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DESIGN OF DISTURBANCE OBSERVERS FOR THE COMPENSATION OF LOW-FREQUENCY DISTURBANCES

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ABSTRACT

When the specifications of the disturbance rejection, imposed on a controlled system, are not met by a feedback controller, the control configuration can be extended with a disturbance observer. For this component a systematic design procedure is given. The design procedures provides in a few rules-of-thumb, which are based on the required additional disturbance rejection, and on the resulting overall robustness and noise rejection properties.

1. INTRODUCTION

An important reason for the use of feedback is the attenuation of disturbances. Disturbances in practical control systems can be divided in two categories. The first category contains stochastic disturbances, which are normally characterized by statistical properties, such as mean value, covariance and power spectral density. The second category contains waveform-structured signals, which show distinguishable patterns, at least over short time intervals. In this paper we focus on waveform-structured signals W, as these are appropriate to describe important disturbances in electromechanical systems and can explicitly be incorporated in the design of the control system.

The internal model principle (IMP) states that "a regulator is structurally stable only if the controller utilizes feedback of the regulated variable, and incorporates in the feedback path a suitably reduplicated model of the dynamic structure of the exogenous signals, which the regulator is required to process" [3]. I.e., a feedback component has to include an internal model of the dynamics of the disturbance it tries to attenuate. A controller structure that includes such a model is the disturbance observer [4]. In this paper we will derive a new systematic design procedure for a disturbance observer, as a part of the control configuration with a feedback component. The design procedure is based on the separation property of the sensitivity functions.

Theory on waveform-mode descriptions of realistic disturbances can be found in [4], where it is described how a disturbance w, consisting of linear waveform descriptions, can be interpreted as the output of an autonomous system (Fig.1), subject to a set of initial conditions, according to:

$$\begin{array}{rcl} \dot{x}_w &=& A_w \times x_w, \;\; x_x(t_0) = x_{w_0} \\ w &=& C_w \times x_w \end{array} \qquad (1)$$

where $x_w \in \mathbb{R}^{x_w}$ is the vector of disturbance states, $w \in \mathbb{R}^w$ is the vector of disturbances. The pair $\{A_w, C_w\}$ constitutes constant matrices with appropriate dimensions, i.e. $A_w \in \mathbb{R}^{x_w \times x_w}$, $C_w \in \mathbb{R}^{w \times x_w}$.

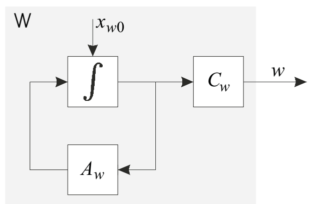

Fig. 1 Disturbance model

To arrive at a design procedure for a disturbance observer that incorporates such a disturbance model, we proceed as follows. First, the general concept of linear state observers is extended to incorporate the model of the disturbances (section 2). Then the resulting disturbance observer is analyzed in terms of the realized transfer matrices (section 3 and 4). The robustness of the resulting closed-loop system is analyzed in section 5, after which the design issues are presented in section 6. The final result (i.e. the design procedure) is presented in section 7. This design procedure incorporates the aspects of robustness, noise- and disturbance- suppression issues in a few rules-of-thumb.

2. DISTURBANCE OBSERVER

The general concepts of observer theory have been described extensively in textbooks, such as [1, 2, 5]. These concepts are also applicable to the disturbance observer that is used in this paper. First, assume that the plant P, without disturbances, is linear time-invariant, such that it can be described as:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ z &= Cx + Du \end{aligned} \tag{2}$$

where $x \in \mathbb{R}^x$ is the vector of (physical) states, $u \in \mathbb{R}^u$ is the vector of plant inputs and $z \in \mathbb{R}^z$ is the vector of plant outputs. The quadruplet $\{A, B, C, D\}$ constitutes constant matrices with appropriate dimensions. This model can be used to construct estimates $\hat{x} \in \mathbb{R}^x$ of the real states. The estimated state vector will equal the vector of (physical) states x, when both the plant model (2) and the estimation are driven by the same input. That is, when the plant model is completely correct and the initial conditions are identical. Generally, this will not be the case and an error signal is used to adjust the estimated states. The error signal is the difference between the measured output z and the estimated output and is called the *innovation signal*. This signal is multiplied by the observer gain matrix $L_p \in \mathbb{R}^{x \times z}$. This type of observer is called an asymptotic state observer. When the throughput matrix D is assumed to be zero, we obtain:

