamental properties of reset control and systems. *Automatica*, 40:905–915, 2004.
[4] V. D. Blondel and J. N. Tsitsiklis. Complexity of stability and controllability of elementary hybrid systems. *Automatica*, 35(3), March 1999.
[5] M.S. Branicky. Multiple lyapunov functions and other analysis tools for switched and hybrid systems. *IEEE Transactions on automatic control*, 43(4), April 1998.
[6] J.P. Hespanha and A.S. Morse. Switching between stabilizing controllers. *Automatica*, 38(11):1905–1917, 2002.
[7] I.M. Horowitz and Rosenbaum. Nonlinear design for cost of feedback reduction in systems with large parameter uncertainty. *International Journal of Control*, 24(6):977–1001, 1975.
[8] K.R. Krishman and I.M. Horowitz. Synthesis of a nonlinear feedback system with significant plant-ignorance for prescribed system tolerances. *International Journal of Control*, 19(4):689–706, 1974.
[9] R. Langerak and J.W. Polderman. Tools for stability of switching linear systems: Gain automata and delay compensation. *44th IEEE Conference on Decision and Control and European Control Conference ECC*, 1:4867–4872, 2005.
[10] Z. G. Li, C. B. Soh, and X. H. Xu. Stability of impulsive differential systems. *Journal of Mathematical Analysis and Application*, 216:644–653, 1997.
[11] S.V. Polenkova, J.W. Polderman, and R. Langerak. Stability criteria for planar linear systems with state reset. In *Proceedings of the 19th International Symposium on Mathematical Theory of Networks and Systems (MTNS 2010)*, Budapest, 5-9 July, 2010.
[12] Hui Ye, Anthony N. Michel, and Ling Hou. Stability analysis of systems with impulse effects. *IEEE Trans. Automat. Control*, 43(12):1719–1723, 1998.