$$\dot{\hat{x}} = A\hat{x} + Bu + L_p(z - C\hat{x}) \tag{3}$$

If we now assume that the plant is disturbed by the autonomous system (1), the plant model (2) can be augmented with the disturbance generator, to result in the dynamics:

$$\begin{bmatrix} \dot{x} \\ \dot{x}_w \end{bmatrix} = \begin{bmatrix} A & BC_w \\ 0 & A_w \end{bmatrix} \begin{bmatrix} x \\ x_w \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u \tag{4}$$

Here, we assumed that the disturbance enters at the plant input. Constructing an asymptotic state observer for this *augmented plant* (4) results in the *augmented observer* given by:

$$\begin{aligned} \dot{\hat{x}}_o &= A_o\hat{x}_o + B_o u + L(z - C_o\hat{x}_o) \\ \hat{w} &= \begin{bmatrix} 0 & C_w \end{bmatrix} \hat{x}_o \end{aligned} \tag{5}$$

with:

$$\begin{aligned} A_o &= \begin{bmatrix} A & BC_w \\ 0 & A_w \end{bmatrix}, \; B_o = \begin{bmatrix} B \\ 0 \end{bmatrix}, \; C_o = \begin{bmatrix} C & 0 \end{bmatrix} \\ L &= \begin{bmatrix} L_p \\ L_d \end{bmatrix}, \; \hat{x}_o = \begin{bmatrix} \hat{x} \\ \hat{x}_w \end{bmatrix} \end{aligned} \tag{6}$$

Now, the innovation signal is multiplied by the observer gain matrices L_p for the plant states and $L_d \in \mathbb{R}^{x_w \times z}$ for the states of the disturbance model (see Fig.2).

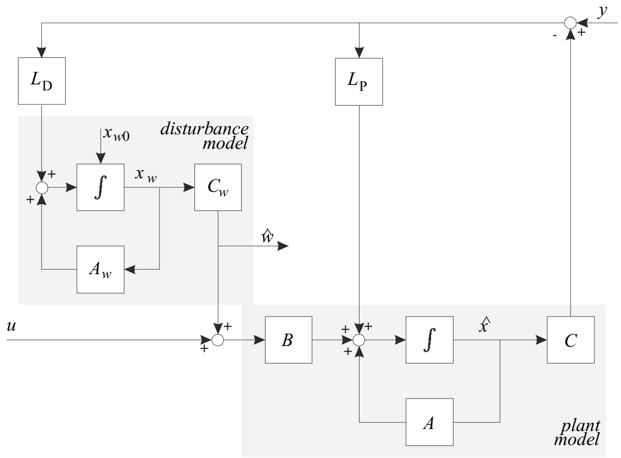

Fig. 2 Observer with disturbance model

The estimated input disturbance can be used to attenuate the real input disturbance. When only the disturbance estimate, and not the complete estimated state vector, is fed back, the observer of Fig.2 is called a disturbance observer. This is shown in Fig.3.

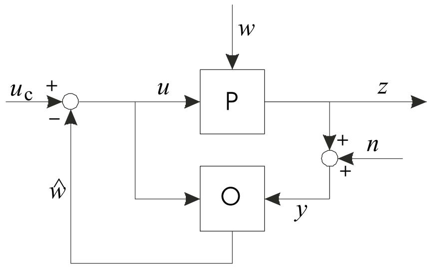

Fig. 3 Plant with disturbance observer

Remark 1 The principle of separation, which applies for an asymptotic state observer, also holds for the disturbance observer. Therefore the poles of the closed-loop system can be placed independently from the poles of the augmented observer [6].

Remark 2 Profeta et al. [6] also prove that, provided that both $\{A, C\}$ and $\{A_w, C_w\}$ are observable, a vector $\begin{bmatrix} L_p^T & L_d^T \end{bmatrix}^T$ can be found such that the observer poles can be placed substantially anywhere desired, subject to the physical limitations of the system and:

1. no pole-zero cancellation takes place [5], i.e. no eigenvalue of A_w is a zero of the plant model the disturbance acts upon.

2. the zero frequency gain of the plant must not be zero

When these conditions are fulfilled, the design of the disturbance observer reduces to a pole-placement problem. The poles have to be placed such that the desired attenuation of the disturbance is obtained.

3. ANALYSIS

Now, we will investigate the influence of the disturbance observer on the plant behaviour. Therefore, we first consider the block diagram of Fig.3. Within this diagram we will consider four transfer matrices:

1. the transfer matrix from input u_c to output z
2. the disturbance compensation transfer matrix from disturbance w to output z
3. the disturbance estimation transfer matrix from disturbance w to the estimated disturbance $\hat{w}$
4. the transfer matrix from the measurement noise n to the output z.

We will start with the first two transfer matrices. From Fig.3 we can see that, when we neglect the measurement noise n, the Laplace transform Z of the output z can be expressed as:

$$Z = P(s) \cdot (W + U_c - \hat{W}) \tag{7}$$

where $P(s)$ is the actual plant transfer matrix, and where the disturbance is assumed to enter at the plant input. When we replace $\hat{W}$ by an expression in terms of U_c, W and Z, using (5), we obtain:

$$\begin{array}{rcl} s\hat{X}_o & = & A_o\hat{X}_o + B_o(U_c - \hat{W}) + L(Z - C_o\hat{X}_o) \\ \hat{W} & = & \left[\begin{array}{cc} 0 & C_w \end{array} \right] \hat{X}_o = G\hat{X}_o \end{array} \tag{8}$$

This expression can be rewritten as:

$$\begin{array}{c} (sI_{x_o} - A_o + LC_o + B_oG)\hat{X}_o = B_oU_c + LZ \\ \Longleftrightarrow \\ \hat{X}_o = (sI_{x_o} - A_o + LC_o + B_oG)^{-1}(B_oU_c + LZ) \end{array} \tag{9}$$

where $I_{x_o} \in \mathbb{R}^{x_o \times x_o}$ denotes the unit matrix. Combining (7) and (9) results in:

$$\begin{array}{rcl} Z & = & P\,(W + U_c) \\ & & -PG(sI_{x_o} - A_o + LC_o + B_oG)^{-1}(B_oU_c + LZ) \end{array} \tag{10}$$

This expression can be written as two separate transfer matrices, i.e. from input U_c to output Z and from disturbance W to output Z. The transfer matrix that has to be inverted can be rewritten using (6):

$$sI_{x_o} - A_o + LC_o + B_oG = \left[\begin{array}{cc} sI_x - A + L_pC & 0 \\ L_dC & sI_{x_w} - A_w \end{array} \right] \tag{11}$$

The inverse of this block matrix can be determined using the matrix identity:

$$\left[\begin{array}{cc} Q & 0 \\ W & V \end{array} \right]^{-1} = \left[\begin{array}{cc} Q_{-1} & 0 \\ -V^{-1}WQ^{-1} & V_{-1} \end{array} \right] \tag{12}$$

provided Q^{-1} and V^{-1} exist [5]. Application to the matrix (11) results in:

$$\left[\begin{array}{cc} (sI_x - A + L_pC)^{-1} & 0 \\ -(sI_{x_w} - A_w)^{-1}L_dC(sI_x - A + L_pC)^{-1} & (sI_{x_w} - A_w)^{-1} \end{array} \right] \tag{13}$$

Additionally, we use the matrix identity [5]:

$$(I_y - C(sI_x - A)_{-1}L_p)^{-1} = I_y - C(sI_x - A + L_pC)^{-1}L_p \tag{14}$$

Substitution of (14) and (13) into (10) leads to the first two desired transfer matrices. From input U_c to output Z:

$$Z = (I_z + PE)^{-1}P(I_w + EC(sI_x - A)B)U_c \tag{15}$$

and from the disturbance W to the output Z:

$$Z = (I_z + PE)^{-1}PW \tag{16}$$

where

$$E = (C_w(sI_w - A_w)^{-1}L_d)(I_z + C(sI_x - A)L_p)^{-1} \tag{17}$$

which is the transfer matrix from Z to $\hat{W}$, i.e. the observer transfer matrix.

When we compose expressions (15) and (17), we obtain the third transfer matrix, from W to $\hat{W}$:

$$\hat{W} = E(I_z + PE)^{-1}P \cdot W \tag{18}$$

and the fourth transfer matrix from N to Z:

$$Z = -(I_z + PE)^{-1}PE \cdot N \tag{19}$$

When we consider the transfer matrices above, we recognize two standard transfer matrices:

$$S_o = (I_z + PE)^{-1} \tag{20}$$

$$T_o = (I_z + PE)^{-1}PE \tag{21}$$

where S_o is called the observer sensitivity function and T_o the complementary observer sensitivity function. The observer sensitivity function is the transfer matrix from the output disturbance w_{out} to the output z.

Statement 1 Separation Principle
Expression (15) shows that a disturbance observer has no influence on the transfer matrix from input U_c to output Z when the plant is modeled correctly, i.e. when:

$$P(s) = C(sI_x - A)^{-1}B \tag{22}$$

4. MODEL UNCERTAINTY

To analyze the influence of modeling errors, write the plant as $P = P_0 + \Delta$ where:

$$P_0 = C(sI_x - A)^{-1}B \tag{23}$$

stands for the nominal plant transfer matrix that is incorporated in the observer and Δ is the modeling error, i.e. the unstructured uncertainty. When we assume $w = 0$, we can write (15) as

$$Z = (I_z + PE)^{-1}PU_c + (I_z + PE)^{-1}PEP_0U_c \tag{24}$$

Rewriting, and using the fact that $S_o + T_o = I$ gives:

$$Z = (S_oP + T_oP_0)U_c = (P_0 + S_o\Delta)U_c \tag{25}$$

We know that the sensitivity function S_o is small at low frequencies and close to one at high frequencies. This means that, at low frequencies, the feedback component sees a plant with a dynamic behavior equal to the dynamic behavior of the nominal plant transfer matrix P_0. At high frequencies, the observer cannot influence the plant anymore and the original plant transfer matrix P is again obtained. As always, the indefinite region is the crossover region, where the behavior of the plant, as seen by the feedback component, will be a transition from the observer model P_0 to the real plant P.

Statement 2 The plant with disturbance observer, seen by the feedback component, behaves as the model of the plant in the disturbance observer P_0, in the frequency range where the disturbance observer is active, i.e. where $|S_o| = 1$

5. SENSITIVITY ANALYSIS

An important issue in controller design in general, is the robustness of the control scheme with respect to modeling errors. One way to analyze robustness, is by examining the sensitivity and complementary sensitivity functions. The sensitivity functions for the disturbance observer alone (i.e. without feedback controller) has been derived in section 3. But when a feedback controller is connected, the overall sensitivity function will be different [7]. To analyze this, we will first switch to a description where the disturbances enter at the output of the plant, in stead of at the input.

As the system is presumed to be linear, the input disturbance w can easily be rewritten to an output disturbance w_{out}:

$$W_{out} = P(s) \cdot W \tag{26}$$

Hence, we obtain a familiar expression:

$$Z = (I_z + PE)^{-1}PW = S_oW_{out} \tag{27}$$

When we combine (15) and (27) and we disregard measurement noise, we obtain an expression for the output Z, according to Fig.3:

$$\begin{aligned} Z &= (I_z + PE)_{-1}P(I_w + EC(SI_x - A)B)U_c \\ &\quad +(I_z + PE)^{-1}W_{out} \end{aligned} \tag{28}$$

When we close the loop with the feedback component $C_c(s)$, and assume $r = 0$, we obtain:

$$\begin{aligned} Z &= (I_z + PE)^{-1}P(I_w + EC(sI_x - A)B)(-C_cZ) \\ &\quad +(I_z + PE)^{-1}W_{out} \end{aligned} \tag{29}$$

When the plant is modeled correctly this expression can be rewritten as:

$$Z = -PC_cZ + (I_z + PE)^{-1}W_{out} \tag{30}$$

Thus the sensitivity function of the overall control configuration can be written as:

$$Z = (I_z + PC)^{-1}(I_z + PE)^{-1}W_{out} = S_c \cdot S_o \cdot W_{out} \tag{31}$$

Thus:

$$S = S_c \cdot S_o \tag{32}$$

where S_c is the sensitivity function of the control configuration with only a feedback component, i.e. a 1-DOF controller, and S is the sensitivity function of the control configuration with both a feedback component and a disturbance compensator.

Statement 3 Separation of sensitivity function
The sensitivity function S of the overall control configuration can be obtained from a multiplication of the sensitivity function S_c of the closed-loop with only a feedback component, and the observer sensitivity function S_o, when the plant is modeled correctly.

Similarly, we can investigate the influence of the measurement noise N. When we assume the input disturbance W to be zero, we can write for the output Z:

$$\begin{aligned} Z &= (I_z + PE)^{-1}P(I_w + EC(SI_x - A)B)U \\ &\quad -(I_z + PE)^{-1}PEN \end{aligned} \tag{33}$$

When we again close the loop with the feedback component $C_c(s)$ and assume $r = 0$, we obtain:

$$\begin{aligned} Z &= (I_z + PE)^{-1}P(I_w + EC(SI_x - A)B)(-C_cZ - C_cN) \\ &\quad -(I_z + PE)^{-1}PEN \end{aligned} \tag{34}$$

When the plant is modeled correctly, this expression can be rewritten as:

$$Z = -PC_cZ - PC_cN - (I_z + PE)^{-1}PEN \tag{35}$$

The measurement noise attenuation of the overall control configuration can be written as:

$$\begin{aligned} Z &= (I_z + PC_c)^{-1}(-PC_c(I_z + PE)^{-1}PE)N \\ &= -(T_c + S_cT_o)N \end{aligned} \tag{36}$$

Statement 4 The disturbance observer does not affect the measurement noise attenuation of the closed-loop system according $-T_c$, under the condition that the product of the sensitivity function S_c and the complementary observer sensitivity function T_o is small compared to T_c.

6. DESIGN ISSUES

In the following it is assumed that the disturbance observer will be designed after the design of a feedback component. Furthermore, it is assumed that the specifications for disturbance attenuation are expressed in terms of the input sensitivity function $S_{wz}(s)$. This means that we want that $|S_{wz}(j\omega)| < s_l$ for $\omega < \omega_l$, for some given S_l, ω_l. This kind of specification requires minimal information about the actual characteristics of the disturbances. The observer should simply provide sufficient attenuation for every possible disturbance, not a specific waveform, up to a certain frequency ω_l.

From (31), we can derive an expression for the input sensitivity function $S_{wz}(s)$ of the control configuration with disturbance observer:

$$S_{wz} = S \cdot P = S_o \cdot S_c \cdot P \tag{37}$$

The feedback component, in the 1-DOF configuration, will give a certain suppression of low-frequency disturbances, expressed in the magnitude of the input sensitivity function $S_c(s)P(s)$. When this is insufficient, a disturbance observer introduces an additional attenuation factor, according to (37):

$$|S(j\omega)| = |S_o(j\omega)| + |S_c(j\omega)| + |P(j\omega)|[dB] \tag{38}$$

For the observer sensitivity function this means that, given the specifications (see Fig.4):

$$|S_o(j\omega)| < (s_l - |S_c(j\omega)| - |P(j\omega)|) = \sigma[dB] \tag{39}$$

for $\omega < \omega_l$

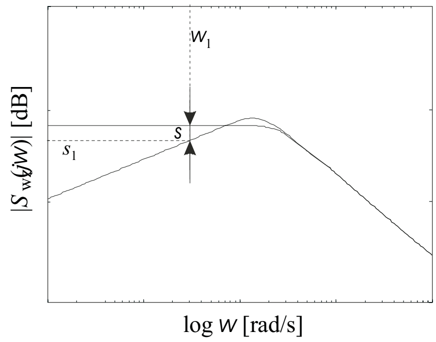

Fig. 4 Extra disturbance suppression due to disturbance observer

Remark 3 The designer should be well aware that, due to the Bode sensitivity integral, an extra suppression at low frequencies will result in extra gain at other frequencies

In order to give the observer sensitivity function, and thus the overall input sensitivity function S, the specified characteristics, we will place all poles of the observer sensitivity function at frequency ω_σ. The coefficients of the characteristic polynomial of S_o:

$$\chi(s) = s^n + a_{n-1}\omega_\sigma s^{n-1} + \cdots + a_1\omega_\sigma^{n-1}s + \omega_\sigma^n \tag{40}$$

will be chosen such that we obtain a Butterworth polynomial or that its roots are all located at $-\omega_\sigma$. Consequently, the observer sensitivity function will peak at about this frequency. The low-frequency slope of the observer sensitivity function is determined by the order of the disturbance model; a disturbance model of order k results in a slope of 20k [dB/decade]. When a Butterworth polynomial is used, the peak of the sensitivity function at ω_σ occurs at about twice the frequency of the crossing of the 0 dB-line. When the poles are placed on the real axis this ratio c is about 3. These observations allow for an approximate frequency ω_σ for the poles of the observer:

$$\omega_\sigma \approx c \cdot \omega_l \cdot 10^{\frac{\sigma}{20k}} \tag{41}$$

This expression shows a trade-off between the order k of the disturbance model and the frequency ω_σ. Experience learned that k should generally not be chosen larger than 3. The frequency ω_σ can also not be increased infinitely. When the peak of the observer sensitivity function S_o is located close to the peak of S_c, the disturbance observer will interfere with the feedback component. This occurs when ω_σ approximates $2 \times \omega_b$. When ω_σ is much smaller, the sensitivity function S_c will damp the peak of the observer sensitivity function. As a rule of thumb we state that ω_σ should be smaller than $0.5 \times \omega_b$.

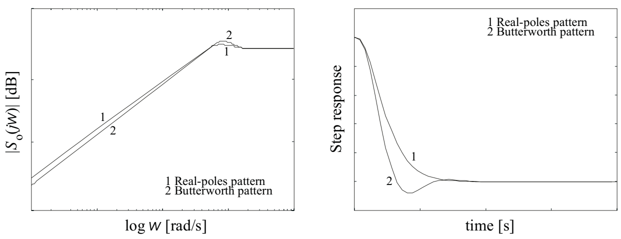

Fig. 5 Real-poles versus Butterworth pattern

In Fig.5 the difference between a real-poles pattern and a Butterworth pattern is illustrated. When the poles are placed on the real axis, we see a relative small peak in the observer sensitivity function and a relative well-damped step response. A Butterworth pattern gives better attenuation in the low-frequency area.

7. DESIGN PROCEDURE

Now, we will summarize the properties and requirements as derived before, in a design procedure for the disturbance observer. Suppose that:

- the feedback component $C_c(s)$ with a bandwidth ω_b is given (it is wise to use a feedback component without integral action, as the disturbance observer will incorporate one or more integral actions).
- the disturbance attenuation of the feedback component is not sufficient.
- the overall specification for input disturbance attenuation is given as: $|S_{wz}(s)| < s_l$ for $\omega < \omega_l$.

then the following design procedure can be used:

Design Procedure

1. Identify the order n of the plant and determine the appropriate state-space model of the plant.
2. Set the order of the disturbance observer k to 1.
3. Determine the suppression provided by the feedback component, $|S_c(j\omega_l)| + |P(j\omega)|$.
4. Determine the suppression required of the disturbance observer:

$$\sigma = s_l - (|S_c(j\omega_l) + |P(j\omega)|) \qquad (42)$$

5. Select a real-pole polynomial or a Butterworth polynomial of order n+k.
6. Determine the location of the observer poles in terms of the frequency ω_σ using:

$$\omega_\sigma \gg c \cdot \omega_l \cdot 10^{\frac{\sigma}{20k}} \qquad (43)$$

   where c is 3 for a real-pole polynomial and 2 for a Butterworth polynomial.
7. Verify whether $\omega_\sigma < 0.5 \times \omega_b$. If not, increase k and go to step 5. Note: k is usually not larger than 3. If this is still not sufficient, the bandwidth ω_b should be reconsidered.
8. Determine the observer gain L

8. CONCLUSIONS

In this paper is systematic design procedure is proposed to construct a disturbance observer using minimal knowledge about the frequency contents of the disturbances. The procedure allows the design of the disturbance observer with a few rules-of-thumb, given the amount of additional disturbance rejection needed. The rules-of-thumb are based on robustness, disturbance- and noise- suppression considerations.

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